Contests: Theory and Topics

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Summary and Keywords

Contest-like competitive situations are ubiquitous in modern economic landscape. In a contest, economic agents expend costly effort to vie for limited prizes, and they are rewarded for "getting ahead" of their opponents instead of their absolute performance metrics. Many social, economic, and business phenomena exemplify such competitive schemes, ranging from college admissions, political campaigns, advertising, and organizational hierarchies, to warfare. The economics literature has long recognized contest/tournament as a convenient and efficient incentive scheme to remedy the moral hazard problem, especially when the production process is subject to random perturbation or the measurement of input/output is imprecise or costly. An enormous amount of scholarly effort has been devoted to developing tractable theoretical models, unveiling the fundamentals of the strategic interactions that underlie such competitions, and exploring the optimal design of contest rules. This voluminous literature has enriched basic contest/tournament models by introducing different variations to the modelling, such as dynamic structure, incomplete and asymmetric information, multi-battle confrontations, sorting and entry, endogenous prize allocation, competitions in groups, contestants with alternative risk attitude, etc.

Keywords: Contests; Tournaments; All-pay Auctions; Moral Hazard; Incentives; Efforts; Contest Design.

Theory of Contest: A Brief Overview

Contests/tournaments are prevalent on social and economic landscapes, because many competitive activities can be viewed as a contest/tournament. In a contest-like situation, the parties are rewarded by their relative performance, instead of absolute output; they forfeit costly resources to vie for a limited number of prizes, while the bids are nonrefundable regardless of the outcome. Such competitive interactions are common in diverse social, political, and economic contexts, including college admissions, politics, promotion competitions between firms to increase market share, rent-seeking activities, R&D races, sporting events, collective decision making, and wars and other conflicts. In particular, contests/tournaments have long been recognized as a convenient and effective scheme to elicit productive effort and address moral hazard problems in firms' internal labor market competitions (e.g., Lazear and Rosen, 1981): Workers compete for bonus packages or strive to climb hierarchical ladders.

The ubiquity of such phenomena has spawned a great wealth of research efforts to explore economic subjects' strategic interactions in contest-like situations. Since the seminal study of Tullock (1967) on rent-seeking competitions, a plethora of contest models have been constructed to shed light on the various aspects of these competitive situations. They are applied broadly and have yielded ample implications for a wide array of research areas, ranging from economics, business, political science, and history to public administration.

The majority of economic studies of contests focus on winner-take-all competitions. A formal contest model can be characterized as follows. A set of contestants, indexed by $i = \{1, \ldots, n\}$, exert their efforts x_i to compete for a prize, with each contestant i having a valuation $v_i > 0$. Central to formally modelling contests is a mechanism that selects the winner and awards the prize. The selection mechanism is conventionally abstracted by a contest success function that maps contestants' effort entries into the likelihood $p_i \in [0, 1]$ of every contestant winning the prize. As a result, each contestant i's expected payoff in the game can be written as

$$\pi_i(\mathbf{x}) = p_i(\mathbf{x})v_i - x_i,$$

where x is the vector of contestants' effort entries, with $x := (x_1, \ldots, x_n)$. The contest is perfectly discriminatory if a higher effort leads to a sure win, in which case the contest translates into a first-price all-pay auction. It is imperfectly discriminatory if the outcome of the contest is also subject to measurement errors, subjective biases, or randomness in the competitive environment. The size of the prize purse is usually assumed to be fixed and exogenously given.[1]

An enormous amount of research effort has been devoted to advancing the theory of the contest game and exploring its strategic substance. The accumulated knowledge about contestants' strategic behavior, in turn, motivates the extensive research on optimal contest design, which strategically manipulates the structural elements of a contest—e.g., contest rules—to promote stated goals. Vojnović (2016) provides a comprehensive review of contest theory and its applications in economics, business, computer science, and statistics. The vast literature has been reviewed in several surveys, including Nitzan (1994), Corchón (2007), Konrad (2009), Connelly, Tihanyi, Crook, and Gangloff (2014), and Corchón and Serena (2018). In addition, Szymanski (2003) provides a survey that stresses the application of contest models in sporting events; Sisak (2009) is dedicated to the studies of contests with multiple prizes; Chowdhury and Gürtler (2015) examine studies of contests with sabotage;

Dechenaux, Kovenock, and Sheremeta (2015) mainly focus on experimental research in this field; and Mealem and Nitzan (2016) provide an account of discrimination in contests. Congleton and Hillman (2015) collect a broad collection of focused studies that summarize the theory of rent seeking and its various applications. Our survey aims to provide an overview of theoretical contributions to the literature on contests.

This survey begins with a summary of major contest modelling frameworks that differ in the prevailing winner-selection mechanisms. It goes on to discuss a wide range of variants to baseline contest models in a broad spectrum of contexts, including (1) contests with dynamics, (2) information in contests, (3) multi-battle contests, (4) contests with entry, (5) contests with heterogeneous players and optimal favoritism, (6) group contests, and (7) contests with non-risk-neutral players.

Contest Modelling Frameworks

This section begins with a descriptive account of perfectly discriminatory contest models, in which one secures a sure win when he outperforms with an infinitesimal margin. Such contest models are conventionally labeled all-pay auctions in the literature. It then discusses imperfectly discriminatory contest models, in which a higher effort can only lead to a higher winning probability and noises could play a role in determining the outcome of the competition. Two types of models are considered: rank-order tournaments (Lazear and Rosen, 1981; Green and Stokey, 1983), and contests with ratio-form contest success functions (Tullock, 1967, 1980).[2,3] Finally, it proceeds to a discussion of the link between the two types of frameworks.

Perfectly Discriminatory Contests: All-pay Auctions

All-pay auction models are applied to a wide variety of contexts, such as lobbying (Hillman and Samet, 1987; Hillman and Riley, 1989); R&D races (Dasgupta, 1986); litigation (Baye, Kovenock, and de Vries, 2005); etc. Consider a typical winner-take-all all-pay auction (Hillman and Riley, 1989; Baye, Kovenock, and de Vries, 1996). For an effort profile $x = (x_1, \ldots, x_n)$, a player i wins the contest with a probability

$$P_i = \begin{cases} 1, & \text{if } x_i > \max\{x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n\}; \\ \frac{1}{m} & \text{if } x_i \text{ is among the } m \text{ highest of } \mathbf{x} \text{ with a tie;} \\ 0 & \text{if } x_i < \max\{x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n\}. \end{cases}$$

A typical all-pay auction has no pure-strategy equilibrium, and yields a mixed-strategy equilibrium. Baye, Kovenock, and de Vries (1996) provide a general characterization of the equilibria in an n-person all-pay auction with complete information. In a standard incomplete-information all-pay auction, contestants' bids are continuously distributed above zero, so a tie is a zero-probability event. Gelder, Kovenock, and Roberson (2016) propose a two-person all-pay auction model with tie: A contestant wins the entire prize if and only if his bid exceeds that of his opponent by a minimum margin; the prize will be shared if their bid differential falls below a required cutoff. Che and Gale (1998) and Kaplan and Wettstein (2006) study complete-information all-pay auctions with players' bids being capped by upper limits. Che and Gale (2003) model an R&D contest in which a buyer attempts to procure an innovative product from two bidders, and the one that offers a higher buyer surplus wins. The surplus is given by the difference between product quality and the price offered. The model differs from a usual all-pay auction model because while quality incurs irreversible investment, price affects only the winner's utility. Kaplan, Luski, Sela, and Wettstein (2002) consider an incomplete-information all-pay auction model in which the value of one's prize endogenously depends on his decision variable.

Clark and Riis (1998b), Barut and Kovenock (1998), Bulow and Levin (2006), Siegel (2009, 2010, 2014a), Xiao (2016), and Fang, Noe, and Strack (2018) allow a number of prizes to be given away through all-pay auctions. In particular, it should be highlighted that Siegel (2009) considers a broad category of "all-pay contests" in which each contestant chooses his "score" to win one of multiple identical prizes. The generic framework encompasses a broad array of variations, allowing for general production technologies, cost functions, conditional and unconditional investments, and risk attitude, with complete-information all-pay auction as a special example. The study, remarkably, provides a closed-form formula for players' equilibrium payoffs in this generic contest game. Siegel (2010) focuses on equilibrium behavior in an all-pay contest and develops an algorithm that constructs the unique equilibrium. Siegel's framework assumes homogeneous prizes; Xiao, in contrast, considers all-pay auctions with risk-neutral players but convex prize series. Fang, Noe, and Strack identify a demoralizing effect of excessive competition—i.e., increasing inequality in prize structure—on contestants' effort supply.

