

Confidence Management in Tournaments*

Shanglyu Deng[†] Hanming Fang[‡] Qiang Fu[§] Zenan Wu[¶]

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Abstract

An incumbent employee competes against a new hire for bonuses or promotions. The incumbent's perception of the new hire's ability distribution is biased. This bias can result in overconfidence or underconfidence. We show that debiasing may be counterproductive in incentivizing efforts. We then explore whether a firm that values employees' efforts should disclose an informative signal about the new hire's type and we characterize the conditions under which transparency or opacity is optimal for the firm. We further consider four extensions to the model. Our results contribute to the extensive discussion of confidence management and organizational transparency in firms.

Keywords: Confidence Management; Information Asymmetry; Tournaments; Effort Incentives; Information Disclosure.

JEL Classification Codes: D21, D23, D82, D91.

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[†]Department of Economics, University of Maryland, 3114 Tydings Hall, 7343 Preinkert Dr., College Park, MD 20742. Email: sdeng1@umd.edu.

[‡]Department of Economics, University of Pennsylvania, 133 S. 36th Street, Philadelphia, PA 19104; School of Entrepreneurship and Management, ShanghaiTech University, China; and the NBER. Email: hanming.fang@econ.upenn.edu.

[§]Department of Strategy and Policy, National University of Singapore, 15 Kent Ridge Drive, Singapore, 119245. Email: bizfq@nus.edu.sg.

[¶]School of Economics, Peking University, Beijing, China, 100871. Email: zenan@pku.edu.cn.

“Attempt easy tasks as if they were difficult, and difficult as if they were easy; in the one case that confidence may not fall asleep, in the other that it may not be dismayed.”

—Baltasar Gracián

“Perhaps a successful life, like a successful company, needs both optimism and at least occasional pessimism, and for the same reason a corporation does.”

—Martin Seligman

1 Introduction

The internal labor markets inside firms are widely viewed to resemble tournaments (Lazear and Rosen, 1981; Rosen, 1986). Workers strive for bonuses or to climb the hierarchical ladder (Brown and Minor, 2014). They are rewarded or punished based on their performance relative to competitors or benchmarks instead of absolute output metrics (Chen and Lim, 2013; Chen, 2016). A plethora of anecdotal and empirical observations have documented the prevalence of tournament incentives and relative performance evaluation (RPE) schemes (see, e.g., Eriksson, 1999; Henderson and Fredrickson, 2001; Belzil and Bognanno, 2008; Connelly, Tihanyi, Crook, and Gangloff, 2014; and Lazear, 2018). Consider, for instance, the popular practice of *vitality curve*—or stack ranking—that was pioneered by Jack Welch and has proliferated in the modern corporate landscape (see, e.g., McGregor, 2006).¹ As argued by DeVaro (2006), promotion tournaments are an integral component of firms’ human resource practices to advance their strategic interests.

The conventional wisdom holds that the incentive of the agents involved in tournament situations crucially depends on their relative competitiveness and their perception of each other’s competency (Brown, 2011). However, one’s knowledge about his opponent is often

¹In performance management, a vitality curve ranks (or rates) individuals against their coworkers. It is also called “stack ranking,” “forced ranking,” and “rank and yank.” The concept of a vitality curve has been used to justify the “rank-and-yank” system of management at GE, whereby 10% of workers are fired after each evaluation.

limited, and his perception can be systematically biased. Consider the usual scenario in which a new hire joins an organization and competes—under an RPE scheme—against incumbent employees for bonuses or promotion. The competency of the incumbents can be inferred from their established track record, while that of the new hire often remains to be ascertained, which gives rise to the typical problem of information asymmetry (see, e.g., Hurley and Shogren, 1998a; Wärneryd, 2003; Zhang and Zhou, 2016; and Denter, Morgan, and Sisak, 2020). Furthermore, incumbent employees may misestimate the new hire. A large body of economics and psychology literature has identified the prevalence of perceptual biases by which people “misplace” themselves in comparison with others or with the population mean, being either overconfident or underconfident (see, e.g., Larwood and Whittaker, 1977; Cooper, Woo, and Dunkelberg, 1988; Malmendier and Tate, 2005, 2015; Moore and Cain, 2007; Moore and Healy, 2008; and Muthukrishna, Henrich, Toyokawa, Hamamura, Kameda, and Heine, 2018)². Such phenomena are pervasive in workplaces. Consider the following examples.

- (i) A startup recruits a high-profile executive poached from an industry leader; incumbent employees may presumably overestimate the external hire.
- (ii) Optimism typically arises in a rapidly growing firm; incumbent employees would arguably underestimate newbies, as they attribute the firm’s success to their own superior competence.
- (iii) A corporate culture that champions workplace Darwinism—e.g., that at Enron—typically boosts employees’ egos and breeds overconfidence, which also leads them to look down on newcomers.³

In this paper, we aim to explore two main questions. Suppose that a firm cares about the aggregate effort supply in the workplace. First, does the firm benefit or suffer from its

²Santos-Pinto and Sobel (2005) outline a mechanism under which self-overestimation arises in subjective assessment of relative abilities.

³See Netessine and Yakubovich (2012).

employee’s perceptual bias? Second, suppose that the firm is able to conduct an evaluation to acquire an informative signal about the new hire’s true ability; is the firm willing to disclose it to employees, which manipulates their beliefs and, in turn, influences the performance of the competition?

To answer these questions, we adopt a standard lottery contest setting—as in Denter, Morgan, and Sisak (2020) and Zhang and Zhou (2016)—to model a promotion tournament in a firm. Two employees—an incumbent and a new hire—are involved in the competition. They differ in their abilities (or strengths). The incumbent’s ability is common knowledge, while that of the new hire is privately known. The new hire’s ability can take either a high or a low value. We allow the incumbent employee to possess a different prior about the new hire than the true underlying distribution. The uncommon priors thus depict the incumbent employee’s misperception of his relative competitiveness in the tournament. A manager—e.g., HR director—acts in the firm’s interest and can secure an informative signal about the new hire’s true ability through an evaluation exercise. She decides on the firm’s information disclosure policy and commits to either disclosing the signal to both employees or concealing it, with the latter being equivalent to foregoing the evaluation exercise.

The questions posed in this paper are not only theoretically interesting, but also practically relevant. First, successful confidence management is broadly viewed in practice as a key to boosting productivity. The economics literature has espoused the motivation effect of (over)confidence, as a positive self-image could incentivize efforts and catalyze success (see, e.g., Bénabou and Tirole, 2002; Compte and Postlewaite, 2004; Gervais and Goldstein (2007); Chen and Schildberg-Hörisch, 2019). However, overconfidence has typically been examined in settings of stand-alone decision making or a principal-agent relationship. We nevertheless demonstrate the more subtle impact of overconfidence on effort supply in a tournament setting. We show that both overconfidence and underconfidence can benefit or harm effort provision. Imagine that the incumbent is the *ex ante* favorite to win the tournament. Overconfidence would stifle the competition, as the complacency entices him to further slack off; in contrast, underconfidence on the part of the incumbent can prevent shirking. Con-

versely, when the incumbent is the ex ante underdog, his overconfidence would help avoid discouragement, and thus debiasing would weaken the competition. The ramifications result from (i) the relative-performance based reward structure in tournaments, and (ii) players' nonmonotone best response correspondence in the strategic interactions that occur in such competitive events (Lazear and Rosen, 1981; and Dixit, 1987). To the best of our knowledge, such effects have yet to be formally delineated in the literature.

Second, firms' internal information management—i.e., the information accessible to their employees—has spawned extensive discussion in both academic studies and practice. A large portion of leading firms in Europe and the United States have established internal knowledge system or built competency models that identify best practices and publicize feedback on employees' performance relative to their peers (Nafziger and Schumacher, 2013; O'Connell, 2008; Vanek Smith, 2015; Song, Tucker, Murrell, and Vinson, 2018). Eli Lilly & Co., for instance, allows its employees to access their rankings in the succession planning system. In the National University of Singapore (NUS) Business School, faculty members are allowed to access colleagues' student feedback reports.⁴ The informative signal, if disclosed, allows the uninformed incumbent to make inferences about his opponent: It not only ameliorates information asymmetry, but also changes his perception of their relative competitiveness. This update, by the same logic laid out above, would indeterminately affect his incentive in the competition and trigger an ambiguous strategic response from the new hire.

Snapshots of Results The results of our analysis can be summarized as follows. We first fully characterize the necessary and sufficient conditions under which the incumbent's misperception benefits/harms the firm in terms of aggregate effort. We then explore the optimal information disclosure policy. When the quality—i.e., the precision—of the signal obtained through the evaluation is fixed, two effects loom large when the incumbent observes the signal with misperception in place. The informative signal serves two roles. First, it catalyzes an *information effect* due to information asymmetry: The update alleviates

⁴NUS conducts annual performance reviews for faculty members. Each department sets aside a bonus pool to reward teaching excellence, and only top-ranked performers receive the monetary reward.

information asymmetry ex post, but ex ante causes more dispersed tournament outcomes across different states. Second, it gives rise to a *morale effect* because of the perceptual bias. The additional information leads the biased incumbent to revise his perception of the relative competitiveness. The *direction* and *magnitude* of his response to the signal depends on the nature of his initial perceptual bias and the realization of the signal. The morale effect concurs with the information effect when the incumbent is overconfident, but it works against the information effect when the incumbent is underconfident. We identify the conditions under which either disclosing the signal or concealing it is optimal, and interpret the underlying logic. Our theoretical results yield novel and useful managerial implications for firms' confidence and internal information management, which we elaborate on in Sections 2.4 and 3.2.

We further explore four variations of the model. First, we examine the case of private disclosure, i.e., allowing the firm to disclose the signal to the incumbent employee only. Second, we endogenize the information structure, allowing the firm to design its evaluation system flexibly. Third, we allow the firm to maximize the expected winner's effort instead of the aggregate effort. Finally, we allow the new hire's ability to take three or more values. We demonstrate that the main findings in the baseline setting are robust to these extensions.

Related Literature Our paper contributes to the literature on information transmission in contests/tournaments. One stream of this literature assumes that a designer possesses superior information about the contenders and explores her optimal disclosure policy, e.g., Fu, Jiao, and Lu (2014), Zhang and Zhou (2016), Serena (2021), Lu, Ma, and Wang (2018), Chen (2021), and Boosey, Brookins, and Ryvkin (2020). The other stream of work studies contenders' strategic action to reveal private information. Denter, Morgan, and Sisak (2020) and Fu, Gürtler, and Münster (2013) let the informed party take a costly action to signal his private type prior to the competition. Kovenock, Morath, and Münster (2015) and Wu and Zheng (2017) study contenders' voluntary information disclosure. These studies

mainly assume common priors and rational beliefs.⁵ Our paper belongs to the former class of studies, as it allows the firm to conduct an evaluation and decide whether or not to disclose an informative signal. However, this strand of literature does not allow for perceptually biased players; as a result, the morale effect due to the perceptual bias in our setting—which plays a subtle and important role in determining the optimum—is absent. Our study thus complements these studies.

Our paper is naturally linked to the literature on the incentive effect of over(under)-confidence, such as Bénabou and Tirole (2002), Compte and Postlewaite (2004), Fang and Moscarini (2005), and Chen and Schildberg-Hörisch (2019).⁶ However, these studies focus on the decision making of a single agent or in a principal-agent setting. Santos-Pinto (2008) examine both single-agent and multi-agent scenarios. Fang (2001) explores the role of perceptual bias in a team-production setting. Gervais and Goldstein (2007) show that overconfidence reduces free-riding and benefits teamwork, as an overconfident agent works harder. Kyle and Wang (1997) demonstrate in a Cournot duopoly setting the commitment value of overconfidence. However, Kyle and Wang (1997) interpret overconfidence as overoptimism, i.e., excessively optimistic perception of the precision of his own signal; in contrast, we focus on over(under)-placement (Moore and Healy, 2008), by which a player over(under)-estimates his relative competitiveness. Grubb (2009) analyzes a model of optimal contracting between firms and overconfident consumers in the cellular phone services market. Fang and Wu (2020) study the welfare effects of secondary markets when consumers are overconfident in the context of life settlement market.

We join the small but growing literature that explores the role played by the perceptual bias in a tournament in which the reward is based on relative performance. Santos-Pinto (2010) considers a tournament inside a firm in which both workers overestimate their own

⁵In an extension (Proposition 5 and Appendix K), Denter, Morgan, and Sisak (2020) consider a case of overconfident players. Their model sharply differs from ours: They allow the player of private type to misperceive himself and assume that both players possess the same biased belief, while in ours, one knows precisely his private type, while the other systematically misestimate his opponent.