Baye, Kovenock and de Vries (2012) provide a unified framework to characterize two-player complete-information contests with various rank-order-dependent externalities. For instance, under British and Continental rules, the loser in a legal battle is required to bear a portion of the winner's legal expense. Klose and Kovenock (2015a) consider a complete-information all-pay auction with identity-dependent externality, in that one's valuation for losing is endogenous because it depends on which of his competitors wins. They identify a sufficient and necessary condition for the existence of equilibria in the model. Surprisingly, they show that these equilibria are not payoff equivalent, and identical players may earn different expected payoffs; this is in contrast to the conventional wisdom of payoff equivalence in complete-information all-pay auctions. In a relatively less general model with similar identity-dependent externalities, Klose and Kovenock (2015b) investigate how the distribution of preferences on equilibrium behaviors affects the outcome of a contest. Konrad (2006) and Fu and Lu (2013) allow for cross-share holding in all-pay auctions, which can be viewed as a specific form of identity-dependent externalities. Konrad allows one contestant to cross-own shares in other bidders, whereas Fu and Lu allow both contestants to have silent interests in competitors.[4]

Incomplete information is widely assumed in contest studies based on all-pay auction models, such as Amann and Leininger (1996), Glazer and Hassin (1988), Singh and Wittman (2001)[5], Moldovanu and Sela (2001, 2006), Moldovanu, Sela, and Shi (2007, 2012), Minchuk and Sela (2014)[6], Siegel (2014b), Liu, Lu, Wang, and Zhang (2018), among many others. In particular, Glazer and Hassin (1988) pioneered in the research on contest design by allowing the prize series to be a design choice. Moldovanu and Sela (2001) demonstrate that the conventional wisdom of the winner-take-all principle may fail if effort costs are convex. Moldovanu and Sela (2006) take a further step by allowing the designer to decide not only the division of prize money, but also whether to embed a two-stage architecture into the contest, i.e., splitting contestants into multiple subcontests and letting the winners of these subcontests compete in a finale. They show that the optimum depends on the design objectives, number of contestants, and shape of the effort cost functions. Moldovanu, Sela, and Shi (2007) study the optimal distribution of status categories in an environment in which contestants value status as an intangible prize. Moldovanu and Sela (2001, 2006) assume that a fixed prize purse can be split into a series of positive prizes. In contrast, Thomas and Wang (2013) assume a single fixed prize for the winner and allow the designer to set a punishment for the lowest performer. Using a mechanism-design approach, Liu, Lu, Wang, and Zhang characterize the optimal prize allocation rule with punishment for a designer with a fixed budget. The studies of Moldovanu, Sela, and Shi (2012) and Thomas and Wang (2013) assume that punishment is costly; in contrast, Liu, Lu, Wang, and Zhang allow the designer to top up her prize purse by the revenue collected through the punishment. This feature leads to almost full surplus extraction in the optimum even from contestants with independent private types and no limited liability. This is intriguing, because full surplus extraction in an adverse-selection setting usually requires correlated agents' types.

Olszewski and Siegel (2016) develop a framework of large contests that allows for many heterogeneous players and prizes, with both complete and incomplete information. Olszewski and Siegel (2018a) continue the research stream of contest design initiated by Moldovanu and Sela (2001) and show that numerous heterogeneous prizes can be optimal in a large-contest framework when contestants are risk averse with convex costs. Olszewski and Siegel (2018b) extend the framework by relaxing the assumption of a strict single-crossing condition on contestants' utility functions. Olszewski and Siegel (2018c) impose caps on contestants' bids and study the effect of rigid and flexible bid caps on equilibrium behavior. Bodoh-Creed and Hickman (2018) and Olszewski and Siegel (2018d) apply the framework to model college admissions.

Rank-order Tournaments with Additive Noise

The outcome of an all-pay auction is determined deterministically, as a slight margin allows a contestant to secure a sure win. In reality, the outcome of a contest often depends not only on contestants' effort entries, but also random perturbations. Consider, for instance, measurement errors, principals' subjective biases in performance evaluation, or random factors inherent to production processes. There are two typical and convenient ways to model contests with a noisy winner-selection mechanism.

Lazear and Rosen (1981) propose a rank-order tournament with additive noise, which is adopted and extended by a number of studies, such as Green and Stokey (1983) and Nalebuff and Stiglitz (1983). Let each contestant i exert an effort x_i, which allows him to produce an effective output y_i, with

$$y_i = f_i(x_i) + \varepsilon_i.$$

The function $f_i(x_i)$ is a deterministic output and increases with the contestant's effort; ε_i is a random shock, which is often assumed to be identically and independently distributed across contestants. Contestants are ordered by their effective output y_i, and the one with the highest output wins.

When only two players are involved in the competition, the condition for player 1 to win can be written as

$$\begin{aligned} f_1(x_1) + \varepsilon_1 &> f_2(x_2) + \varepsilon_2 \Leftrightarrow \\ \varepsilon_2 - \varepsilon_1 &< f_1(x_1) - f_2(x_2). \end{aligned}$$

Let the noise term $(\varepsilon_2 - \varepsilon_1)$ be distributed with a cumulative distribution function $G(\cdot)$. The probability that contestant 1 wins is thus given by

$$\begin{aligned} P_1 &= G(\varepsilon_2 - \varepsilon_1) \\ &= G\big(f_1(x_1) - f_2(x_2)\big), \end{aligned}$$

which gives the well-known probit winning probability specification (Dixit, 1987).

This modelling framework has been applied in a number of studies. Krishna and Morgan (1998) explore the optimal prize structure in small tournaments with sequential elimination and establish a winner-take-all principle in this context. In contrast to Lazear and Rosen (1981), they impose limited liability constraints, which allow them to demonstrate that optimal tournaments do not necessarily induce first-best outcomes. Hvide (2002) studies contestants' incentive for risk taking in tournaments. Aoyagi (2010) and Ederer (2010) consider dynamic versions of the model and investigate optimal interim feedback policies in this framework. Akerlof and Holden (2012) allow a block of prizes to be given away based on contestants' rankings, explore the optimal prize structure, and show that the optimum could involve punishment imposed on bottom-ranked contestants. Drugov and Ryvkin (2018) explore how the distribution of additive shocks affects the optimal prize structure. They show that a winner-take-all tournament remains optimal with light-tailed shocks, whereas prize sharing that reduces inequality can be more appealing in the presence of heavy tails in the distribution of the shocks. Balafoutas, Dutcher, Lindner, and Ryvkin (2017) investigate the optimal prize allocation in tournaments of heterogeneous players. They show that a loser-prize tournament that awards a low prize to bottom performers can be optimal. Miklós-Thal and Ullrich (2015) study the relationship between belief precision and contestants' effort incentives. They show that in a tournament, a more precise belief does not necessarily reduce effort, as is the case in standard career-concern models (Holmström, 1982, 1999), and an intermediate-level precision could maximally incentivize effort supply.

In parallel to Gelder, Kovenock, and Roberson (2016), who study all-pay auctions with ties, Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), Eden (2007), and Imhof and Kräkel (2014, 2016) study rank-order tournaments in which one can win only if he outperforms his opponents by a minimum margin.

Contests with Ratio-form Success Functions

Perhaps the most widely adopted modelling approach that allows for a noisy winner-selection mechanism is the one that assumes a ratio-form contest success function, with the Tullock contest model as its most popular special case. In a winner-take-all contest with a ratio-form contest success function, the likelihood that a contestant i wins, P_i, is given by the ratio of the output of his effort to the total output supplied by the entire cohort

$$P_i = \begin{cases} \frac{g_i(x_i)}{\sum_{j=1}^{n} g_j(x_j)} & \text{if } \sum_{j=1}^{n} g_j(x_j) > 0; \\ \frac{1}{n} & \text{if } \sum_{j=1}^{n} g_j(x_j) = 0, \end{cases}$$

where the output production function $g_i(x_i)$ is usually an increasing function of effort x_i and is conventionally called the *impact function*.

Assume that all contestants bear a linear effort cost function $c(x_i) = x_i$. This framework provides an intuitive and tractable specification for the winning probability as a function of effort in winner-take-all imperfectly discriminatory contests. Tullock (1980) assumes $g_i(x_i) = x_i^r$, with $r > 0$, and applies the model to rent-seeking competitions, and the model under this specification is called Tullock contests. A standard *lottery contest*, as a popular variant in this family of models, would result when r is set to one. The parameter r is viewed as a convenient and intuitive measure for the level of noisiness in the winner-selection mechanism. A large r implies a higher marginal return for effort and that factors other than effort count less in selecting the winner. As a result, in a symmetric contest, a contestant tends to be compelled to increase his bid when r increases. The properties of the equilibrium in Tullock contests are thoroughly studied by Cornes and Hartley (2005). More generally, suppose that the impact functions $g_i(x_i)$ are strictly increasing, concave, and twice differentiable—i.e., $g_i'(x_i) > 0$ and $g_i''(x_i) \leq 0$—and $g_i(0) \geq 0$. Assuming $g_i(0) = 0$, Szidarovszky and Okuguchi (1997) demonstrate that a unique Nash equilibrium in pure strategy exists in the game. Fu and Wu (2018a) show that the existence and uniqueness remain when $g_i(0) > 0$.

It is well known in the literature that a standard symmetric Tullock contest game—i.e., with $g_i(x_i) = x_i^r$— yields a unique Nash equilibrium in pure strategy if and only if the contest is not excessively discriminatory, i.e., $r \leq \frac{n}{n-1}$. When r goes to infinity, the contest converges to an all-pay auction, as an infinitesimal difference in bid would determine the outcome of the contest.

However, the literature has produced limited knowledge about the equilibria of contests when the discriminatory power is at an intermediate level, i.e., for $r \in (\frac{n}{n-1}, \infty)$ in Tullock contests, in which case pure-strategy equilibria no longer exist. Baye, Kovenock, and de Vries (1994) verify the existence of a symmetric mixed-strategy equilibrium that fully dissipates the rent when strategy space is discrete. However, a closed-form solution to the equilibrium is unavailable. Alcade and Dahm (2010) demonstrate that in many such contests an equilibrium exists, which resembles that in a standard all-pay auction. Such an equilibrium is thus called an all-pay auction equilibrium. Ewerhart (2017a) shows that other payoff-nonequivalent equilibria could also exist in a wide variety of such contests. Ewerhart (2015) further explores the structural properties of equilibria in such contests. In Tullock contests with an intermediate level of r, it is shown that in the mixed-strategy equilibria, the probability mass is "concentrated on countably infinitely many mass points, which form a discrete set in strategy space." Such equilibria differ fundamentally from both the pure-strategy equilibrium when r is small and the continuous mixed-strategy equilibrium in all-pay auctions. Ewerhart (2017b) and Feng and Lu (2017) establish the uniqueness of the equilibrium in a two-player asymmetric Tullock contest when r is between 1 and 2.

Technically, a contest with nonlinear impact functions and linear effort costs is equivalent to a standard lottery contest—i.e., $g_i(x_i) = x_i$—with nonlinear effort cost function. Ryvkin (2013) considers such a setting with convex costs and shows that a more dispersed distribution of contestants' effort cost parameters can either increase or decrease efforts, depending on the curvature of the effort cost function.

A typical Tullock contest assumes players' winning odds add to one, and a single winner is always picked as the prize recipient. A handful of studies propose variants of the conventional models by allowing for draws/ties, including Nti (1997), Blavatskyy (2010), Jia (2012), Jia, Skaperdas, and Vaidya (2013), Vesperoni and Yıldızparlak (2019), and Deng, Wang, and Wu (2018), among many others.

Multiple-prize Extensions

The ratio-form contest success function can readily be applied to modelling winner-take-all competitions. However, it does not directly address the widely observed multi-prize contests, in which case more than one prize is available to contestants. Consider, for instance, that firms set aside a number of bonus packages to reward their top-performing workers, employees may strive to fill multiple vacancies on a higher rung of an organizational hierarchy, and silver and bronze medals are typically awarded to runners-up in sporting events.

To fill this gap, Clark and Riis (1996b, 1998a) provide an intuitive "generalization" of the basic Tullock contest model that allows a block of prizes to be given away: multi-winner nested contests. A formal description follows.

Let $\mathcal{C}(n, m)$ denote a static contest with $n \geq 2$ participants and $m \in \{1, \ldots, n\}$ prizes to be given away. Clark and Riis (1996b, 1998a) conveniently depict the contest as a sequential lottery process that consists of a series of independent draws. Let contestants be indexed by $i \in \{1, \ldots, n\}$. They simultaneously submit their effort entries x_i. Once a contestant is picked as the recipient of a prize, he is immediately removed from the pool of contestants eligible for the rest of the prizes, as each contestant is eligible for at most one prize. Let Ω^k be the set of contestants who remain eligible for the kth-draw and $\mathbf{x}^{\Omega^k}$ be the effort profile of all contestants in the set Ω^k, with $k \in \{1, \ldots, m\}$. Then the probability of a contestant i's receiving the kth prize *conditional* on his not having been picked in the previous $m-1$ draws is given by

$$p_i^k\left(\mathbf{x}^{\Omega^k}; \Omega^k\right) {}^{7}= \begin{cases} \frac{(x_i)^r}{\sum_{j \in \Omega^k}(x_j)^r}\ , & \text{if} \quad \mathbf{x}^{\Omega^k} \neq \mathbf{0}\ , \\ \frac{1}{n-k+1}\ , & \text{if} \quad \mathbf{x}^{\Omega^k} = \mathbf{0}\ . \end{cases}$$

The process continues until all m prize winners have been drawn. Obviously, the usual winner-take-all Tullock contest is a special case of the multi-winner nested model, with $m = 1$.

By adopting ratio-form success functions as its building block, this winner-selection mechanism hypothetically conducts a series of conditionally independent (single-winner) "lotteries." As a result, the conditional probability that a remaining contestant will be selected in a draw does not depend on the effort exerted by contestants selected in previous draws. It should be noted, however, that the contest remains a simultaneous game despite the "sequential" prize allocation process, as players exert one-shot efforts. Thus far, the nested contest model offers the most popular and convenient alternative for determining multiple prize recipients in imperfectly discriminatory contests.[8] Further, Lu and Wang (2015) provide an axiomatic foundation for the model.

Based on the multi-winner nested contest model, Schweinzer and Segev (2012) establish the conditions under which the winner-take-all principle applies. In contrast to Schweinzer and Segev, who assume homogeneous contestants, Szymanski and Valletti (2005) allow for heterogeneity among contestants and demonstrate that a runner-up prize can motivate underdogs and therefore stimulate competition in the contest.

Berry (1993) suggests another multi-winner contest model based on a ratio-form success function. Consider an n-player contest with m identical prizes. Set the discriminatory power r to one. The probability of one subset of contestants' being chosen to each receive one prize is given by the ratio of the sum of these contestants' efforts to that of the entire pool of competitors.[9]

The two prize-allocation mechanisms generate contrasting implications for contest design. Fu and Lu (2009) consider the following problem. Let there be n homogeneous contestants and $m \leq n$ prizes. The contest designer can either run a grand contest that pools all contestants and lets them compete for the entire set of prizes, or she can create a set of $k \geq 2$ subcontests: Each subcontest i distributes m_i prizes among n_i contestants, with $\sum_{i=1}^{k} m_i = m$, $\sum_{i=1}^{k} n_i = n$ and $m_i \leq n_i$, $i \in \{1, \ldots, k\}$. Assuming a multi-winner nested contest model (Clark and Riis, 1996b, 1998a), Fu and Lu show that a grand contest always generates more effort, regardless of how subcontests are constructed. The opposite, however, was obtained by Chowdhury and Kim (2017) when Berry's (1993) approach is adopted.

Economic Foundations

The popularity of a ratio-form contest success function has catalyzed a large research effort to uncover its economic foundations. There are two major perspectives on this issue: an axiomatic approach and a noisy ranking approach. Our discussion mainly focuses on these approaches.

However, notable alternative approaches can be found in the contributions of Corchón and Dahm (2010, 2011). Corchón and Dahm (2011) show that many popular contest success functions can be derived as an optimal choice of the contest organizer with some appropriately chosen objective functions, given contestants' effort profile. In particular, the ratio-form success function can be rationalized when the contest organizer is a non-expected utility maximizer whose preference is delineated by prospect theory. Corchón and Dahm (2010) interpret contest success functions as sharing rules, and establish a connection to Nash bargaining and claims problems. They show that the ratio-form contest success function (sharing rule) maximizes the n-player Nash product $\prod_{i=1}^{n} p_i^{x_i}$, where $x_i \geq 0$ is contestant

i's effort level.

Axiomatic Foundation The seminal study of Skaperdas (1996) shows that a ratio-form contest function, i.e.,

$$P_i = \begin{cases} \frac{g(x_i)}{\sum_{j=1}^{n} g(x_j)} & \text{if } \sum_{j=1}^{n} g(x_j) > 0; \\ \frac{1}{n} & \text{if } \sum_{j=1}^{n} g(x_j) = 0, \end{cases}$$

where $g(\cdot)$ is a positive increasing function of its argument, is the only alternative that satisfies a series of axiomatic properties. First, a positive effort leads to one's positive winning probability. Second, an increase in one's effort leads to an increase in his winning odds, while reducing those of his opponents. Third, the contest is *anonymous* in the sense that one's winning probability depends only on his effort and not his identity. Fourth, the contest is *consistent* in the sense that the contest remains qualitatively similar when additional players enter. Fifth, the probability of one's winning in the competition among a subset of the n contestants does not depend on the effort entries of those who are not included in the subset, which is commonly labeled as the *independence of irrelevant alternatives* (IIA) property. Further, it is shown that impact function $g(x_i) = \alpha x_i^r$, $\alpha > 0, r > 0$, is the only continuous function that satisfies the homogeneity of degree zero property, i.e., the distribution of winning probabilities remains the same when all contestants' efforts are scaled up or down by a given percentage. This paper also formally links the contest success function to the probabilistic choice models in social science (Luce and Suppes, 1965; Suppes, Krantz, Luce, and Tversky, 1989).