⁶Santos-Pinto and de la Rosa (2020) provide a thorough survey of the literature on the incentive effect of self-perceptual bias. Also see Kőszegi (2014), Grubb (2015a,b), and Heidhues and Kőszegi (2018) for surveys on behavioral industrial organization and overconfident consumers in the marketplace.

productivity; he finds that under plausible conditions, workers' positive self-image accrues to the benefit of the firm. Santos-Pinto (2021) examines a tournament between an overconfident worker and a rational one and investigates whether the former is more likely to prevail. Both papers assume complete-information settings, while we allow for one-sided asymmetric information and examine the optimal information disclosure.⁷

Crutzen, Swank, and Visser (2013) demonstrate that manager may refrain from differentiation among employees, as differentiation may lead them to downgrade their self-ratings and dampen incentives. Nafziger and Schumacher (2013) show that revealing peer performance can be counterproductive as an employee can infer the impact of his effort on the probability of success. However, these settings do not involve competition or perceptual biases.

The rest of our paper is organized as follows. In Section 2, we set up an asymmetric-information tournament model with uncommon priors, characterize the equilibrium, and elaborate on the impact of perceptual bias. In Section 3, we explore the optimal information disclosure policy in the tournament and interpret the results. In Section 4, we explore four variations to the baseline setting and demonstrate the robustness of our results. In Section 5, we conclude.

2 Asymmetric-Information Tournament with Uncommon Priors

We model the competition between two employees inside a firm as a tournament. In this part, we spell out the fundamentals of the tournament model and solve for the equilibrium, which lays a foundation for the analysis of optimal information policy.

⁷Park and Santos-Pinto (2010) document empirical evidence of overestimation bias in field.

2.1 Model

We consider a firm with a manager and two risk-neutral employees, indexed by $i \in \{A, B\}$. The two employees compete for a prize—e.g., a promotion—by exerting irreversible efforts simultaneously. We assume a lottery *contest success function* (CSF) to model the tournament competition in the firm’s internal labor market: For an effort profile $(x_A, x_B) \geq (0, 0)$, employee i wins with a probability⁸

$$p_i(x_A, x_B) = \begin{cases} x_i/(x_A + x_B) & \text{if } x_A + x_B > 0, \\ 1/2 & \text{if } x_A + x_B = 0. \end{cases}$$

We assume that employee i ’s effort x_i entails a unity marginal effort cost, and he chooses his effort to maximize his expected payoff

$$\pi_i(x_i, x_j) = p_i(x_A, x_B)v_i - x_i, \quad i, j \in \{A, B\}, i \neq j,$$

where $v_i > 0$ is the value that employee i receives from winning the prize. For ease of exposition, we interpret each employee’s valuation of the win as his ability: A more motivated employee has greater incentive to exert effort. Remark 1 below elaborates on this interpretation.

Remark 1 *The model is isomorphic to an alternative setting in which employees equally value the prize—e.g., a bonus package—from winning the tournament—i.e., $v_i = v$ for $i \in \{A, B\}$ —but bear different (linear) effort costs (e.g., Moldovanu, Sela, and Shi, 2007; Taylor and Yildirim, 2011; Brown and Minor, 2014). The payoff function is given by*

$$\tilde{\pi}_i(x_i, x_j) = p_i(x_A, x_B)v - c_i x_i, \quad i, j \in \{A, B\}, i \neq j,$$

⁸A closed-form equilibrium solution to the model is not available if we assume a CSF in the form of $(x_i)^\gamma / [(x_A)^\gamma + (x_B)^\gamma]$, with $\gamma \in (0, 1]$. Simulation shows that our results remain qualitatively unchanged if $0 < \gamma < 1$. The analysis is available from the authors upon request.

and maximizing $\tilde{\pi}_i(x_i, x_j)$ is equivalent to maximizing

$$\frac{\tilde{\pi}_i(x_i, x_j)}{c_i} = p_i(x_A, x_B) \frac{v}{c_i} - x_i,$$

which restores the original game considered in our paper.

Importantly, we assume that the incumbent worker's value v_A is commonly known, but the new hire's value v_B is B 's private information. Specifically, v_B is a random variable on the set $\{v_B^L, v_B^H\}$ with $0 < v_B^L < v_B^H$ and $\Pr(v_B = v_B^H) = \mu \in (0, 1)$.⁹ We impose the following assumption throughout the paper:

Assumption 1 $v_B^L \geq v_A/4$.

Assumption 1 is intuitive. It ensures that the competition will not be excessively lopsided even if employee B is of the low type, which rules out the possibility of a corner solution in which a low-ability employee B is discouraged from exerting any effort in equilibrium.

The manager possesses the prior μ , while employee A believes that $\Pr(v_B = v_B^H) = \tilde{\mu} \in (0, 1)$.¹⁰ When $\tilde{\mu} < \mu$, employee A underestimates his opponent, and we say that employee A exhibits overconfidence; when $\tilde{\mu} > \mu$, he overestimates his opponent, and we say that he is underconfident. Misplacement may stem from an employee's misperception of himself, or from his misperception of others. Our setting focuses on the latter, e.g., Moore and Schatz (2017). One may arguably have more precise knowledge about himself than about others, while a bias about oneself is less likely to persist.¹¹

Two remarks are in order before we carry out the analysis. Remark 2 concerns players' priors and beliefs imposed in the contest game.

⁹In Section 4.4, we allow v_B to take three or more values and show that the main results are robust.

¹⁰Note that employee B 's belief about v_B does not matter in our model because (i) he has private information about v_B ; and (ii) he only cares about employee A 's effort.

¹¹However, it should be noted that our analysis can seamlessly incorporate a case in which employee A also systematically over(under)estimates his own ability, v_A . We omit this case because the firm monotonically benefits from employee A 's biased perception of his *own* ability, which refers to the usual motivational effect.

Remark 2 *Employee A’s perception of the newcomer—i.e., his prior $\tilde{\mu}$ about v_B —is common knowledge to both employees and the manager. However, our analysis does not require that the prior μ held by the manager be known to either employee.*

As will be shown in the subsequent analysis, only employee A’s belief affects the strategic interaction in the tournament.

Remark 3 *The setting can be interpreted in a wide array of ways. For instance, one may view $\tilde{\mu}$ as the common perception held about workers’ ability distribution on the labor market. The manager nevertheless possesses superior information about the newcomer, obtained through the recruiting process. Alternatively, the bias may arise from employees’ inability (relative to the manager) to make accurate inferences about others from common observations, as in Zbojnik (2004).^{12;13}*

2.2 Equilibrium in Tournament

We derive the equilibrium in the model by standard technique.¹⁴ Employee A exerts effort

$$x_A = \left(\frac{\frac{1-\tilde{\mu}}{\sqrt{v_B^L}} + \frac{\tilde{\mu}}{\sqrt{v_B^H}}}{\frac{1}{v_A} + \frac{1-\tilde{\mu}}{v_B^L} + \frac{\tilde{\mu}}{v_B^H}} \right)^2,$$

and employee B has a type-dependent effort strategy, which is given as follows:

$$x_B(v_B) = \sqrt{v_B x_A} - x_A, \text{ for } v_B \in \{v_B^H, v_B^L\}.$$

¹²The economics literature has broadly embraced the notion that uncommon beliefs about the underlying states can arise from a Bayesian process even when individuals hold common priors (see, e.g., Van den Steen, 2011; Benoît and Dubra, 2011).

¹³Similarly, Santos-Pinto (2010) and Santos-Pinto (2021) both assume workers in tournaments “agree to disagree,” such that the biased beliefs are commonly known, while the firm correctly estimates their abilities and knows their biases.

¹⁴Hurley and Shogren (1998a) and Zhang and Zhou (2016) fully characterize the equilibrium of a lottery contest game with one-sided incomplete information, and their analysis extends to our setting.

For notational convenience, we define $K(\tilde{\mu}) := \sqrt{x_A}$. The ex ante expected total effort of the tournament, which we denote by $TE(\mu, \tilde{\mu})$, is given by

$$TE(\mu, \tilde{\mu}) = \mathbb{E}_\mu [x_B(v_B) + x_A] = \mathbb{E}_\mu [\sqrt{v_B x_A}] = \left[(1 - \mu) \sqrt{v_B^L} + \mu \sqrt{v_B^H} \right] K(\tilde{\mu}), \quad (1)$$

where we use the notation $\mathbb{E}_\mu[\cdot]$ to denote the expectation under belief μ . It is noteworthy that employees' equilibrium efforts, x_A and $x_B(v_B)$, involve only employee A 's perceived belief $\tilde{\mu}$. However, both μ and $\tilde{\mu}$ enter the expression of the an ex ante expected total effort $TE(\mu, \tilde{\mu})$, as it is aggregated over the true distribution described by μ .

We now explore the property of $K(\tilde{\mu})$. Taking the derivative of $K(\tilde{\mu})$ with respect to $\tilde{\mu}$ yields

$$K'(\tilde{\mu}) = \frac{\left(\sqrt{v_B^H} - \sqrt{v_B^L} \right) \left(v_A - \sqrt{v_B^H v_B^L} \right)}{v_A v_B^L v_B^H \left[\frac{1}{v_A} + \frac{v_B^H(1-\tilde{\mu}) + v_B^L \tilde{\mu}}{v_B^L v_B^H} \right]^2}.$$

The sign of $K'(\tilde{\mu})$ depends on that of $v_A - \sqrt{v_B^H v_B^L}$. Note that $\sqrt{v_B^H v_B^L}$ is the geometric mean of employee B 's ability; the sign of $v_A - \sqrt{v_B^H v_B^L}$ thus indicates the ex ante comparison of the employees' abilities. Further,

$$K''(\tilde{\mu}) = \frac{2 \left(\sqrt{v_B^H} - \sqrt{v_B^L} \right)^2 \left(v_A - \sqrt{v_B^H v_B^L} \right)}{v_A \left(v_B^L v_B^H \right)^2 \left[\frac{1}{v_A} + \frac{v_B^H(1-\tilde{\mu}) + v_B^L \tilde{\mu}}{v_B^L v_B^H} \right]^3}.$$

Again, its sign depends on that of $v_A - \sqrt{v_B^H v_B^L}$. It is straightforward to obtain the following.

Lemma 1 *The function $K(\cdot)$ is strictly increasing with its argument and convex if $v_A > \sqrt{v_B^H v_B^L}$, and is strictly decreasing and concave if $v_A < \sqrt{v_B^H v_B^L}$.*

2.3 Desirability of Persistent Misperception

Employees' efforts accrue to the firm's benefit. The equilibrium result allows us to explore one natural question: Does the firm benefit from employee A 's misperception, i.e., $\mu \neq \tilde{\mu}$? Specifically, does the persistence of the uncommon priors boost the firm's productivity in

terms of its expected total effort $TE(\mu, \tilde{\mu})$? Recall by (1) that the tournament generates an expected total effort

$$TE(\mu, \tilde{\mu}) = \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K(\tilde{\mu}).$$

With common prior, the expected total effort boils down to

$$TE(\mu, \mu) = \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K(\mu),$$

as in Zhang and Zhou (2016). Therefore, the comparison hinges on the monotonicity of $K(\cdot)$. We obtain the following.

Proposition 1 (Value of Persistent Misperception) *Suppose that the firm aims to maximize the expected total effort in the tournament. Then the following statements hold:*

- (i) *When $v_A < \sqrt{v_B^H v_B^L}$, the firm strictly benefits from employee A's misperception—i.e., $TE(\mu, \tilde{\mu}) > TE(\mu, \mu)$ —if and only if employee A exhibits overconfidence—i.e., $\tilde{\mu} < \mu$;*
- (ii) *When $v_A > \sqrt{v_B^H v_B^L}$, the firm strictly benefits from employee A's misperception—i.e., $TE(\mu, \tilde{\mu}) > TE(\mu, \mu)$ —if and only if employee A exhibits underconfidence—i.e., $\tilde{\mu} > \mu$;*
- (iii) *When $v_A = \sqrt{v_B^H v_B^L}$, employee A's belief does not affect the expected total effort, i.e., $TE(\mu, \tilde{\mu}) = TE(\mu, \mu)$.*

Proposition 1 states that the firm may either benefit or suffer from the incumbent employee's perceptual bias; neither overconfidence nor underconfidence necessarily harms the firm. To interpret its logic, let us explore the impact of the incumbent employee's belief on the total effort $TE(\mu, \tilde{\mu})$, as given by (1). Clearly, it suffices to focus on how x_A varies with $\tilde{\mu}$, i.e., the property of $K(\tilde{\mu})$. The conventional wisdom in the contest/tournament literature is that a more level playing field leads to more fierce competition. We conveniently interpret employee A as an ex ante underdog in the case of $v_A < \sqrt{v_B^H v_B^L}$.¹⁵ Proposition 1(i) shows

¹⁵Employee A is more likely to be the weaker (stronger) contender when v_A is smaller (larger) relative to $\sqrt{v_B^H v_B^L}$. Players' relative competency also depends on the perceived belief $\tilde{\mu}$.

that in this case, his overconfidence boosts his morale, which narrows the gap in terms of ability and fuels the competition. Conversely, Proposition 1(ii) states that, if employee A is the favorite in the sense that $v_A > \sqrt{v_B^H v_B^L}$, then the firm suffers from his overconfidence: Employee A underestimates his opponent, which softens the competition and entices employee A to slack off. In the knife-edge case of $v_A = \sqrt{v_B^H v_B^L}$, these balancing forces cancel out in the ex ante even race.