Clark and Riis (1998c) provide a generalization of Skaperdas (1996) by axiomatizing asymmetric Tullock contest success function, i.e.,

$$P_i = \begin{cases} \frac{\alpha_i x_i^r}{\sum_{j=1}^{n} \alpha_j x_j^r} & \text{if } \sum_{j=1}^{n} \alpha_j x_j^r > 0; \\ \frac{1}{n} & \text{if } \sum_{j=1}^{n} \alpha_j x_j^r = 0. \end{cases}$$

Rai and Sarin (2009) and Arbatskaya and Mialon (2010) provide axiomatic foundations for contest success functions when contestants invest in multiple activities. Münster (2009) axiomatizes contest success functions when contestants compete in groups. Blavatskyy (2010)[10], Jia (2012), Jia, Skaperdas, and Vaidya (2013), and Vesperoni and Yıldızparlak (2019) axiomatize Tullock contests with the possibility of draws/ties. Bozbay and Vesperoni (2018) axiomatize a success function that introduces networks to contests.

Noisy Ranking Approach Konrad (2009) points out in his thorough survey of economic studies on contests that a contest can naturally be regarded as a competitive event in which contestants expend costly effort to "get ahead of their rivals." We demonstrate that there exists a unique noisy ranking system that underpins ratio-form contest success functions.

The usual axiomatic foundation—i.e., Skaperdas (1996) and Clark and Riis (1996a, 1998c)—alludes to the hidden connection between this class of contest models, the probabilistic choice models (Luce and Suppes, 1965), and the discrete choice econometric models (McFadden, 1973, 1974).[11]

Let $n \geq 2$ contestants, indexed by $i \in \mathbf{N} \triangleq \{1, 2, ..., n\}$, simultaneously commit to their effort $\mathbf{x} = (x_1, \ldots x_n)$, to compete for $l \in \{1, 2, ..., n\}$ prizes. A contestant's effort allows him to produce a perceivable output y_i, which is subject to random perturbation. Contestants are evaluated through the noisy signals of their performance y_i. Following the discrete choice framework of McFadden (1973, 1974),[12] noisy signal y_i is assumed to be described by

$$\log y_i = \log g_i(x_i) + \varepsilon_i, \quad \forall i \in \mathbf{N}, \tag{1}$$

where the deterministic and strictly increasing function $g_i(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ measures the output of contestant i's effort x_i,[13] and the additive noise term ε_i reflects the randomness in the production process or the imperfection of the measurement and evaluation process. $g_i(\cdot)$ is the production function of contestant i. Define $\mathbf{g} \triangleq \{g_i(\cdot), i \in \mathbf{N}\}$, which denotes the set of production technologies. Idiosyncratic noise terms $\boldsymbol{\varepsilon} \triangleq \{\varepsilon_i, i \in \mathbf{N}\}$ are independently and identically distributed.

The l prizes are ordered by their values, with $v_1 \geq v_2 \geq \ldots \geq v_l$. Each contestant is eligible for at most one prize. These contestants are ranked based on their perceivable performance (i.e., y_i) in descending order. Prizes are allocated among contestants based on their ranks, given the availability of the prizes. That is, the contestant who contributes the highest output y_i receives v_1, the contestant who contributes the second highest output receives v_2, and so on until all prizes have been awarded.

For any given effort entries $\mathbf{x}$**,** a complete ranking among contestants immediately results from any realization of the noise terms $\boldsymbol{\varepsilon}$. Assume that a tie is broken randomly and fairly. The probability of a contestant i winning prize v_l is simply given by the probability that he is ranked at the l-th position.

As in McFadden (1973, 1974), the random component ε_i is assumed to be drawn from a type I extreme-value (maximum) distribution. The cumulative distribution function of ε_i is

$$\Psi(\varepsilon_i) = e^{-e^{-\varepsilon_i}}, \varepsilon_i \in (-\infty, +\infty), \quad \forall i \in \mathbf{N}, \tag{2}$$

and the corresponding density function is

$$\psi(\varepsilon_i) = e^{-\varepsilon_i - e^{-\varepsilon_i}}, \varepsilon_i \in (-\infty, +\infty), \ \forall i \in \mathbf{N}. \tag{3}$$

As shown by McFadden (1973, 1974), for any given $\mathbf{x} \geq 0$ such that $\sum_{j \in \mathbf{N}} g_j(x_j) > 0$, the ex ante likelihood that a contestant i achieves the top rank is

$$\Pr\left(y_i = \max\{\mathbf{y}\} \middle| \mathbf{x}\right) = \frac{g_i(x_i)}{\sum_{j \in \mathbf{N}} g_j(x_j)}, \quad \forall i \in \mathbf{N}. \tag{4}$$

This winning probability coincides with the popularly assumed ratio-form contest success function in winner-take-all contests. Both Clark and Riis (1996a) and Jia (2008) point out this microeconomic underpinning of ratio-form contest success functions.

Fu and Lu (2012b) consider a multi-prize contest, i.e., $n \geq l \geq 2$. Let the sequence $\{i_k\}_{k=1}^{n}$ denote a complete ranking of the n contestants, where i_k is the index of the kth ranked contestant. For any given effort entries $\mathbf{x} \geq 0$ such that $g_i(x_i) > 0, \forall i \in \mathbf{N}$, the term $g_{i_k}(x_{i_k}) \Big/ \sum_{k'=k}^{n} g_{i_{k'}}(x_{i_{k'}})$ gives the probability of a contestant i_k being ranked in the kth place, conditional on his rank falling below $k-1$.[14] As a result, the ex ante likelihood of any complete ranking outcome $\{i_k\}_{k=1}^{n}$ can be expressed as

$$\Pr\left(\{i_k\}_{k=1}^{n}\right) = \prod_{k=1}^{n} \frac{g_{i_k}(x_{i_k})}{\sum\limits_{k'=k}^{n} g_{i_{k'}}(x_{i_{k'}})}. \tag{5}$$

A stochastic equivalence is thus established between this noisy ranking model and the multi-winner nested contest (Clark and Riis, 1996b, 1998a). As summarized above, this nested model literally implements a sequential lottery process. Contestants simultaneously submit their one-shot effort entries. The recipient of each prize is selected through a "lottery" among all remaining eligible candidates, with each "lottery" represented by a ratio-form contest success function. Assuming $g_i(x_i) = x_i^r$, a contestant i, provided that he has not been picked in the previous $k-1$ draws, is selected to receive the kth prize with a probability $(x_i)^r / \sum_{j \in \Omega^k}(x_j)^r$, where Ω^k is the set of contestants who have not been selected and remain eligible for the kth prize.

The (unique) equivalence illuminates the stochastic nature of the multi-winner nested contest model. More importantly, the nested contest is reduced to a standard winner-take-all contest when only one prize is available. From the perspective of noisy rankings, it is clear that the multiple-winner nested contest model and usual winner-take-all contest with ratio-form contest success function are integrated into a unified framework through a unique underlying ranking system.

Based on the statistical nature of the distribution of ε, Fu and Lu (2012b) label the ranking system a "*favorable performance ranking*": It honors the most favorable performance of each contestant, i.e., the "best shot" of each contestant within multiple independent realizations that follow *any* given distribution. Contests that are underpinned by such a ranking system are then interpreted as "best-shot" contests, which uniquely generate winning probabilities that are identical to those of a multiple-winner nested contest model (Clark and Riis, 1996b, 1998a).

Baye and Hoppe (2003) reveal that research tournament models (Fullerton and McAfee, 1999) and patent race models (Loury, 1979; Dasgupta and Stiglitz, 1980) are strategically equivalent to a winner-take-all Tullock contest. Fu and Lu (2012b) further demonstrate that both the research tournament model and the patent race model are underpinned by such a ranking system and fall in the category of best-shot contests.

The survey next elaborates on the numerous variations that have been introduced into contest models to depict competitions in various scenarios.

Contests with Dynamics

The standard contest models assume one-shot interactions: Contestants submit their effort entries; a selection mechanism maps their bids into winning probabilities; and prizes are distributed accordingly. Many competitions, however, involve dynamics, and the interactions cannot be encapsulated in a simple Cournot-Nash structure. Success requires more than a single stroke of effort but rather continuous input, and the outcome of the contest is determined by contestants' aggregate performance during the entire series. There are five main categories of dynamic contest models: (1) sequential-move contests, (2) contests in which contestants can add to their previous effort entries, (3) sequential-elimination contests, (4) contests with inter-period spillovers, and (5) dynamic multi-battle contests. This part focuses on (1)-(4), and (5) will be discussed separately, together with simultaneous multi-battle contests.[15]

Sequential Moves in Contests

Dixit (1987) is the first to introduce Stackelberg structure to contest models that allow contestants to commit to their efforts sequentially. He identifies the condition under which one benefits as the first mover, since he can strategically affect late mover's behavior by his action. Konrad and Leininger (2007) consider a complete-information all-pay auction with general cost functions. They assume that some contestants make their moves earlier than the rest. It is shown that only the contestant with the lowest cost receives a positive payoff. Segev and Sela (2014) characterize the unique perfect Bayesian equilibrium of a sequential-move all-pay auction with privately known cost parameters. They demonstrate that a head start awarded to the first mover can affect the performance of the contest. Kahana and Klunover (2018) verify the existence of a unique subgame perfect equilibrium in a sequential lottery contest with multiple players. Klunover (2018) further shows that sequential moves lead to higher rent dissipation when more than two players are involved.