The contest/tournament literature has conventionally espoused the productive role played by various design instruments that manipulate the balance of competition—e.g., favoritisms (Epstein, Mealem, and Nitzan, 2011; Franke, Kanzow, Leininger, and Schwartz, 2013, 2014; Fu and Wu, 2020, among others), headstarts (Kirkegaard, 2012; Konrad, 2002; Siegel, 2009; Drugov and Ryvkin, 2017, among others), and bidding caps (Che and Gale, 1998; Gavious, Moldovanu, and Sela, 2002; Olszewski and Siegel, 2019, among others).¹⁶ Our analysis implies that the same can alternatively be achieved by a perceptual bias, and debiasing may turn out to weaken the competition and mute employees’ incentives.

2.4 Implications of Proposition 1

Our analysis demonstrates the subtle roles played by employees’ perceptual biases. It is broadly championed that confidence catalyzes success and that managers should foster confidence in their staff members. The economics and psychology literature has also identified the motivational effect that advocates the positive incentive effect of overconfidence. We nevertheless show that employees’ incentives and productivity depend indeterminately on their (mis)perception about relative competitiveness when they engage in internal competitions, which are pervasive in modern workplace (Netessine and Yakubovich, 2012).

Proposition 1 demonstrates that employees’ (mis)perceptions can be either productive or counterproductive, depending on the actual relative competitiveness between the incumbent and the new hire. The firm may sometimes benefit from persistent underconfidence.

¹⁶See Mealem and Nitzan (2016), Chowdhury, Esteve-González, and Mukherjee (2019), and Fu and Wu (2019) for comprehensive surveys on discrimination in contests.

Consider, for instance, a startup that rose from successful grassroots innovations. Its early employees could underestimate their own abilities relative to better-educated junior recruits, despite the extensive experience and know-how they possess. Proposition 1 suggests that the firm may not want to “debias” the incumbent even if it is able to: For instance, if the firm is confident in the value of its early employees’ human capital—i.e., $v_A > \sqrt{v_B^H v_B^L}$ —which might have been critical in helping the firm navigate the startup stages, then underconfidence would incentivize employees and fuel greater competition. In contrast, consider an ambitious academic institution in the process of an aggressive expansion by recruiting from more prestigious peers. Its faculty members may be on average disadvantaged in their research capacity, i.e., $v_A < \sqrt{v_B^H v_B^L}$, but also underconfident about their skills relative to the new hires. Proposition 1 then suggests that it is helpful to restore the confidence of the incumbent faculty.

3 Internal Evaluation and Information Disclosure

In this section, we expand the model to explore the optimal information disclosure policy that modifies the information environment. The manager sets an information disclosure policy prior to the competition. For the moment, we assume that the firm equally values employees’ contributions, so the manager aims to maximize the expected total effort.¹⁷

The manager conducts an internal evaluation of the new hire B and obtains a noisy signal $s \in \{H, L\}$ regarding his ability. Specifically, we assume that the signal is drawn as follows:

$$\Pr\left(s = H \mid v_B = v_B^H\right) = \Pr\left(s = L \mid v_B = v_B^L\right) = q, \quad (2)$$

where $q \in (\frac{1}{2}, 1]$ indicates the quality of the signal.¹⁸ When $q = 1$, the signal perfectly reveals employee B ’s ability. In the extreme case that $q = 1/2$, the signal is completely

¹⁷We consider an extension in which the manager cares about the expected winner’s effort in Section 4.3.

¹⁸Note that q is exogenous in this section. We will generalize the model and endogenize the information structure using a Bayesian persuasion approach (e.g., Kamenica and Gentzkow, 2011, Alonso and Camara, 2016) in Section 4.2.

uninformative. Before the competition, the manager commits to her disclosure policy, i.e., whether the result of her private evaluation of employee A 's ability—i.e., the realized signal s —is to be disclosed publicly or concealed.¹⁹

The signal would allow the manager and employee A to update their beliefs based on their own prior. For the manager, she would infer that employee B is of high type with a posterior probability μ_s , as given by

$$\mu_s = \frac{\mu \Pr(s|v_B = v_B^H)}{\mu \Pr(s|v_B = v_B^H) + (1 - \mu) \Pr(s|v_B = v_B^L)}, \text{ for } s = H, L. \quad (3)$$

Similarly, employee A 's posterior belief, denoted by $\tilde{\mu}_s$, is given by

$$\tilde{\mu}_s = \frac{\tilde{\mu} \Pr(s|v_B = v_B^H)}{\tilde{\mu} \Pr(s|v_B = v_B^H) + (1 - \tilde{\mu}) \Pr(s|v_B = v_B^L)}, \text{ for } s = H, L. \quad (4)$$

It is straightforward to verify that both μ_s and $\tilde{\mu}_s$ strictly increase with the priors, μ and $\tilde{\mu}$, respectively, for $q < 1$. When the signal is perfectly informative—i.e., $q = 1$ —both parties' posterior beliefs would jump to one upon receiving $s = H$ and would drop to zero upon receiving $s = L$, independent of their priors.

3.1 Optimal Information Disclosure Policy

We denote by $TE^C(\mu, \tilde{\mu})$ the expected total effort when the signal s is withheld, where the superscript C indicates “concealment.” The expected total effort $TE^C(\mu, \tilde{\mu})$ is the same as (1) and given by

$$TE^C(\mu, \tilde{\mu}) = \left[(1 - \mu) \sqrt{v_B^L} + \mu \sqrt{v_B^H} \right] K(\tilde{\mu}). \quad (5)$$

When the signal $s \in \{H, L\}$ is disclosed, the expected total effort is given by

$$TE(\mu_s, \tilde{\mu}_s) = \left[(1 - \mu_s) \sqrt{v_B^L} + \mu_s \sqrt{v_B^H} \right] K(\tilde{\mu}_s).$$

¹⁹We consider an extension in which the manager is able to privately inform employee A of the signal in Section 4.1.

where μ_s and $\tilde{\mu}_s$, with $s \in \{H, L\}$, are given by (3) and (4), respectively.

Further, the actual probabilities that $s = H$ and $s = L$ occur amount to $\mu q + (1 - \mu)(1 - q)$ and $\mu(1 - q) + (1 - \mu)q$, respectively. This allows us to calculate the expected equilibrium total effort when the manager commits to disclosing the signal, $TE^D(\mu, \tilde{\mu})$, where we use superscript D to indicate “disclosure”:

$$TE^D(\mu, \tilde{\mu}) = \left[\mu q + (1 - \mu)(1 - q) \right] \times \left[(1 - \mu_H) \sqrt{v_B^L} + \mu_H \sqrt{v_B^H} \right] K(\tilde{\mu}_H) \\ + \left[\mu(1 - q) + (1 - \mu)q \right] \times \left[(1 - \mu_L) \sqrt{v_B^L} + \mu_L \sqrt{v_B^H} \right] K(\tilde{\mu}_L). \quad (6)$$

We then investigate the manager’s choice of disclosure policy. For expositional convenience, we define Θ as

$$\Theta := \left[\sqrt{v_B^H v_B^L} - v_A \right] \times \left[\frac{(v_B^L)^{\frac{3}{2}} (v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}} (v_A + v_B^L)} - \frac{\mu(1 - \tilde{\mu})}{\tilde{\mu}(1 - \mu)} \right]. \quad (7)$$

Proposition 2 (Concealment vs. Disclosure) *Suppose $q \in (\frac{1}{2}, 1]$ and that the manager aims to maximize the expected total effort in the tournament. Then the following statements hold:*

- (i) *When $\Theta > 0$, it is optimal for the manager to commit to disclosing her private signal, i.e., $TE^D(\mu, \tilde{\mu}) > TE^C(\mu, \tilde{\mu})$;*
- (ii) *When $\Theta < 0$, it is optimal for the manager to conceal the signal, i.e., $TE^D(\mu, \tilde{\mu}) < TE^C(\mu, \tilde{\mu})$;*
- (iii) *When $\Theta = 0$, the manager is indifferent between disclosing the signal and concealing it, i.e., $TE^D(\mu, \tilde{\mu}) = TE^C(\mu, \tilde{\mu})$.*

Proposition 2 states that the optimal information disclosure policy hinges on the sign of Θ .²⁰ To interpret this proposition, it is key to identify the condition that determines the sign of Θ . Note that that the second term in (7) is always negative when employee A exhibits

²⁰It is useful to point out that Θ is independent of $q \in (1/2, 1]$.

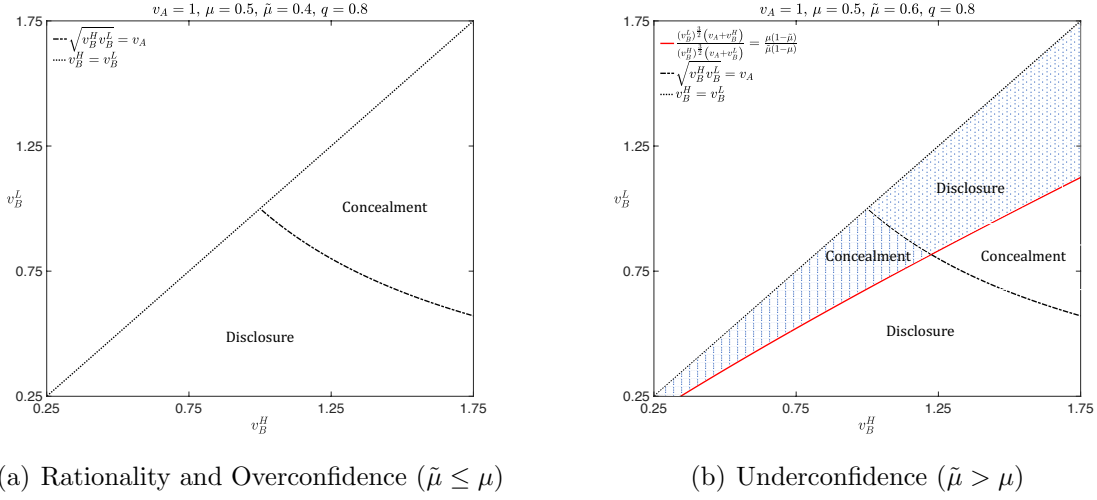


Figure 1: Optimal Effort-Maximizing Information Disclosure Policy

(weak) overconfidence, i.e., $\tilde{\mu} \leq \mu$. To see that, note that $[\mu(1 - \tilde{\mu})]/[\tilde{\mu}(1 - \mu)] \geq 1$ in this case, which in turn implies

$$\frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1 - \tilde{\mu})}{\tilde{\mu}(1 - \mu)} \leq \frac{v_B^L(v_A + v_B^H)}{v_B^H(v_A + v_B^L)} - 1 = \frac{v_A(v_B^L - v_B^H)}{v_B^H(v_A + v_B^L)} < 0.$$

This observation allows us to infer that with overconfidence ($\tilde{\mu} < \mu$) or rational belief ($\tilde{\mu} = \mu$), disclosure is optimal if $\sqrt{v_B^H v_B^L} - v_A < 0$, or equivalently, employee A is the ex ante favorite; conversely, concealment is optimal if $\sqrt{v_B^H v_B^L} - v_A > 0$, or equivalently, employee A is the ex ante underdog.

The optimum is illustrated in Figure 1(a). In the figure, the horizontal axis traces v_B^H , while the vertical axis measures v_B^L . Therefore, the area under the diagonal collects all the relevant parameterizations with $v_B^L < v_B^H$. Assuming $(v_A, \mu, \tilde{\mu}, q) = (1, 0.5, 0.4, 0.8)$, the dashed curve splits the area into two regions: The upper portion depicts the case of $\sqrt{v_B^H v_B^L} - v_A > 0$ such that $\Theta < 0$, in which concealment is preferred; while the lower portion represents $\sqrt{v_B^H v_B^L} - v_A < 0$ such that $\Theta > 0$, in which case full disclosure prevails.