There has been a continuous effort to endogenize contestants' timing of moves. Baik and Shogren (1992) and Leininger (1993) study a three-period model. Contestants can sink their efforts in either period 1 or period 2, and a winner is selected after both have committed to their effort entires. Prior to the contest, they simultaneously commit to the timing of their moves in the subsequent contest. The resulting contest can be either simultaneous or sequential. It is shown that when contestants are asymmetric in terms of their valuations, the underdog always chooses an early timing, while the favorite chooses to be the late mover. In an all-pay auction setting, Konrad and Leininger (2007) demonstrate that the strongest contestant, i.e., the one with the lowest cost, always chooses the later timing. In the following subgame of the contest, he wins the prize with zero aggregate cost.

While asymmetry leads to the sequential-move structure in the contests in the studies of Baik and Shogren (1992), Leininger (1993) and Konrad, and Leininger (2007), Morgan (2003) demonstrates that a sequential-move contest can emerge in the equilibrium even between ex ante symmetric players. He assumes that a contestant's prize valuations follow a common two-point distribution, with one's valuation being either high or low. Their valuations are realized at the beginning of period 1 and will be commonly known. It is shown that contestants' timing choices in period 0 still lead to a sequential-move contest. These studies assume complete information in the contest. Fu (2006a) allows for information asymmetry. The object in dispute has a common value to both players. But one is informed, and the other knows only its distribution. Fu shows that in the unique equilibrium, the uninformed player has a dominant strategy to perform in period 1, while the informed one's best response is to perform in period 2. Ludwig (2012) considers a model of two ex ante symmetric players and compares four institutional settings, under which players move either sequentially or simultaneously, and their types can be either public or private information. It is shown that sequential moves are not only ex ante preferred by the designer, but also Pareto dominant.

The observations of endogenous sequential moves challenge the usual Cournot assumption in the contest literature. Morgan and Värdy (2007) argue that these results critically rely on the assumption that a contestant's early move (effort choice) is perfectly observable. Assuming a contest with a ratio-form contest success function, they show that the value of commitment vanishes entirely when the second mover has to bear a cost to observe the first mover's action. As a result, the prediction obtained in the Nash equilibrium of a standard simultaneous contest is restored despite players' sequential moves. It is further shown that in a rank-order tournament model (Lazear and Rosen, 1981), the value of commitment can be preserved if the second mover can pay a small cost to observe the first mover's realized performance.

Hinnosaar (2018) considers a model in which n contestants sequentially arrive in the competition and exert efforts to vie for a prize. The game proceeds in T periods; contestants are thus partitioned into T groups based on the time they arrive. One observes the effort choices of those who arrive in earlier periods, but cannot observe those of his peers. He fully characterizes the equilibria. Further, he demonstrates that efforts increase when contestants observe previous contestants' actions. As a result, when a designer can choose the amount of information about past plays, the optimal policy must be either to fully disclose it or completely conceal it, depending on the desirability of efforts.

Multi-stage Contests with Addable Efforts

Players' competitive actions remain one shot in the models that have been discussed in this section. In another strand of literature, the contest lasts multiple rounds, and contestants are given the flexibility to add to their early effort entries in later rounds. The contest is resolved based on players' aggregate efforts over the multiple rounds. Such a situation is not uncommon in reality; consider an R&D race in which the competition takes place over time, and firms accumulate their input continuously.

Yildirim (2005) considers a two-player two-period model. Let x_i^t denote a player i's effort sunk in period t, with $i \in \{1, 2\}$ and $t \in \{1, 2\}$. A player wins the contest with a probability

$$P_i = \frac{f(X_i)}{f(X_1) + f(X_2)}, i \in \{1, 2\},$$

where $X_i := \sum_{t=1}^{2} x_i^t$ is the player's aggregate effort across two periods. Arbatskaya and Mialon (2012) consider a similar setup in which efforts do not simply add up, but compound over time. Specifically, they employ the Cobb-Douglas-type contest success function axiomatized by Arbatskaya and Mialon (2010) and assume $f(x_i^1, x_i^2) = \prod_{t=1}^{2} (x_i^t)^{\alpha^t}$, with $\alpha^t > 0$. They show that effort expenditures are lower in the sequential contest than in the simultaneous contest. Hirata (2014) considers a similar game, but he assumes that the contest is resolved through an all-pay auction. Joffrion and Parreiras (2013) adopt a noisy rank-order tournament model. They show that with strictly convex effort cost functions, the early frontrunner tends to slack off in the late stage.

All these studies consider contests between two individuals. Fu and Lu (2018), in contrast, consider a dynamic contest between two teams. The contest lasts two periods. Each team consists of two players, and each performs in one period. In each period, two players from rival teams simultaneously exert their efforts, and early performance is revealed before late players commit to their efforts. The two players' effort is summed for each team, and the team with the higher aggregate effort wins the prize, which is a public good to its players. Assuming that players on a team are heterogeneous, with one stronger and the other weaker, Fu and Lu allow the planner for each team to decide on the placement of her players—i.e., in the early position or the late position—before the contest begins. It is shown that when planners aim to maximize their teams' winning odds, they must assign the stronger players to the late leg for nonexcessive intra-team heterogeneity.

In Yildirim (2005), Hirata (2014), and Fu and Lu (2018), players' early performance is assumed to be commonly known. A handful of studies view the observability of early performance to contenders as a structural element of the contest and endogenize it as a designer's choice. Midterm reviews, for instance, are often provided to junior faculty members at the halfway point on their tenure clocks. It remains unclear whether a contest designer should provide feedback on contenders' interim performance.

Aoyagi (2010) addresses this issue in a two-player two-period rank-order tournament model (Lazear and Rosen, 1981). In each period $t, t \in \{1, 2\}$, players simultaneously exert their efforts $x_i^t, i \in \{1, 2\}$. His output z_i^t is a random variable, with $z_i^t = x_i^t + \varepsilon_i^t$, where ε_i^t is a noise term. The player with higher total aggregate output $z_i^1 + z_i^2$ wins the prize. At the end of period 1, the designer observes the performance differential between players, i.e., $z_1^1 - z_2^1$; she decides whether to provide feedback to contenders about their standings before the contest begins. Aoyagi shows that the optimal feedback policy that maximizes total effort in the contest largely depends on the shapes of contenders' effort functions.

Ederer (2010) identifies novel effects caused by interim performance feedback in an alternative setting. He assumes that a contestant's output in period t depends not only on his effort and random noise, but also his innate ability, i.e., $z_i^t = f(a_i, x_i^t) + \varepsilon_i^t$, where a_i measures the contestant's ability and is ex ante unknown to both players. When one's ability and his effort are complementary—i.e., the production function $f(\cdot, \cdot)$ takes the form of $f(a_i, x_i^t) = a_i x_i^t$—contestants, upon receiving interim feedback, can partially infer their own abilities and the differential between themselves and the other players. This setting differs substantially from one in which effort and ability are substitutes, i.e., $f(a_i, x_i^t) = a_i + x_i^t$: When ability enters additively in contestants' productions, the marginal benefit of effort is unaffected by the information acquired by a contestant; however, with complementarity, one expects a higher level of marginal output when he perceives himself as a more capable contender. Hence, interim feedback is a double-edged sword, because a favorable feedback to a contestant incentivizes him and discourages his opponent. The optimum depends on the distribution of the noise term and the property of the effort cost function.

Gershkov and Perry (2009) consider a different design problem. In addition to deciding whether to conduct a midterm review or review contestants' performance only at the end of the second period, they let the designer choose how to aggregate the outcomes of the two reviews in determining the final ranking when a midterm review is conducted.

In the model of Goltsman and Mukherjee (2011), a contestant performs a task in each period, and its outcome can be either a success or a failure, which can be observed by neither contestant; the contestant who achieves a larger number of successes wins a prize. This is in contrast to the usual setting in which performance is measured and aggregated continuously; Goltsman and Mukherjee demonstrate that partial disclosure can emerge as the optimum.

Sequential-elimination Contests

Many competitive events have a hierarchical structure. The contest proceeds in multiple stages, and contestants are eliminated successively along the hierarchical ladder; they compete for advancement in the early stages, and often do not receive an actual reward until the finale. Such contests can be intuitively exemplified by NBA playoffs. Alternatively, consider firms' succession planning processes. For instance, when GE searched for a successor for Jack Welch, 23 candidates were initially identified. The slate was then narrowed to eight, and finally to three.

The literature has long recognized such hierarchical features common to many contest-like situations. This strand of literature dates back to Rosen (1986). Assuming a fixed hierarchical contest structure, Rosen explores the incentive effect of the prevailing reward structure in the dynamics, and argues that a disproportionate share of the prize money should be concentrated at the top of the hierarchy. Krishna and Morgan (1998) provide a rationale for the winner-take-all principle in small two-stage tournament models: In the optimum, the entire prize money is allocated to the top prize in the finale.

The hierarchical structure of a sequential elimination contest is endogenized as a designer's choice by Gradstein and Konrad (1999) and Moldovanu and Sela (2006). The former considers a setting with 2^N homogeneous players. The remaining contestants in each stage are divided and matched into different groups, with one surviving the competition in a group—which is modeled as a Tullock contest—and advancing to the next stage. Gradstein and Konrad let the designer choose how many stages the contest should last when maximizing the total effort of the contest. It is shown that an N-stage contest is optimal when the discriminatory power of the contest success function is less than one, while a static winner-take-all contest emerges as the optimum when it exceeds one. Assuming an all-pay auction as the winner-selection mechanism and allowing for incomplete information, Moldovanu and Sela explore whether a contest should involve a preliminary stage that selects finalists.

In the studies of Gradstein and Konrad (1999) and Moldovanu and Sela (2006), a set of parallel subcontests take place in the preliminary stages and a single winner is selected from each subcontest for advancement, with each subcontest being a winner-take-all competition. In contrast, Fu and Lu (2012a) assume a "pooling" approach to model the elimination process: The remaining contestants in each stage compete against all others. The competition in each preliminary stage is modeled as a multi-prize contest (Clark and Riis, 1996b and 1998a). Fu and Lu explore both the optimal hierarchical structure of the contest and the optimal prize allocation rule. It is shown that with N contestants, the optimal contest lasts for $N - 1$ stages, with each stage eliminating one contestant, and concentrates the entire prize purse on the top prize in the finale.

In a two-stage model, Amegashie (2000) compares the two ways to select finalists: *pooling*

(Fu and Lu, 2012a) and *splitting and matching* (Gradstein and Konrad, 1999; Moldovanu and Sela, 2006). He shows that the former can elicit a higher level of total effort. Cohen, Maor, and Sela (2018) also assume a pooling approach, but consider a novel design issue. In a two-stage elimination contest, the designer can choose to favor the finalist who is top ranked in the first stage.

In all of these studies, contestants are fully informed of interim results—i.e., elimination or survival—before they sink their efforts to compete for further advancement. Fu and Wu (2018b) relax this assumption and view the observability of interim results as a design variable: The designer decides whether to disclose it to contestants before the contest begins. It is shown that the optimum depends on the designer's objective function. Further, Fu and Wu consider a design problem that generalizes that of Cohen, Maor, and Sela. The designer is given full flexibility to assign individualized weights to all contestants' second-stage effort entries based on their early rankings. As a result, the structure of the two-stage contest is also endogenized: The number of finalists is chosen endogenously, because eliminating a contestant is equivalent to imposing a very small weight on his second-stage effort entry.

Zhang and Wang (2009) consider a different issue regarding optimal disclosure policy in a two-stage model with contestants' prize valuations being privately known. The designer chooses whether to disclose contestants' bids in the preliminary stages, which could reveal their underlying types and change the information structure of the second-stage competitions.

The models discussed in the subsection so far assume homogeneous or ex ante homogeneous contestants. Groh, Moldovanu, Sela, and Sunde (2012) consider a game of four heterogeneous players. They explore the optimal seeding rule that sorts contestants into two groups for elimination in the preliminary stage. In a two-stage model, Mendel, Pieroth, and Seel (2018) also assume a pooling approach, but—in contrast to Fu and Lu (2012a)—allow for heterogeneous players and model the competition in each stage as an all-pay auction.

Contests with Inter-period Spillovers

One strand of literature studies contest series in which a pair of contenders meet head-to-head repeatedly, and the outcomes of or players' behavior in early contests determine the characteristics of late competitions. Sela (2012), for instance, studies a two-stage contest between two contestants. In each stage, a competition takes place in the form of an all-pay auction, and one player wins a prize. One's valuation for the second-stage prize depends on the outcome—i.e., his win or loss—in the first stage. Megidish and Sela (2014) study a similar question, but model each contest in the series as a Tullock contest. Further, they consider the case of resource-constrained contestants. Sela (2017) investigates a setting in which one's valuation for the second-stage prize depends on his first-period effort.

Clark and Nilssen (2013) allow for learning by doing, in the sense that one's early effort helps reduce his late effort cost. Allowing for learning by doing, DeVaro and Gürtler (2016) study a firm's task assignment in a dynamic employment relationship. Kovenock and Roberson (2009), Beviá and Corchón (2013), and Clark and Nilssen (2018a, 2018b) allow the winner of early contests to secure advantage in late competitions.

Information in Contests

Incomplete information or information asymmetry are prevalent in competitive situations. For instance, an athlete in a sporting event knows his own skill level, while he has only a rough notion of how well his opponent has been prepared. An incumbent firm has a more precise estimate of the value of a market, while a new entrant presumably faces more uncertainty. A growing number of studies have incorporated an information element in contest modelling.

Symmetric incomplete information has been assumed by a number of studies based on all-pay auction models, including Amann and Leininger (1996), Moldovanu and Sela (2001, 2006), and Moldovanu, Sela, and Shi (2007, 2012). They typically assume that contestants' valuations for prizes, or their effort costs, are privately known and drawn identically and independently from continuous distributions. In Krishna and Morgan (1997), contestants partially know the value of the object in dispute, and each receive a signal that affects the value of the project. They allow their signals to be affiliated. Siegel (2014b) considers a two-player asymmetric all-pay auction in which each player receives a signal that affects players' valuations for the prize. Players' private signals, however, are drawn from asymmetric joint distributions and affect their valuations asymmetrically. Lu and Parreiras (2017) extend Amann and Leininger (1996) to allow for interdependent values and correlated signals. Kotowski and Li (2014) consider all-pay auctions in which players have two-dimensional private information: Each player receives an interdependent, affiliated signal regarding the item for purchase, and has privately known budget constraints.

Incomplete information has received limited attention in Tullock contest models because of analytical difficulty. The imperfectly discriminatory nature of the winner-selection mechanism substantially complicates the analysis because of the compounded uncertainty. Hurley and Shogren (1998a, 1998b) allow for one-sided asymmetric information in a two-player Tullock contest. Wärneryd (2003) assumes a more general ratio-form contest success function. Wärneryd (2012) further extends his previous work to a multi-player setting, in which some contestants know the value of the prize and the rest are uninformed. In Tullock contest settings, Hurley and Shogren (1998a), Malueg and Yates (2004), and Fey (2008) allow for two-sided incomplete information, assuming that players' types are drawn independently from discrete distributions.

Fey (2008) also verifies the existence of a symmetric Bayesian Nash equilibrium in a case of uniform distribution. Ryvkin (2010a) generalizes the setting of Fey in three important dimensions. First, he allows for a more general ratio-form contest success function, without restricting the impact function to the form of the power function. Second, his setting accommodates more general distributions for contestants' marginal effort costs. Third, the contest may host more than two contestants. The study verifies the existence of a Bayesian Nash equilibrium in the game. In addition, the findings challenge the conventional wisdom obtained in two-player settings, by which private information leads to lower effort supply. Wasser (2013) considers a modified version of a Tullock lottery contest model. In contrast to Ryvkin, he allows contestants' types to be drawn from different distributions. In contests with highly general success functions, Einy, Haimanko, Moreno, Sela, and Shitovitz (2015) establish the existence of a pure-strategy Bayesian Nash equilibrium, assuming that contestants' private information is described by a countable partition. Brookins and Ryvkin (2016) verify equilibrium existence in group contest settings, in which a contestant's effort cost can be unknown, either outside his group or to all other players, inside or outside his group.

Ewerhart and Quartieri (2016) establish that under general conditions, a unique pure-strategy Nash equilibrium exists in contests with ratio-form contest success functions. They consider the setting of finite state spaces and allow for budget constraints. Assuming continuously and independently distributed types, Ewerhart (2014) verifies that a lottery contest yields a unique pure-strategy Bayesian Nash equilibrium with arbitrary forms of convex cost functions.

In view of the important role information can play in impacting contestants' behavior and the performance of the contest, a growing literature focuses on the acquisition and transmission of information. In a two-player all-pay auction model, Morath and Münster (2013) study contenders' incentives to pay a cost to learn about their own valuations for the object in dispute. Kovenock, Morath, and Münster (2015) and Wu and Zheng (2017) consider players' incentives to share their private information regarding prize valuations.