Complexity arises in the scenario of underconfidence. The sign of $\sqrt{v_B^H v_B^L} - v_A$ alone cannot predict the sign of Θ , as the second term in the expression of (7) is indeterminate when $\tilde{\mu} > \mu$. The optimal disclosure policy is depicted in Figure 1(b). The terms of $\sqrt{v_B^H v_B^L} - v_A$

and $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$ jointly determine the sign of Θ , allowing four scenarios to arise in this context. Fixing $(v_A, \mu, \tilde{\mu}, q) = (1, 0.5, 0.6, 0.8)$, the solid curve in the figure traces all parameterizations that satisfy $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] = [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$. The area above the curve depicts the case with $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)] > 0$, in which case the optimum overturns that under overconfidence or rational belief. In the area below the solid curve, the term continues to be negative, which affirms the prediction of what happens under overconfidence or rational belief.

Proposition 2 and Equation (7) enable comparative statics with respect to the degree of employee underconfidence. Fix v_A and $\tilde{\mu} > \mu$. Let us define

$$\Upsilon(\tilde{\mu}) := \left\{ (v_B^H, v_B^L) \left| \frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1 - \tilde{\mu})}{\tilde{\mu}(1 - \mu)} > 0, v_B^H > v_B^L \geq \frac{v_A}{4} \right. \right\},$$

as the set of parameters (v_B^H, v_B^L) under which the optimal information disclosure policy with underconfidence differs from that with overconfidence or rational belief. The following proposition can be obtained:

Proposition 3 (*Impact of Increasing Underconfidence*) *Suppose that $\tilde{\mu}^\dagger > \tilde{\mu} > \mu$. Then $\Upsilon(\tilde{\mu}) \subset \Upsilon(\tilde{\mu}^\dagger)$ and the inclusion is strict.*

For given (v_A, v_B^L, v_B^H) , the sign of Θ is determined by the size of $\tilde{\mu}$ relative to μ in the case of underconfidence. For a $\tilde{\mu}$ that is mildly above μ , i.e., moderate underconfidence, the optimum is more likely to coincide with that under overconfidence or rational belief, as the sign of $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$ remains negative. In the case of severe underconfidence, i.e., a large $\tilde{\mu}$ relative to μ , the sign would turn positive, and the optimum under overconfidence or rational belief would be overturned, which is formally stated as $\Upsilon(\tilde{\mu}) \subset \Upsilon(\tilde{\mu}^\dagger)$ for $\tilde{\mu}^\dagger > \tilde{\mu} > \mu$ by Proposition 3.

Figure 2 illustrates how a change in the degree of underconfidence affects the optimal information disclosure policy, which confirms the observation from Proposition 3. Figure 2(a)

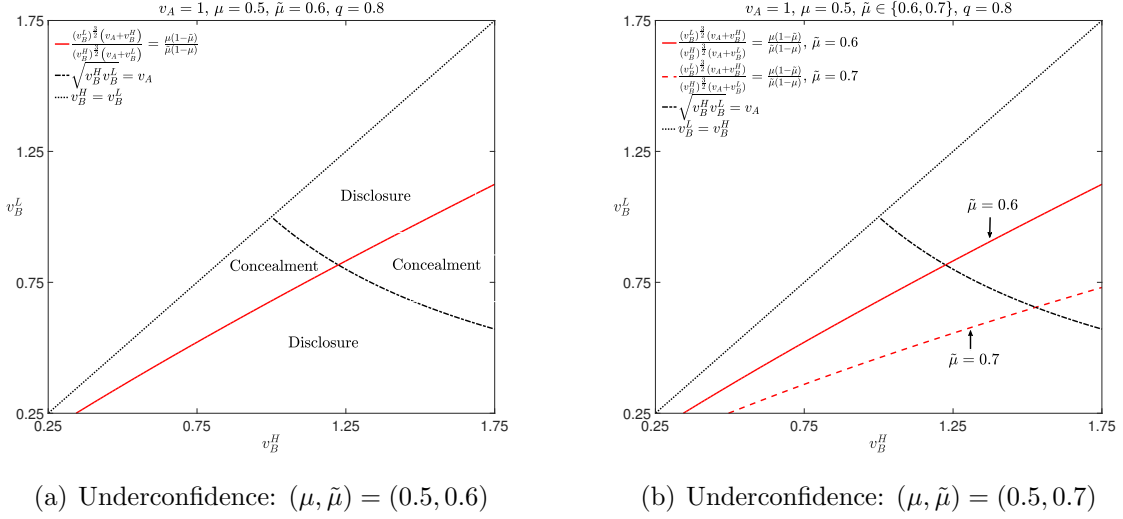


Figure 2: Impact of Underconfidence on Optimal Information Disclosure Policy

depicts the same scenario as Figure 1(b), which shows the optimum with underconfidence with $(\mu, \tilde{\mu}) = (0.5, 0.6)$. Recall that the area above the solid curve depicts the case with $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)] > 0$, which causes the optimum to divert from that with overconfidence and rational belief. In Figure 2(b), we demonstrate the comparative statics when $\tilde{\mu}$ increases from 0.6 to 0.7. The curve that defines $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)] = 0$ is shifted downward, with the lower dashed curve representing the case with $\tilde{\mu} = 0.7$. Because $[\mu(1 - \tilde{\mu})]/[\tilde{\mu}(1 - \mu)]$ strictly decreases with $\tilde{\mu}$, the term $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$ is more likely to be positive following such an increase in $\tilde{\mu}$; this enlarges the set of parameterizations under which the optimal information disclosure policy differs from that in the case of overconfidence or rational belief.

3.2 Implications of Propositions 2 and 3

Our results in Section 3.1 provide a playbook for firms' internal information management practices. The optimal information disclosure policy is sensitive to the specific environment, which can be summarized as follows:

	Overconfidence	Moderate Underconfidence	Significant Underconfidence
Weak Incumbent ($v_A < \sqrt{v_B^H v_B^L}$)	Concealment	Concealment	Disclosure
Strong Incumbent ($v_A > \sqrt{v_B^H v_B^L}$)	Disclosure	Disclosure	Concealment

The table demonstrates that the optimal disclosure policy depends solely on employees’ ex ante relative competitiveness—i.e., the comparison between v_A and $\sqrt{v_B^H v_B^L}$ —when the incumbent employee is overconfident or has rational beliefs. However, additional cautions are required when the incumbent is underconfident: Mild underconfidence preserves the optimum under the previous case, while significant underconfidence overturns it.

Let us first consider the scenario of overconfidence. Imagine a rapidly-growing firm whose employees excessively attribute the firm’s success to their own talent and contributions, and thus exhibit overconfidence. If the firm is confident in the quality of its search effort, i.e., $v_A < \sqrt{v_B^H v_B^L}$, then Proposition 2 would recommend that the firm refrain from granting employees access to the information about their peers, as the table shows. Conversely, imagine a seasoned teaching star in a business school: The wealth of classroom experience and industry knowledge accumulated over the years not only ensures reliable delivery in teaching, but also breeds complacency. Proposition 2, as well as the table, clearly indicates that allowing the faculty members to access peers’ teaching feedback reports may increase the school’s aggregate teaching quality.²¹

Next, consider a case of underconfidence. Imagine a startup that poaches a veteran executive from an industry leader to upgrade its managerial talent. The early employees may grossly overestimate the external hire who possesses a stellar career record, thereby exhibiting severe underconfidence; by Propositions 2 and 3, the firm should embrace transparent internal information management. At first, the recommendation appears to be counterintuitive. In this scenario, an existing employee suffers from both deficiency in competence and a severe lack of confidence. When additional observation from the evaluation allows him to

²¹The practice of NUS business school exemplifies a system of transparent internal feedback and competitive performance evaluation. See Introduction and Footnote 4 for details.

infer more about relative competitiveness, his morale can either be elevated or degraded, depending on the realization of the signal. Ex ante, however, the possible boost in his confidence outweighs the possible “bust.” The logic will be further unveiled when we delve in depth into the underlying logic for our results in the next subsection.

3.3 Intuition for Propositions 2 and 3

We now interpret the logic that underlies Propositions 2 and 3. Recall that by Proposition 2, the optimal information disclosure policy under rational belief coincides with that under overconfidence, but may not for the case of underconfidence. We begin with the benchmark case of common prior and expound the role played by information disclosure, which gives rise to an *information effect* without the complications caused by misperception. We then elaborate on the role played by misperception, which catalyzes a *morale effect*. Their combination determines the optimum. Recall that the sign of Θ defined by expression (7) predicts the optimum: The two terms included in Θ each reflect one effect:

$$\Theta := \underbrace{\left[\sqrt{v_B^H v_B^L} - v_A \right]}_{\text{information effect}} \times \underbrace{\left[\frac{(v_B^L)^{\frac{3}{2}} (v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}} (v_A + v_B^L)} - \frac{\mu (1 - \tilde{\mu})}{\tilde{\mu} (1 - \mu)} \right]}_{\text{morale effect}}.$$

A rationale about the morale effect would explain how and why the optimum with underconfidence may depart from that with overconfidence or rational belief.

Common Prior: Information Effect The additional information conveyed by the signal $s \in \{H, L\}$ can lead employee A 's belief to be revised either upward or downward, depending on the realization of the signal. The update causes the equilibrium in the tournament to diverge across states.

The dispersion across states triggered by the signal occurs regardless of the perceptual bias. We thus focus on the case of common prior—i.e., $\mu = \tilde{\mu}$ —to illustrate its implications.

Our rationale is largely aligned with that of Zhang and Zhou (2016). Define

$$TE_R^C(\mu) := TE^C(\mu, \mu) = \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K(\mu),$$

to be the expected total effort in the case of concealment, where the subscript R indicates the rational benchmark. When the signal is revealed, employee A 's belief will be revised to either μ_H or μ_L , and the expected total effort of the tournament ends up as either $TE_R^C(\mu_H)$ or $TE_R^C(\mu_L)$; the corresponding ex ante expected total effort with common prior—which is similarly defined as $TE_R^D(\mu, \tilde{\mu}) := TE^D(\mu, \mu)$ —aggregates over the two states. Simple algebra would verify that

$$\frac{dTE_R^C}{d\mu} = \left(\sqrt{v_B^H} - \sqrt{v_B^L} \right) K(\mu) + \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K'(\mu),$$

and

$$\frac{d^2 TE_R^C}{d\mu^2} = 2 \left(\sqrt{v_B^H} - \sqrt{v_B^L} \right) K'(\mu) + \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K''(\mu).$$

Recall from Lemma 1 that (i) $K(\mu)$ is increasing if $v_A > \sqrt{v_B^H v_B^L}$, and decreasing if $v_A < \sqrt{v_B^H v_B^L}$; and (ii) $K(\mu)$ is convex if $v_A > \sqrt{v_B^H v_B^L}$ and concave if $v_A < \sqrt{v_B^H v_B^L}$. Hence, $TE_R^C(\mu)$ perfectly preserves the concavity/convexity of $K(\mu)$.

Carrying out the algebra, we can obtain the ex ante expected total effort

$$TE_R^D(\mu) = [\mu q + (1 - \mu)(1 - q)] TE_R^C(\mu_H) + [\mu(1 - q) + (1 - \mu)q] TE_R^C(\mu_L).$$

Because $[\mu q + (1 - \mu)(1 - q)] \mu_H + [\mu(1 - q) + (1 - \mu)q] \mu_L \equiv \mu$ by the martingale property of beliefs, the function $TE_R^D(\mu)$ is simply a weighted average of $TE_R^C(\mu)$ over two different states. As a result, the comparison depends on the concavity/convexity of the function $TE_R^C(\mu)$. We can immediately infer the following by Jensen's inequality.

Remark 4 $TE_R^D(\mu) > (<)TE_R^C(\mu)$ if and only if $TE_R^C(\mu)$ is strictly convex (concave).

That is, full disclosure (concealment) outperforms concealment (full disclosure) if and

only if employee A is the ex ante favorite (underdog), which explains Proposition 2 for the case of $\mu = \tilde{\mu}$.

Uncommon Priors: Morale Effect We now explore the case of uncommon priors, i.e., $\mu \neq \tilde{\mu}$. We need to compare $TE^C(\mu, \tilde{\mu})$ as in (5) to $TE^D(\mu, \tilde{\mu})$ as in (6). For the sake of expositional convenience, we focus on the case of $\mu = 1/2$, in which case the ex ante probabilities of receiving $s = H$ and $s = L$ are simply $1/2$ and do not depend on q . As a result, $TE^C(\mu, \tilde{\mu})$ and $TE^D(\mu, \tilde{\mu})$ can be simplified (respectively) as

$$\begin{aligned} TE^C\left(\frac{1}{2}, \tilde{\mu}\right) &= \frac{1}{2} \left[\sqrt{v_B^L} + \sqrt{v_B^H} \right] K(\tilde{\mu}), \text{ and} \\ TE^D\left(\frac{1}{2}, \tilde{\mu}\right) &= \frac{1}{2} \left[(1-q)\sqrt{v_B^L} + q\sqrt{v_B^H} \right] K(\tilde{\mu}_H) + \frac{1}{2} \left[q\sqrt{v_B^L} + (1-q)\sqrt{v_B^H} \right] K(\tilde{\mu}_L). \end{aligned}$$

The comparison boils down to

$$TE^D\left(\frac{1}{2}, \tilde{\mu}\right) - TE^C\left(\frac{1}{2}, \tilde{\mu}\right) = \frac{1}{2} \left\{ \begin{array}{l} \left[(1-q)\sqrt{v_B^L} + q\sqrt{v_B^H} \right] \times \left[K(\tilde{\mu}_H) - K(\tilde{\mu}) \right] \\ - \left[q\sqrt{v_B^L} + (1-q)\sqrt{v_B^H} \right] \times \left[K(\tilde{\mu}) - K(\tilde{\mu}_L) \right] \end{array} \right\}.$$

Upon observing the signal s , employee A revises his belief, which affects his effort incentive in the tournament. His morale can be either boosted—i.e., $\tilde{\mu}$ dropping to $\tilde{\mu}_L$ —or be busted—i.e., $\tilde{\mu}$ rising to $\tilde{\mu}_H$. The comparison highlighted above hinges on the change of $[K(\tilde{\mu}_H) - K(\tilde{\mu})]$ vis-à-vis $[K(\tilde{\mu}) - K(\tilde{\mu}_L)]$. The magnitude of his belief adjustment in response to a given signal depends on the nature of his initial misperception, i.e., whether employee A exhibits overconfidence or underconfidence.

Suppose that employee A is overconfident, so he underestimates his opponent, i.e., $\tilde{\mu} < \mu$. His posterior tends to respond to a high signal more sensitively—i.e., with a significant jump from the initially underestimated $\tilde{\mu}$ to $\tilde{\mu}_H$ —compared to the response to a low signal, i.e., a relatively mild decrease from $\tilde{\mu}$ to $\tilde{\mu}_L$. This follows from the properties of Bayesian updating: A new signal impacts the posterior more if it is more unexpected under the

prior.²² Thus, in the case of overconfidence, the incumbent's perception of the competitor would be substantially revised upward when a high signal refutes his initial underestimate of the competitor, while the revision would be more incremental when a low signal simply reinforces the existing bias. The opposite holds for the case of underconfidence with $\tilde{\mu} > \mu$, but the intuition is analogous. The upward revision of the posterior in response to a high signal tends to be muted compared to that in the presence of a low signal. A low signal would sharply overturn the initial overestimates, causing a significant drop from $\tilde{\mu}$ to $\tilde{\mu}_L$; in contrast, a high signal only confirms the initial overestimate, so the rise from $\tilde{\mu}$ to $\tilde{\mu}_H$ tends to be moderate.

For expositional efficiency, let us focus on the case with $v_A > \sqrt{v_B^H v_B^L}$, as the case with $v_A < \sqrt{v_B^H v_B^L}$ is simply its mirror image. Recall that in this case $K(\cdot)$ is strictly increasing in its argument by Lemma 1, and $K(\tilde{\mu}_H) - K(\tilde{\mu})$ and $K(\tilde{\mu}) - K(\tilde{\mu}_L)$ are both positive. Further, $TE^D(\frac{1}{2}, \tilde{\mu}) - TE^C(\frac{1}{2}, \tilde{\mu})$ is positive when $\tilde{\mu} = \mu = 1/2$ by the information effect.

With overconfidence, the argument laid out above implies that $K(\tilde{\mu}_H) - K(\tilde{\mu})$ tends to outweigh $K(\tilde{\mu}) - K(\tilde{\mu}_L)$: A high signal overturns employee A 's initial misperception, while a low signal marginally confirms his bias. This implies that $TE^D(\frac{1}{2}, \tilde{\mu}) - TE^C(\frac{1}{2}, \tilde{\mu})$ tends to be positive, and thus information disclosure outperforms concealment. The effect of his asymmetric morale response to high and low signals coincides with the information effect laid out above. The comparison between disclosure and concealment under overconfidence remains the same as that under rationality, as Figure 1(a) shows.

Consider, alternatively, the case of underconfidence. Although both $K(\tilde{\mu}_H) - K(\tilde{\mu})$ and $K(\tilde{\mu}) - K(\tilde{\mu}_L)$ are positive, $K(\tilde{\mu}_H) - K(\tilde{\mu})$ tends to be outsized by $K(\tilde{\mu}) - K(\tilde{\mu}_L)$: In this case, a low signal tends to overturn the initial underconfidence, whereas a high signal only mildly endorses the misperception. As a result, $TE^D(\frac{1}{2}, \tilde{\mu}) - TE^C(\frac{1}{2}, \tilde{\mu})$ is less likely to be positive, and thus concealment is more likely to prevail. The morale effect runs into conflicts with the aforementioned information effect and could outweigh the latter and overturn the optimum predicted by information effect, as Figure 1(b) shows.

²²This property of Bayesian updating is also exploited in Fang and Moscarini (2005) in a principal-agent setting, in which they refer to this effect the *morale hazard*.

Intuitively, the more biased the belief, the stronger this morale effect. This rationale thus sheds light on the observation of Proposition 3: A larger $\tilde{\mu}$ relative to μ —which corresponds with greater underconfidence—would amplify the morale effect and could more than offset the information effect, diverting the optimum away from that under overconfidence and rational belief, as shown in Proposition 3.

4 Extensions

In this section, we consider four variations to the baseline model. Section 4.1 extends the model to allow for private disclosure, in which the manager can choose to disclose the signal to the incumbent only. Section 4.2 applies a Bayesian persuasion approach to endogenize the information structure of the internal evaluation. Section 4.3 explores a setting in which the manager is concerned with the expected winner’s effort instead of total effort. Section 4.4 allows the new hire’s ability to take three or more values.

4.1 Private Disclosure

We have so far assumed that the signal is revealed to both the incumbent and the newbie when the manager chooses to disclose it. Next, we consider an alternative disclosure format: The manager can disclose the signal $s \in \{H, L\}$ as specified in Section 3 to the incumbent only. With private disclosure, the baseline model with one-sided incomplete information turns into one with two-sided incomplete information à la Fang and Morris (2006), in which employee A ’s estimate about employee B ’s type upon receiving the signal is privately known to himself.^{23,24}

Denote by $\langle (x_A^H, x_A^L), (x_B^H, x_B^L) \rangle$ the equilibrium effort profile: x_A^H and x_A^L are employee A ’s effort supply upon receiving a high and a low signal respectively; while x_B^H and x_B^L are

²³This kind of private disclosure scheme is also considered by Chen (2021) in all-pay auctions.

²⁴It is noteworthy that the contest game under private disclosure differs from the usual independent private value (IPV) frameworks assumed in auction and contest literature. Employee A ’s private type concerns the additional information he receives from the signal, while employee B ’s is about his true ability; their types differ in nature but are correlated.

employee B 's effort level given that his valuation from winning is v_B^H and v_B^L respectively.

Upon observing the signal s , employee A updates his belief $\tilde{\mu}_s$ according to (4) and chooses his signal-dependent effort, which we denote by $x_A^s \geq 0$, to maximize his expected payoff

$$\left[\tilde{\mu}_s \frac{x_A^s}{x_A^s + x_B^H} + (1 - \tilde{\mu}_s) \frac{x_A^s}{x_A^s + x_B^L} \right] v_A - x_A^s.$$

Note that employee A 's effort decision under private disclosure is signal-dependent as under public disclosure.

Recall that under public disclosure, employee B 's equilibrium effort depends on both the public signal $s \in \{H, L\}$ and his private type $v_B \in \{v_B^H, v_B^L\}$. In contrast, under private disclosure, his equilibrium effort can only depend on his own valuation v_B . Specifically, a type- v_B^z employee B , with $z \in \{H, L\}$, chooses $x_B^z \geq 0$ to maximize

$$\left\{ \Pr(s = H | v_B = v_B^z) \frac{x_B^z}{x_A^H + x_B^z} + \left[1 - \Pr(s = H | v_B = v_B^z) \right] \frac{x_B^z}{x_A^L + x_B^z} \right\} v_B^z - x_B^z,$$

where $\Pr(s = H | v_B = v_B^H) = 1 - \Pr(s = H | v_B = v_B^L) = q \in (1/2, 1)$ by (2).²⁵

A closed-form solution to the contest game is unavailable in general when it involves two-sided incomplete information (see, e.g., Hurley and Shogren, 1998b; and Serena, 2021), which substantially complicates the model.²⁶ However, it should be noted that the information effect and the morale effect featured in our baseline model also exist under this alternative information structure. The information effect arises when the additional information revealed by the signal s causes employee A to have diverging posterior beliefs and thus diverging effort decisions in the tournament, depending on the specific realization of the signal. This effect prevails regardless of whether the signal is publicly or privately disclosed. The morale effect is driven by the fact that the biased incumbent adjusts his perception of the new hire asymmetrically in response to high vis-à-vis low signal, depending on the nature of his initial

²⁵In the extreme case of $q = 1$, private disclosure is equivalent to public disclosure.

²⁶Several papers overcome the lack of a closed-form solution by either imposing more structure on the type distribution (Malueg and Yates, 2004; Fey, 2008; and Ewerhart, 2010) or modifying the CSF (Wasser, 2013).

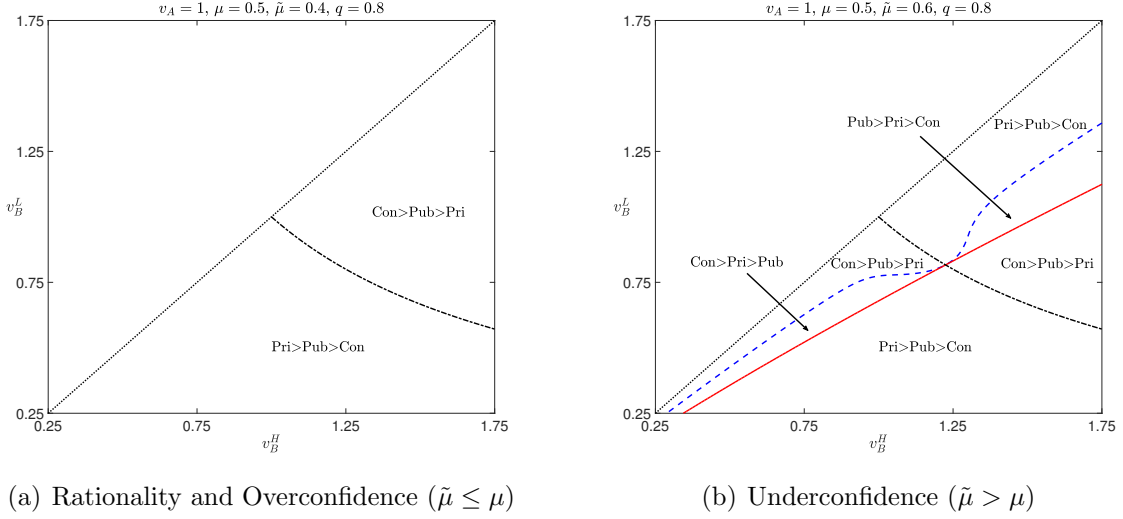


Figure 3: Public Disclosure vs. Private Disclosure vs. Concealment

misperception. This effect is preserved under private disclosure, given that the incumbent conducts the same Bayesian updating that he would under public disclosure. As a result, the trade-off between disclosure and concealment should not be sensitive to the specific mode of disclosure. This conjecture is confirmed by our numerical exercises, which are presented in Figure 3.

Figure 3 assumes $(v_A, \mu, \tilde{\mu}, q) = (1, 0.5, 0.4, 0.8)$, as in Figure 1. It provides a ranking among private disclosure, public disclosure, and concealment for different combinations of (v_B^H, v_B^L) . As in Figure 1(a), the relevant parameterizations in Figure 3(a)—i.e., those below the diagonal—are split into two regions by the dashed curve defined by $v_A = \sqrt{v_B^H v_B^L}$; the solid curve in Figure 3(b)—as in Figure 1(b)—traces all parameterizations that satisfy $\left[(v_B^L)^{\frac{3}{2}} (v_A + v_B^H) / (v_B^H)^{\frac{3}{2}} (v_A + v_B^L) \right] = [\mu(1 - \tilde{\mu}) / \tilde{\mu}(1 - \mu)]$. Comparing Figure 3 with Figure 1, we observe that the ranking between disclosure and concealment is *independent* of the mode of disclosure: Whenever public disclosure generates more (less) expected total effort than concealment, so does private disclosure. This confirms the rationale laid out above, although the comparison between public and private disclosure depends subtly on the specific setting and parameterization.