One strand of literature views information disclosure as one part of contest rules, and allows a designer—who observes contestants' types—to decide whether and how to disclose the information to contestants in order to influence their beliefs and behaviors. Fu, Jiao, and Lu (2014) study, in a multi-prize all-pay auction, whether the designer should disclose contestants' abilities—i.e., cost parameters—to contestants. In a two-player Tullock contest, Serena (2017) explores the optimal disclosure policy when the designer can observe contestants' effort cost parameters, and he shows that partial disclosure could outperform full disclosure or full concealment. Lu, Ma, and Wang (2018) extend the Serena's study into an all-pay auction setting and provide a ranking of alternative disclosure policies. All of these studies assume that contestants are ex ante symmetric, and each possesses private information about his own type. Zhang and Zhou (2016) consider a setting with one-sided asymmetric information, with one player's prize valuation being privately known. Using a Bayesian persuasion approach, they show that it is without loss of generality to focus on the comparison between full disclosure and full concealment when the informed contestant's valuation follows a binary distribution.

The section has so far focused on a designer's strategy for disclosing information about contestants' characteristics. Kaplan (2012) considers a procurement tournament in which the buyer's preference is privately known to herself, and explores the buyer's optimal strategy for communicating with bidders about her preference.

Einy, Moreno, and Shitovitz (2017) consider a relatively rarely studied setting that involves incomplete information, but not asymmetric information. In a symmetric Tullock contest, all contenders have state-dependent common value for the prize and the same state-dependent effort cost function. Einy, Moreno, and Shitovitz examine how the level of public information about the realized state affects the performance of the contest. It should be pointed out that a Tullock contest with uncertain common value and deterministic entry is, to a large extent, equivalent to an alternative Tullock contest with deterministic common value and symmetric random entry. As a result, Einy, Moreno, and Shitovitz's study bears a subtle relation to Fu, Jiao, and Lu (2011), who study the optimal disclosure policy of contests with symmetric random entry.

A signaling game arises when a player possesses private information and makes observable sequential moves. Signaling elements are embedded in several studies that include contest modelling blocks. Fu (2008) studies a two-stage litigation game. One party is privately informed of the exact value of the damage, while the other is not. In the first stage, the informed party proposes a settlement offer to split the rent; the game ends if the offer is accepted, and proceeds to a legal battle—modeled as a Tullock contest—if the offer is rejected. The offer can thus be viewed as a signaling device. In a two-stage game, Corchón and Yıldızparlak (2013) let two countries declare war in the first stage. As one country's valuation for the victory is privately known, its decision on declaring war allows the other to update its belief before a war erupts. Fu, Gürtler, and Münster (2013) allow a privately informed player to signal his strength by making a public statement about his confidence to win prior to a contest. Denter, Morgan, and Sisak (2018) consider a setting that involves a new entrant whose talent is privately known and an incumbent whose talent is commonly known. The new entrant can send a costly signal before a contest. In the equilibrium, a high-ability new entrant can either "show off" to reveal his superior talent or mimic his low-ability counterpart. It is shown that showing off can benefit both parties in the competition.

Multi-battle Contests

A multi-battle contest requires that contenders meet each other on multiple battlefields, with a discrete battle taking place on each front, and the party that wins a sufficient number of battles is the ultimate winner. Such competitions can intuitively be exemplified by the usual best-of-k tournaments in various sporting events, with k being the maximum number of component battles in the series. A typical men's singles tennis match adopts a best-of-five format: It consists of a maximum of five sets, and the winner is required to prevail in three. Examples can also be found in politics. For instance, in U.S. presidential primaries, electoral battles are held in a number of states sequentially, and the one who wins the majority secures the nomination as his party's candidate (Klumpp and Polborn, 2006).

These competitions may take place either sequentially or simultaneously. Klumpp and Polborn (2006), for instance, study two versions of multi-battle contests: sequential or simultaneous. Snyder (1989) models an electoral competition as a simultaneous multi-battle contest, with two political parties competing for seats in different constituencies.

Studies of multi-battle contests also differ in terms of winning rules, reward structures, contenders' goals, and the way that battles are linked. Snyder (1989) considers two (typical) scenarios: (1) parties seek to gain control of the legislature, i.e., by winning the majority of seats; (2) parties seek to win more seats in the legislature—i.e., the sum of the winning probabilities over all constituencies. In the former, a single prize is awarded to the contender who prevails in the majority of component battles; in the latter, each component battle has a rent, and a contender maximizes the expected sum of rents he receives from all battlefields. Snyder demonstrates that equilibrium behaviors differ sharply in the two scenarios. Assuming the second goal, Kovenock and Roberson (2009) extend the Snyder's study by repeating the competition in another period and introducing an intertemporal interdependence: A player's win on one battlefield in the first period grants him a head start when competitors meet again on the same front. Kovenock, Sarangi, and Wiser (2015) allow for interdependence between component battles by allowing for "complementarity": A contestant wins the contest if he wins a certain combination of component battles. Lu, Shen, and Wang (2019) consider an alternative multi-battle contest that allows for performance bundling: Two contestants meet each other in a series of competitions, and one receives the prize only if he prevails in all component battles.[16]

Contests between Individual Contenders and the Discouragement Effect

The literature on dynamic multi-battle contests dates to Harris and Vickers (1987), who model the R&D race for an innovative product as a continuous competition on a series of component technologies: A firm must make a sufficient number of advances ahead of its competitors to secure the patent for the final product.

Conventional wisdom holds that a discouragement effect or strategic momentum effect often looms large in dynamic multi-battle contests. Consider two homogeneous players. Suppose that one accumulates a lead in early battles due to pure luck, which further motivates the frontrunner in late battles, and discourages the laggard. This renders the subsequent competition asymmetric despite the ex ante homogeneity of contestants. The discouragement effect or strategic momentum effect are formalized in a number of studies. Malueg and Yates (2010) demonstrate theoretically the distortion caused by an early outcome in a dynamic best-of-three contest with symmetric players and provide strong empirical evidence based on observations from tennis matches. Klumpp and Polborn (2006) provide a rationale—based on the discouragement effect—for the *New Hampshire Effect* in U.S. presidential primaries, which says that outcomes in early states often predict the ultimate winners. Further, they demonstrate that a sequential scheme—e.g., primaries—is a more efficient electoral format than a simultaneous one, as a laggard gives up as the series unfolds. Strumpf (2002) also considers a contest consisting of a series of disjoint battles, but the competitive balance between the two contenders is determined by the specific characteristics of each battlefield. He shows that the outcome of the contest depends on the sequence of these battles; for instance, a presidential candidate could have failed to win his/her nomination if California were scheduled for an early slot instead of New Hampshire or Iowa.

A few studies have identified the various factors that could nullify the discouragement effect. Konrad and Kovenock (2009a) provide a complete characterization of the unique subgame perfect equilibrium for a generalized sequential multi-battle contest between two players. They show that when a component battle awards a positive intermediate prize, even a large lead by one player does not fully discourage the other. In a later study, Konrad and Kovenock (2010) demonstrate that the discouragement effect may also fade away when players' bidding costs are uncertain. Gelder (2014) identifies another important context of complications, by introducing a cost for the loser and time discounting on contenders' payoffs. It is shown that when the loser is subject to a sufficiently severe penalty, last-stand behavior could occur in which the laggard fights excessively hard to avoid immediately losing. In contrast, when the prize for winning is sufficiently large, a landslide victory could occur because the winner is compelled to step up effort to secure a quick win, which allows him to avoid discounting on his reward and prevents the loser's comeback. Klumpp and Konrad (2018) demonstrate that contenders' budget constraints may also eliminate the discouragement effect.

Ryvkin (2011a) studies a dynamic best-of-k contest and investigates the role played by contenders' fatigue, by which one's effort costs in late battles increase with his early input.

Multi-battle Contests between Teams

The section has so far focused on contests between two individual players who meet head to head on all battlefields. Such competitions may also take place between adversarial teams, alliances, or groups instead of individual players. As intuitively exemplified by many sporting events—e.g., the Davis Cup for men's tennis—players from rival teams form pairwise matches and compete head-to-head; one's win contributes to the team's success, while all players on a team share the team trophy regardless of individual performance. Fu, Lu, and Pan (2015) label such contentions "team contests with multiple pairwise battles." Senate elections in the United States also resemble such a contest: Candidates compete for legislative seats in each constituency on behalf of their parties, and a party can form a government or set the political agenda in the legislature if it secures majority status. Also, R&D races can take place between cross-functional alliances formed by individual firms.

Equilibrium plays in contests between individual players stand in stark contrast to those in contests between teams. Fu, Lu, and Pan (2015) demonstrate that the strategic momentum effect or discouragement effect does not carry over in team contests. They demonstrate that when the contest is a rivalry between two teams, and battles are contested by different pairs of contestants from rival teams, the outcome of each battle depends purely on the characteristics of the matched players and is independent of the past outcome of earlier battles. In contrast to Strumpf (2002), they show that the sequence of heterogeneous battles does not affect the outcome of the contest. Further, the temporal format of the contest—simultaneous, sequential, or partially sequential—does not affect the stochastic outcome of the contest or the expected total effort supplied in the contest. The sharp contrast to the predication obtained in contests between individual players is largely due to the fact that a player on a team is responsible for only his own battle and does not factor in the cost for his teammates who show up in future battles when the series continues. Based on the setting of Fu, Lu, and Pan, Feng and Lu (2018) study optimal prize-sharing rules with teams.