4.2 Optimal Design of Internal Evaluation

Thus far, we have assumed that the quality of the internal evaluation—i.e., q —is exogenous. In practice, a firm has the discretion to set the scope and format of the evaluation in the workplace or choose the evaluator, which presumably affects the quality of the exercise. For instance, a more experienced supervisor can assess his employee’s ability more accurately.

We now allow the firm to flexibly design and precommit to the information structure of the evaluation exercise before the tournament begins, which is referred to as the Bayesian persuasion approach in the literature, and was pioneered by Kamenica and Gentzkow (2011). Zhang and Zhou (2016) study the optimal information design in a similar setting but with common prior. Alonso and Camara (2016) explore Bayesian persuasion while allowing the sender and (single) receiver to possess heterogeneous beliefs. We borrow their approach and apply it to a tournament setting.

An information structure consists of a signal space \mathcal{S} and a pair of likelihood distributions $\{\pi(\cdot|v_B^H), \pi(\cdot|v_B^L)\}$ over \mathcal{S} . We allow the manager to freely set the information structure of the evaluation; she is thus endowed with full control over the amount of information to be revealed through the evaluation and the form of the signal to be disclosed to employees. Obviously, the evaluation exercise depicted in Section 3 involves a simple information structure with a binary signal space $\mathcal{S} = \{H, L\}$ and a conditional likelihood distribution for each underlying state—i.e., v_B^H or v_B^L —parametrized by a variable q [see Equation (2)].

In their seminal paper, Kamenica and Gentzkow (2011) show that searching for the optimal disclosure policy is equivalent to solving for the concave closure of a value function defined on the set of all posteriors—i.e., μ_s in our notation—assuming that all agents share a common prior (i.e., $\tilde{\mu} = \mu$) over the underlying states. Alonso and Camara (2016) generalize the tools in Kamenica and Gentzkow (2011) and allow for heterogeneous priors. According to Alonso and Camara (2016), it is without loss of generality to consider a binary signal space in our setting, i.e., $\mathcal{S} = \{H, L\}$; the search for the optimal effort-maximizing signal

structure $\{\pi(\cdot|v_B^H), \pi(\cdot|v_B^L)\}$ can be reduced to the following optimization problem:

$$\max_{\{\lambda, \mu_H, \mu_L\}} \lambda TE(\mu_H, \tilde{\mu}_H) + (1 - \lambda) TE(\mu_L, \tilde{\mu}_L) \quad (8)$$

subject to

$$\lambda \mu_H + (1 - \lambda) \mu_L = \mu, \quad (9)$$

$$\tilde{\mu}_s = \frac{t \mu_s}{t \mu_s + r(1 - \mu_s)}, \text{ for } s \in \{H, L\}, \quad (10)$$

$$0 \leq \lambda, \mu_H, \mu_L \leq 1, \quad (11)$$

where r and t are defined as $r := (1 - \tilde{\mu})/(1 - \mu)$ and $t := \tilde{\mu}/\mu$ respectively and capture the likelihood ratios of prior beliefs. As defined above, the variable μ_s in the objective function (8) is the manager's posterior about employee B 's ability as inferred upon observing signal $s \in \{H, L\}$; $\tilde{\mu}_s$ in expression (10), accordingly, refers to employee A 's posterior.

Given the priors $(\mu, \tilde{\mu})$ and manager's belief (μ_H, μ_L) , employee A 's posterior belief can be derived from (10). To be more specific, it follows from (3) and (4) that

$$\tilde{\mu}_s = \frac{t \frac{\mu \Pr(s|v_B=v_B^H)}{\mu \Pr(s|v_B=v_B^H) + (1-\mu) \Pr(s|v_B=v_B^L)}}{\frac{\mu \Pr(s|v_B=v_B^H)}{\mu \Pr(s|v_B=v_B^H) + (1-\mu) \Pr(s|v_B=v_B^L)} + r \frac{(1-\mu) \Pr(s|v_B=v_B^L)}{\mu \Pr(s|v_B=v_B^H) + (1-\mu) \Pr(s|v_B=v_B^L)}} = \frac{t \mu_s}{t \mu_s + r(1 - \mu_s)}.$$

When the manager and the employees share a common prior (i.e., $\tilde{\mu} = \mu$), we have $r = t = 1$ and they share the same Bayesian update (i.e., $\mu_s = \tilde{\mu}_s$ for $s \in \{H, L\}$).

Condition (9) requires $\mathbb{E}_\mu(\mu_s) = \mu$, which is identical to the one in Kamenica and Gentzkow (2011) and is commonly referred to as the Bayes-plausibility constraint. Condition (11) simply requires that the posterior belief μ_H and μ_L and the probability λ be bounded between zero and one. It is useful to point out that a perfectly informative evaluation corresponds to $(\mu_H, \mu_L) = (1, 0)$ with $\lambda = \mu$, and a completely uninformative evaluation (i.e., no information disclosure) corresponds to $(\mu_H, \mu_L) = (\mu, \mu)$ with $\lambda \in [0, 1]$.

Kamenica and Gentzkow (2011) and Alonso and Camara (2016) show that the indirect

value function from the above maximization problem boils down to the value of the concave closure of $TE(\mu_s, \tilde{\mu}_s(\mu_s))$ at the firm's prior μ . Simple algebra yields

$$TE(\mu_s, \tilde{\mu}_s(\mu_s)) = \left[(1 - \mu_s)\sqrt{v_B^L} + \mu_s\sqrt{v_B^H} \right] \times K \left(\frac{t\mu_s}{t\mu_s + r(1 - \mu_s)} \right).$$

Proposition 4 (*Optimal Design of Evaluation with Heterogeneous Priors*) *Suppose that the manager aims to maximize the expected total effort in the tournament and can flexibly design the internal evaluation. Then the following statements hold:*

- (i) *When $\Theta > 0$, full disclosure with a perfectly revealing evaluation—i.e., $(\mu_H, \mu_L) = (1, 0)$ —is optimal;*
- (ii) *When $\Theta < 0$, a completely uninformative evaluation—i.e., $(\mu_H, \mu_L) = (\mu, \mu)$ —is optimal;*
- (iii) *When $\Theta = 0$, the expected total effort is the same across all evaluation designs.*

Proposition 4 states that the optimal evaluation is either perfectly revealing or completely uninformative. The firm has a polarized preference regarding its evaluation, either maximizing the transparency in the tournament or simply minimize it, i.e., forgoing the evaluation. The condition for perfect revelation or no evaluation coincides with that for fully disclosing or concealing a noisy signal of quality $q \in (\frac{1}{2}, 1]$ in Proposition 2.

4.3 Maximizing the Expected Winner's Effort

Next, we consider an alternative context in which the manager is concerned about the expected winner's effort and not about the the total effort (e.g., Moldovanu and Sela, 2006; Serena, 2017; and Barbieri and Serena, 2019). This objective is sensible in many scenarios. For instance, when a firm solicits a technical solution internally, only the quality of the chosen entry accrues to its benefit. A CEO succession race motivates candidates to develop their managerial skills when carrying out assigned tasks: Large public firms—e.g., GE and

HP—often have difficulty retaining losing candidates, which would lead them to focus only on the acquisition of human capital from the winner (Fu and Wu, 2021).

Denote the expected winner’s effort, fixing $(\mu, \tilde{\mu})$, by $WE(\mu, \tilde{\mu})$. Similar to Equation (1), $WE(\mu, \tilde{\mu})$ can be derived as

$$WE(\mu, \tilde{\mu}) = \mathbb{E}_\mu \left[\frac{(x_A)^2 + [x_B(v_B)]^2}{x_A + x_B(v_B)} \right] = \mathbb{E}_\mu \left[x_A + x_B(v_B) - 2 \frac{x_A \cdot x_B(v_B)}{x_A + x_B(v_B)} \right].$$

Because total effort $TE(\mu, \tilde{\mu})$ is simply given by $\mathbb{E}_\mu [x_A + x_B(v_B)]$, the expression can alternatively be written as

$$WE(\mu, \tilde{\mu}) = TE(\mu, \tilde{\mu}) - 2\mathbb{E}_\mu \left[\frac{x_A \cdot x_B(v_B)}{x_A + x_B(v_B)} \right].$$

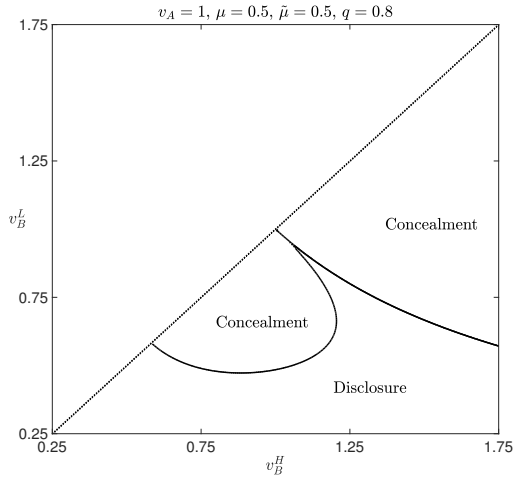
Thus, maximizing $WE(\mu, \tilde{\mu})$ is equivalent to maximizing the total effort minus the term $2\mathbb{E}_\mu \left[\frac{x_A \cdot x_B(v_B)}{x_A + x_B(v_B)} \right]$. The additional non-linear term adds complications. However, we show below that the prediction under total effort maximization remains qualitatively robust to a large extent.

We first evaluate the desirability of persistent misperception, as in Section 2.3. The following result can be obtained.

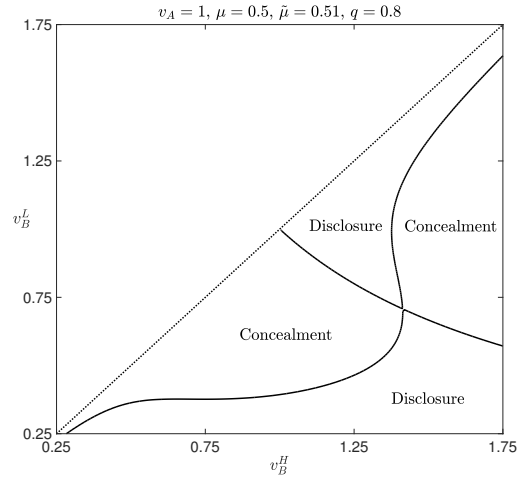
Proposition 5 (Value of Persistent Misperception) *Suppose that the firm is concerned about the expected winner’s effort in the tournament. Then the following statements hold:*

- (i) *When $v_A < \sqrt{v_B^H v_B^L}$, the firm strictly benefits from employee A’s misperception—i.e., $WE(\mu, \tilde{\mu}) > WE(\mu, \mu)$ —if and only if employee A exhibits overconfidence—i.e., $\tilde{\mu} < \mu$;*
- (ii) *When $v_A > \sqrt{v_B^H v_B^L}$, the firm strictly benefits from employee A’s misperception—i.e., $WE(\mu, \tilde{\mu}) > WE(\mu, \mu)$ —if and only if employee A exhibits underconfidence—i.e., $\tilde{\mu} > \mu$;*
- (iii) *When $v_A = \sqrt{v_B^H v_B^L}$, employee A’s prior does not affect the expected total effort, i.e.,*

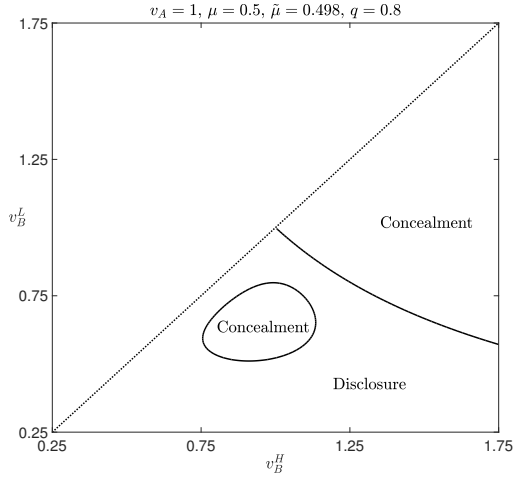
$$WE(\mu, \tilde{\mu}) = WE(\mu, \mu).$$



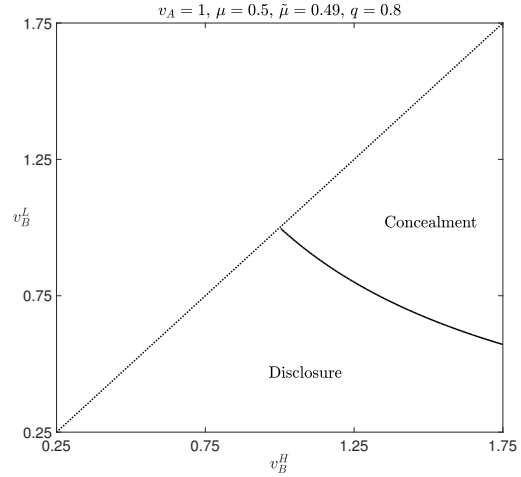
(a) Rationality ($\tilde{\mu} = 0.5$)



(b) Underconfidence ($\tilde{\mu} = 0.51$)



(c) Slight Overconfidence ($\tilde{\mu} = 0.498$)



(d) Significant Overconfidence ($\tilde{\mu} = 0.49$)

Figure 4: Optimal Information Disclosure Policy: Maximizing Winner's Effort

Proposition 5 states that the prediction of Proposition 1 is perfectly preserved in this alternative setting. Further, we explore the question that leads to Proposition 2: Suppose that an informative signal of quality $q \in (\frac{1}{2}, 1]$ is available. Would the manager disclose it to the employees? We resort to numerical exercises and hereby report the observations. Specifically, we compare the expected winner's effort under disclosure and under concealment. To proceed, we set $(v_A, \mu, q) = (1, 0.5, 0.8)$.

Figure 4 illustrates our numerical results for different cases. Three observations are worth highlighting. First, a comparison between Figure 4(a) and Figure 1(a) shows that the manager is more likely to hide information under the rational benchmark when the objective is to maximize the expected winner’s effort, than when the objective is to maximize total effort. Second, as employee A becomes more overconfident, the manager tends to disclose information more often, which can be seen by comparing Figure 4(c) to Figure 4(d), i.e., $\tilde{\mu}$ dropping from 0.498 to 0.49: In the latter case, the resultant pattern for the optimum coincides with that in the case of maximizing total effort as is depicted in Figure 1(a). Third, when employee A exhibits underconfidence, the pattern for the optimum is similar to that in the case of total effort, which can be seen by comparing Figure 4(b) to Figure 1(b). In summary, the result of Section 3 qualitatively remains in place, despite the fact that the objective function of expected winner’s effort causes nonlinearity.

4.4 Multiple Types

The baseline setting assumes that the new hire B ’s prize valuation v_B follows a Bernoulli distribution and can take two values. Next, we generalize the model to allow v_B to take multiple values and show that our main results remain qualitatively unchanged. Specifically, we allow v_B to take $N \geq 3$ values, $0 < v_B^1 < \dots < v_B^N$, with $\Pr(v_B = v_B^i) = \mu^i > 0$ for all $i \in \{1, \dots, B\}$. The manager’s prior about v_B is described by a distribution $\boldsymbol{\mu} := (\mu^1, \dots, \mu^N) \in \text{int}(\Delta^{N-1})$, and employee A ’s belief is denoted by $\tilde{\boldsymbol{\mu}} := (\tilde{\mu}^1, \dots, \tilde{\mu}^N) \in \Delta^{N-1}$.

This generalization complicates the analysis. With multiple types of possible, employee A ’s belief is a vector instead of a single variable as in the baseline model. Definition of overconfidence or underconfidence is thus more nuanced. To simplify the modeling of the incumbent’s perceptual bias, we focus on the case of extreme overconfidence—i.e., employee A ’s belief is given by $\tilde{\boldsymbol{\mu}}_O := (1, 0, \dots, 0)$ —and that of extreme underconfidence—i.e., employee A ’s belief is $\tilde{\boldsymbol{\mu}}_U := (0, \dots, 0, 1)$. Further, when comparing the performance of different information disclosure policies, we assume that the manager receives a perfectly informative

signal about the incumbent's ability.²⁷

In parallel with Assumption 1, we assume $v_B^1 \geq v_A/4$ throughout the subsection. Simple algebra would then verify that

$$TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}) = \frac{\mathbb{E}_{\boldsymbol{\mu}}[\sqrt{v_B}] \mathbb{E}_{\tilde{\boldsymbol{\mu}}} \left[\frac{1}{\sqrt{v_B}} \right]}{\frac{1}{v_A} + \mathbb{E}_{\tilde{\boldsymbol{\mu}}} \left[\frac{1}{v_B} \right]}.$$

In particular,

$$TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O) = \frac{\mathbb{E}_{\boldsymbol{\mu}}[\sqrt{v_B}] v_A \sqrt{v_B^1}}{v_A + v_B^1}, \text{ and } TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_U) = \frac{\mathbb{E}_{\boldsymbol{\mu}}[\sqrt{v_B}] v_A \sqrt{v_B^N}}{v_A + v_B^N}.$$

The following result can then be obtained.

Proposition 6 (Value of Persistent Misperception with Multiple Types) *Suppose that $v_B^1 \geq v_A/4$. Then the following statements hold:*

- (i) *If employee A exhibits extreme overconfidence—i.e., $\tilde{\boldsymbol{\mu}} = \tilde{\boldsymbol{\mu}}_O$ —then there exists a threshold $v^\dagger \in (0, 4v_B^1]$ such that $TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O) > TE(\boldsymbol{\mu}, \boldsymbol{\mu})$ if and only if $v_A < v^\dagger$.*
- (ii) *If employee A exhibits extreme underconfidence—i.e., $\tilde{\boldsymbol{\mu}} = \tilde{\boldsymbol{\mu}}_U$ —then there exists a threshold $v^{\dagger\dagger} \in (0, 4v_B^1]$ such that $TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_U) > TE(\boldsymbol{\mu}, \boldsymbol{\mu})$ if and only if $v_A > v^{\dagger\dagger}$.*

Proposition 6 is in line with Proposition 1: Employee A's overconfidence (underconfidence) benefits an effort-maximizing firm when he is weak (strong) relative to the new hire.

Next, we consider how the incumbent's perceptual bias affects the firm's disclosure policy. As previously mentioned, we assume that the manager has access to a perfectly informative signal and decides whether to publicly disclose or conceal it. Disclosure would turn the posterior game into a complete-information lottery contest, given that the vector of the incumbent's belief $\tilde{\boldsymbol{\mu}}$ is in the interior of Δ^{N-1} .²⁸ The expected total effort can be

²⁷A comprehensive analysis under general beliefs and information structures is definitely worthwhile and should be attempted in future studies.

²⁸In the extreme case that $\tilde{\boldsymbol{\mu}} = \tilde{\boldsymbol{\mu}}_O$ or $\tilde{\boldsymbol{\mu}} = \tilde{\boldsymbol{\mu}}_U$, information disclosure will not affect the incumbent's belief.

obtained as follows:

$$TE^D(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}) = \mathbb{E}_{\boldsymbol{\mu}} \left[\frac{v_A v_B}{v_A + v_B} \right], \text{ for all } \tilde{\boldsymbol{\mu}} \in \text{int}(\Delta^{N-1}).$$

If the manager commits to concealing the signal, the corresponding expected total effort amounts to

$$TE^C(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}) = TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}).$$

Let $\{\tilde{\boldsymbol{\mu}}_O^k\}_{k=1}^\infty$ and $\{\tilde{\boldsymbol{\mu}}_U^k\}_{k=1}^\infty$ be two sequences of beliefs that converge to $\tilde{\boldsymbol{\mu}}_O$ and $\tilde{\boldsymbol{\mu}}_U$ in norm, respectively. The following result ensues.

Proposition 7 (*Concealment vs. Disclosure with Multiple Types*) *Suppose that $v_B^1 \geq v_A/4$. Then the following statements hold:*

- (i) *There exists a threshold $v^* \in (0, 4v_B^1]$ such that $\lim_{k \rightarrow \infty} TE^D(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O^k) > \lim_{k \rightarrow \infty} TE^C(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O^k)$ if and only if $v_A > v^*$.*
- (ii) *There exists a threshold $v^{**} \in (0, 4v_B^1]$ such that $\lim_{k \rightarrow \infty} TE^D(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_U^k) > \lim_{k \rightarrow \infty} TE^C(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_U^k)$ if and only if $v_A < v^{**}$.*

By Proposition 7, the comparison between (public) disclosure and concealment with multiple types resembles that in Proposition 2. Note that from Proposition 2, if employee A is sufficiently overconfident (i.e., if $\tilde{\boldsymbol{\mu}}$ is close to 0), disclosure outperforms concealment when the incumbent is relatively strong, i.e., $v_A > \sqrt{v_B^H v_B^L}$. The optimal disclosure policy is reversed if employee A is sufficiently underconfident (i.e., if $\tilde{\boldsymbol{\mu}}$ is close to 1): Disclosure generates a larger amount of expected total effort than concealment when the incumbent is relatively weak, i.e., $v_A < \sqrt{v_B^H v_B^L}$. Proposition 7 demonstrates that these predictions can be preserved when multiple types are allowed.

5 Concluding Remarks

In this paper, we investigate the impact of perceptual bias—i.e., overconfidence or underconfidence on an opponent’s ability—on a promotion tournament and on the optimal information disclosure in a firm. Rich implications can be inferred from our results.

First, we demonstrate that a persistent misperception may either benefit or harm the firm’s performance. As a result, debiasing its employees can potentially be counterproductive. Second, we fully characterize the conditions under which disclosing an informative signal of an employee’s ability, or concealing it, can prevail.

The intricate role played by the perceptual bias sheds light on the extensive discussion of confidence or morale management and workplace culture building, which casts doubt on any universal recipe given the complexity. The analysis also speaks to the debate about organizational transparency. The information fed to employees changes their belief and perception, which in turn affects their incentives subtly and indeterminately.

In this paper, we focus on the impact of perceptual bias on contenders’ effort incentive and its implications for the optimal information disclosure policy. A firm can be subject to other concerns in practice, e.g., selecting and promoting a more competent candidate (Ryvkin and Ortmann, 2008; Brown and Minor, 2014). It would be interesting to extend our analysis to such an alternative context. That is, the manager’s objective can be modeled as a weighted sum of total effort and the benefit of selection efficiency—i.e., the probability of selecting the more competent employee.

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Appendix: Proofs

Proof of Proposition 1

Proof. Recall that

$$TE(\mu, \tilde{\mu}) = \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K(\tilde{\mu}).$$

The result immediately follows from the monotonicity of $K(\cdot)$, which is characterized by Lemma 1. ■

Proof of Proposition 2

Proof. For notational ease, we include q as an argument of $TE^D(\mu, \tilde{\mu})$

$$\begin{aligned} TE^D(\mu, \tilde{\mu}; q) &= \left[\mu q + (1 - \mu)(1 - q) \right] \times \left[(1 - \mu_H)\sqrt{v_B^L} + \mu_H\sqrt{v_B^H} \right] K(\tilde{\mu}_H) \\ &\quad + \left[\mu(1 - q) + (1 - \mu)q \right] \times \left[(1 - \mu_L)\sqrt{v_B^L} + \mu_L\sqrt{v_B^H} \right] K(\tilde{\mu}_L). \end{aligned}$$

Note that concealment is equivalent to disclosure with $q = \frac{1}{2}$: $TE^C(\mu, \tilde{\mu}) = TE^D(\mu, \tilde{\mu}; \frac{1}{2})$.

Define $G(q)$ as

$$G(q) := \left[\mu q + (1 - \mu)(1 - q) \right] \times \left[(1 - \mu_H(q))\sqrt{v_B^L} + \mu_H(q)\sqrt{v_B^H} \right] K(\tilde{\mu}_H(q)).$$

Recall that $\mu_H = \frac{\mu q}{\mu q + (1 - \mu)(1 - q)}$ and $\tilde{\mu}_H = \frac{\tilde{\mu} q}{\tilde{\mu} q + (1 - \tilde{\mu})(1 - q)}$. In defining $G(\cdot)$, we treat μ_H and $\tilde{\mu}_H$ as functions of q .

It is easy to verify that $TE^D(\mu, \tilde{\mu}; q) = G(q) + G(1 - q)$. Then,

$$\frac{\partial TE^D(\mu, \tilde{\mu}; q)}{\partial q} = G'(q) - G'(1 - q), \text{ and } \frac{\partial^2 TE^D(\mu, \tilde{\mu}; q)}{\partial q^2} = G''(q) + G''(1 - q).$$

Simple algebra yields that

$$\begin{aligned}
G''(q) &= \left[K'(\tilde{\mu}_H(q)) \tilde{\mu}''_H(q) + K''(\tilde{\mu}_H(q)) [\tilde{\mu}'_H(q)]^2 \right] \times \left[\mu q \sqrt{v_B^H} + (1-\mu)(1-q) \sqrt{v_B^L} \right] \\
&\quad + 2K'(\tilde{\mu}_H(q)) \tilde{\mu}'_H(q) \left[\mu \sqrt{v_B^H} - (1-\mu) \sqrt{v_B^L} \right] \\
&= - \underbrace{\frac{2\sqrt{v_B^H} \left(\frac{1}{v_B^L} + \frac{1}{v_A} \right) \tilde{\mu}(1-\mu)}{(1-\tilde{\mu})(1-q) + \tilde{\mu}q + \frac{\tilde{\mu}q}{v_B^L}}}_{>0} \times \underbrace{\tilde{\mu}'_H(q)}_{>0} \times K'(\tilde{\mu}_H(q)) \times \left[\frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1-\tilde{\mu})}{\tilde{\mu}(1-\mu)} \right].
\end{aligned}$$

It can be verified that $\tilde{\mu}'_H(q) = \frac{(1-\tilde{\mu})\tilde{\mu}}{[(1-\tilde{\mu})(1-q) + \tilde{\mu}q]^2} > 0$. Moreover, it follows from Lemma 1 that $K'(\tilde{\mu}_H) \geq 0$ is equivalent to $v_A - \sqrt{v_B^H v_B^L} \geq 0$. Therefore, $G''(q) \geq 0$ is equivalent to

$$\Theta := \left[\sqrt{v_B^H v_B^L} - v_A \right] \times \left[\frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1-\tilde{\mu})}{\tilde{\mu}(1-\mu)} \right] \geq 0.$$

Similarly, we can show that $G''(1-q) \geq 0$ is equivalent to $\Theta \geq 0$. Therefore, we can obtain that

$$\frac{\partial^2 TE^D(\mu, \tilde{\mu}; q)}{\partial q^2} \geq 0 \Leftrightarrow \Theta \geq 0.$$

Next, note that $\frac{\partial TE^D(\mu, \tilde{\mu}; \frac{1}{2})}{\partial q} = G'(\frac{1}{2}) - G'(\frac{1}{2}) = 0$. Consequently, when $\Theta > 0$, $TE^D(\mu, \tilde{\mu}; q)$ is strictly increasing in q and hence $TE^D(\mu, \tilde{\mu}; q) > TE^D(\mu, \tilde{\mu}; \frac{1}{2}) = TE^C(\mu, \tilde{\mu})$ for all $\frac{1}{2} < q \leq 1$. When $\Theta < 0$, $TE^D(\mu, \tilde{\mu}; q)$ is strictly decreasing in q and hence $TE^D(\mu, \tilde{\mu}; q) < TE^D(\mu, \tilde{\mu}; \frac{1}{2}) = TE^C(\mu, \tilde{\mu})$ for all $\frac{1}{2} < q \leq 1$. When $\Theta = 0$, $TE^D(\mu, \tilde{\mu}; q)$ is constant in q and thus the firm is indifferent between disclosure and concealment. ■

Proof of Proposition 3

Proof. In the case of underconfidence, for every given $\mu \in (0, 1)$, the term $[\mu(1-\tilde{\mu})]/[\tilde{\mu}(1-\mu)]$ strictly decreases with $\tilde{\mu}$, with $[\mu(1-\tilde{\mu})]/[\tilde{\mu}(1-\mu)]|_{\tilde{\mu}=\mu} = 1$ and $[\mu(1-\tilde{\mu})]/[\tilde{\mu}(1-\mu)]|_{\tilde{\mu}=1} = 0$. Note that the term $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] < 1$. Therefore, fixing (v_A, v_B^L, v_B^H) ,

there exists a unique cutoff $\tilde{\mu}^* \in (\mu, 1)$ such that

$$\frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1 - \tilde{\mu})}{\tilde{\mu}(1 - \mu)} \begin{matrix} \leq \\ \geq \end{matrix} 0, \text{ if and only if } \tilde{\mu} \begin{matrix} \leq \\ \geq \end{matrix} \tilde{\mu}^*. \quad (12)$$

Proposition 3 follows instantly from (12) and Proposition 2. ■

Proof of Proposition 4

Proof. Recall that

$$\tilde{\mu}_s(\mu_s) = \frac{t\mu_s}{t\mu_s + r(1 - \mu_s)}.$$

It follows immediately that

$$\tilde{\mu}'_s(\mu_s) = \frac{rt}{[t\mu_s + r(1 - \mu_s)]^2} > 0, \text{ and } \tilde{\mu}''_s(\mu_s) = \frac{-2(t - r)rt}{[t\mu_s + r(1 - \mu_s)]^3}.$$

Denote $TE(\mu_s, \tilde{\mu}_s(\mu_s))$ by $\widehat{TE}_s(\mu_s)$. The second-order derivative of $\widehat{TE}_s(\mu_s)$ with respect to μ_s is

$$\begin{aligned} \widehat{TE}_s''(\mu_s) &= \left\{ K''(\tilde{\mu}_s(\mu_s)) [\tilde{\mu}'_s(\mu_s)]^2 + K'(\tilde{\mu}_s(\mu_s)) \tilde{\mu}''_s(\mu_s) \right\} \times \left[(1 - \mu_s)\sqrt{v_B^L} + \mu_s\sqrt{v_B^H} \right] \\ &\quad + 2K'(\tilde{\mu}_s(\mu_s)) \tilde{\mu}'_s(\mu_s) \left(\sqrt{v_B^H} - \sqrt{v_B^L} \right) \\ &= - \underbrace{\frac{2\tilde{\mu}'_s(\mu_s)\sqrt{v_B^H} \left(\frac{1}{v_B^L} + \frac{1}{v_A} \right) t}{\frac{t\mu_s + r(1 - \mu_s)}{v_A} + \frac{r(1 - \mu_s)}{v_B^L} + \frac{t\mu_s}{v_B^H}}_{>0} \times K'(\tilde{\mu}_s(\mu_s)) \times \left[\frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1 - \tilde{\mu})}{\tilde{\mu}(1 - \mu)} \right]. \end{aligned}$$

It follows from Lemma 1 that $K'(\tilde{\mu}_s(\mu_s)) \begin{matrix} \geq \\ \leq \end{matrix} 0$ is equivalent to $v_A \begin{matrix} \geq \\ \leq \end{matrix} \sqrt{v_B^H v_B^L}$. Therefore, $\widehat{TE}_s''(\mu_s) \begin{matrix} \geq \\ \leq \end{matrix} 0$ is equivalent to

$$\Theta = \left[\sqrt{v_B^H v_B^L} - v_A \right] \times \left[\frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1 - \tilde{\mu})}{\tilde{\mu}(1 - \mu)} \right] \begin{matrix} \geq \\ \leq \end{matrix} 0.$$

When $\Theta > 0$, $TE_s(\mu_s)$ is strictly convex in μ_s , indicating the optimality of perfectly revealing

signals. When $\Theta < 0$, $TE_s(\mu_s)$ is strictly concave in μ_s , indicating the optimality of completely uninformative signals. When $\Theta = 0$, $TE_s(\mu_s)$ is linear in μ_s , and thus all information disclosure policies lead to the same expected total effort. ■

Proof of Proposition 5

Proof. First we simplify $WE(\mu, \tilde{\mu})$.

$$\begin{aligned} WE(\mu, \tilde{\mu}) &= \mathbb{E}_\mu \left[x_A + x_B(v_B) - 2 \frac{x_A \cdot x_B(v_B)}{x_A + x_B(v_B)} \right] \\ &= \mathbb{E}_\mu \left[\sqrt{v_B x_A} - 2 \frac{x_A (\sqrt{v_B x_A} - x_A)}{\sqrt{v_B x_A}} \right] \\ &= \mathbb{E}_\mu \left[F(v_B, K(\tilde{\mu})) \right], \end{aligned}$$

where $F(v_B, K) := \frac{2K^3}{\sqrt{v_B}} + \sqrt{v_B}K - 2K^2$. Note that

$$\frac{\partial F(v_B, K)}{\partial K} = \frac{6K^2}{\sqrt{v_B}} + \sqrt{v_B} - 4K \geq (2\sqrt{6} - 4)K > 0.$$

Therefore, $WE(\mu, \tilde{\mu})$ is increasing in K . From Lemma 1, $K(\cdot)$ is strictly decreasing in $\tilde{\mu}$ if $\sqrt{v_B^H v_B^L} > v_A$ and $K(\tilde{\mu})$ is strictly increasing in $\tilde{\mu}$ otherwise. This completes the proof. ■

Proof of Proposition 6

Proof. We focus on the case of overconfidence. The analysis for the case of underconfidence is similar and omitted for brevity. Carrying out the algebra, we have that

$$TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O) - TE(\boldsymbol{\mu}, \boldsymbol{\mu}) = \frac{v_A \mathbb{E}_\mu \sqrt{v_B}}{1 + v_A \mathbb{E}_\mu \left[\frac{1}{v_B} \right]} \left\{ \sqrt{v_B^1} \mathbb{E}_\mu \left[\frac{1}{v_B} \right] \frac{v_A + \frac{1}{\mathbb{E}_\mu \left[\frac{1}{v_B} \right]}}{v_A + v_B^1} - \mathbb{E}_\mu \left[\frac{1}{\sqrt{v_B}} \right] \right\}.$$

Therefore, $TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O) > TE(\boldsymbol{\mu}, \boldsymbol{\mu})$ is equivalent to

$$\phi(v_A) := \sqrt{v_B^1} \times \mathbb{E}_\mu \left[\frac{1}{v_B} \right] \times \frac{v_A + \frac{1}{\mathbb{E}_\mu \left[\frac{1}{v_B} \right]}}{v_A + v_B^1} - \mathbb{E}_\mu \left[\frac{1}{\sqrt{v_B}} \right] > 0.$$

Note that $\frac{1}{\mathbb{E}_\mu \left[\frac{1}{v_B} \right]} > v_B^1$ implies that $\phi(v_A)$ decreases with v_A . Evaluating $\phi(v_A)$ at $v_A = 0$ yields

$$\phi(0) = \sqrt{v_B^1} \times \mathbb{E}_\mu \left[\frac{1}{v_B} \right] \times \frac{0 + \frac{1}{\mathbb{E}_\mu \left[\frac{1}{v_B} \right]}}{0 + v_B^1} - \mathbb{E}_\mu \left[\frac{1}{\sqrt{v_B}} \right] = \frac{1}{\sqrt{v_B^1}} - \mathbb{E}_\mu \left[\frac{1}{\sqrt{v_B}} \right] > 0.$$

Therefore, there exists a cutoff $v^\dagger \in (0, 4v_B^1]$ such that $TE(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O) > TE(\boldsymbol{\mu}, \boldsymbol{\mu})$ if and only if $v_A < v^\dagger$. This concludes the proof. ■

Proof of Proposition 7

Proof. We focus on the case of overconfidence. The analysis for the case of underconfidence is similar and omitted for brevity. Note that

$$\lim_{k \rightarrow \infty} TE^D(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O^k) - \lim_{k \rightarrow \infty} TE^C(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O^k) = \frac{v_A}{v_A + v_B^1} \left\{ \mathbb{E}_\mu \left[v_B \times \frac{v_A + v_B^1}{v_A + v_B} \right] - \sqrt{v_B^1} \times \mathbb{E}_\mu \left[\sqrt{v_B} \right] \right\}.$$

Therefore, $\lim_{k \rightarrow \infty} TE^D(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O^k) - \lim_{k \rightarrow \infty} TE^C(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O^k) > 0$ is equivalent to

$$\psi(v_A) := \mathbb{E}_\mu \left[v_B \times \frac{v_A + v_B^1}{v_A + v_B} \right] - \sqrt{v_B^1} \times \mathbb{E}_\mu \left[\sqrt{v_B} \right] > 0.$$

It is straightforward to verify that $\psi(v_A)$ strictly increases with v_A . Moreover, we have that

$$\psi(0) = \mathbb{E}_\mu \left[v_B^1 \right] - \sqrt{v_B^1} \times \mathbb{E}_\mu \left[\sqrt{v_B} \right] < 0.$$

Therefore, there exists a threshold $v^* \in (0, 4v_B^1]$ such that $\lim_{k \rightarrow \infty} TE^D(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O^k) > \lim_{k \rightarrow \infty} TE^C(\boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}_O^k)$ if and only if $v_A > v^*$. This concludes the proof. ■