Tug-of-war Games

The discussion has so far focused on studies in which the number of component battles is exogenously fixed. In contrast, a tug-of-war model depicts a dynamic conflict that potentially lasts infinitely many periods. A single battle takes place in each period. Winning the battle allows one to push the state of the contest one step closer to victory, while losing it pulls it back by offsetting a previous win. A party secures the ultimate victory if and only if the contest reaches a certain state—i.e., it accumulates a sufficiently large lead. Tug-of-war models between individual players are studied by Harris and Vickers (1987), McAfee (2000), Agastya and McAfee (2006), and Konrad and Kovenock (2005).

Häfner and Konrad (2016) and Häfner (2017) extend tug-of-war models to a team setting that resembles that of Fu, Lu, and Pan (2015). The studies demonstrate that players behave drastically differently when they compete on teams and when they compete on their own. In a typical two-player tug-of-war game, it is well known that a peaceful equilibrium exists in which players remain inactive and neither attempts to terminate the series. It is shown that in a team setting, such a peaceful outcome cannot be sustained.

Contests with Budget Constraints

In Snyder (1989), parties bear linear bidding costs, but spending is not constrained ex ante. A large class of models instead assume that contenders are subject to budget constraints and must allocate their limited resources among parallel battles to maximize the sum of the expected rent they can receive from the whole set of battlefields. This class of models is generically named a Colonel Blotto game of duopoly conflicts in multiple battlefields. The game was first proposed by Borel (1921) and analyzed by Borel and Ville (1938) in a special case of three markets. Gross and Wagner (1950) generalize the analysis to a finite number of markets for symmetric players and solve an asymmetric case with battlefields. Roberson (2006) provides a general characterization of the equilibrium payoffs and the equilibrium univariate marginal distributions in n-battlefield Colonel Blotto games with symmetric and asymmetric players. Thomas (2018) proposes the irregular N-gon solution, an innovative approach to constructing equilibrium distributions in Colonel Blotto games with heterogeneous rents among battlefields.

Kovenock and Roberson (2012) study a coalitional Colonel Blotto game, in which two players form an alliance and each competes against a common rival in a disjoint Colonel Blotto game; allied players are allowed to transfer resources between each other.

All of these studies share a "constant-sum" feature, in that each battlefield provides a fixed rent and contenders' resources are valueless otherwise. Kvasov (2007) relaxes this assumption. He assumes that players maximize the difference between expected total rents and the total resources he sinks, while his overall bids are capped by an upper limit. The model thus resembles an all-pay auction with a bidding cap (Che and Gale, 1998). Kvasov considers symmetric players, while Roberson and Kvasov (2012) allow for asymmetric budget constraints. Fu and Iyer (2019) allow rents to be endogenously determined, as contenders can allocate a portion of their resources to productive activities that build up the rent on a battlefield.

The model has been applied to a broad spectrum of contexts, including campaign financing allocations (Lake, 1979; Laslier, 2002; Laslier and Picard, 2002); advertising (Friedman, 1958; Fu and Ganesh, 2019); military defense (Clark and Konrad, 2007; Arce, Kovenock and Roberson, 2012; Kovenock and Roberson, 2018); and political economy (Roberson, 2008);

etc.

These studies usually assume that contestants simultaneously allocate their budgets across battlefields. Harbaugh and Klumpp (2005) study a two-stage tournament with each player's total effort constrained by an upper limit. They show that the underdog tends to behave more aggressively in the first period, while the favorite saves it for the late period. Konrad (2018) and Klumpp and Konrad (2018) construct general dynamic Colonel Blotto games and show that the discouragement effect fades away because of players' budget constraints.

Multi-battle Contests in Networks

A burgeoning literature imposes network structures on multi-battle contests. Franke and Öztürk (2015) study a conflict network in which players engage in a series of bilateral conflicts. A player's cost is a quadratic function of his total investment in all of the conflicts in which he participates. As a result, one's marginal cost in one conflict depends on his investment in other battlefields. Franke and Öztürk focus on how the network structure affects conflict intensity. They demonstrate that a peaceful resolution—which removes the conflictive linkage between two neighboring players—may not soften the conflict. Franke and Öztürk assume a lottery contest—i.e., $r = 1$—in modelling each bilateral battle. Focusing on a particular class of networks (bipartite networks), Jiao, Shen, and Sun (2019) extend the framework of Franke and Öztürk by allowing for a general Tullock contest with return-to-scale technology, i.e., allowing for $r \in (0, 1]$. A higher return to scale—i.e., a larger r— is well known to incentivize more aggressive bidding in a stand-alone contest. They nevertheless demonstrate that in the conflict network, a higher return to scale may reduce conflict intensity.

Xu and Zhou (2018) construct a more general model in a setting of conflicts network, with many existing single- or multi-battle contest models being its special case. Each player can participate in multiple heterogeneous battles, without restriction on the number of players on each battlefield; the structure of conflicts can be represented by a hyper graph. Battles are interdependent through each player's efforts and costs, and the model allows for a general cost structure. In particular, it includes (1) a case of costless investment with a budget constraint, (2) a case of purely costly effort, and (3) a mixed case in which effort is costly and also bounded by a bid cap. Xu and Zhou show that the set of pure-strategy equilibria is nonempty and convex, and identify the condition under which the equilibrium is unique. Despite the unavailability of a closed-form equilibrium solution, the paper adopts a variational inequality (VI) approach, which enables equivalent characterization and extensive comparative statics analysis.

Dziubiński, Goyal, and Minarsch (2018) develop a dynamic model of conflicts in networks. The model consists of a set of kingdoms; a ruler is able to attack a neighboring kingdom, and winning allows him attack more kingdoms in his expanded neighborhood. The neighborhood structure is abstracted as a contiguity network. Based on a pure-strategy Markov Perfect equilibrium, the paper explores how the dynamics of war and peace are affected by rulers' resources, contest technologies, and the network structure. In addition, the study conducts case studies on major historical episodes of expansion and empire formation to lay empirical foundation for the model and predictions.

Kovenock and Roberson (2018) investigate the attack and defense of multiple networks of targets with intranetwork strategic complementarity among targets: In a weakest-link network, an attack is successful as long as the attacker sinks more resources on one target than the defender; in contrast, in a best-shot network, an attack is successful only if the attacker defeats the defender on all targets. The survival of the supranetwork requires that all networks be successfully defended.

Contests with Entry

In many competitive situations, contenders can often be uncertain about the number or identities of the opponents they will meet. Contenders can often be absent from competitions due to external shocks. For instance, a coder can be unavailable to a crowdsourcing contest because of difficulty with scheduling concurrent tasks. A firm may have to forgo a procurement competition due to fluctuating demand.

One strand of literature studies contests with an uncertain number of contestants, assuming that contestants enter the competition with exogenously given probabilities. Myerson and Wärneryd (2006) study a contest with an infinite number of potential contestants whose entry follows a Poisson process. Münster (2006), Lim and Matros (2009), and Fu, Jiao, and Lu (2011) assume finite numbers of potential contestants, with each entering the contest with a fixed and independent probability.

A natural question arises for optimal contest design. Suppose that potential contestants enter randomly with exogenous probabilities, and the designer is able to observe their actual entry outcome. Does it pay for her to disclose the number of actual participants? Lim and Matros (2009) consider a setting with symmetric contestants and uniform entry probability. Denote Ω_e the set of potential contestants who have actually entered the competition. The winner is selected through a Tullock contest success function

$$P_i = \frac{x_i^r}{\sum_{j\in\Omega_e} x_j^r}, r \in (0, 1].$$

The designer commits to her disclosure policy prior to the entry stage: She either fully discloses the entry outcome or fully conceals it. It is shown that the expected total effort is independent of the prevailing disclosure policy. Fu, Jiao, and Lu (2011) assume a more general ratio-form contest success function, with one winning the competition with probability

$$P_i = \frac{f(x_i)}{\sum_{j \in \Omega_e} f(x_j)}.$$

They define $H(\cdot) = f(\cdot)/f'(\cdot)$ as the characteristic function of the contest and find that the optimum depends on the property of the characteristic function: Full disclosure (full concealment) generates a higher total effort if the function is strictly concave (convex). The problem is reexamined by Feng and Lu (2016) by using a Bayesian persuasion approach, which allows for a broad and unconstrained set of candidate disclosure schemes. They show that partial disclosure is always suboptimal, and it is thus without loss of generality to focus on the comparison between full disclosure and full concealment, as in Fu, Jiao, and Lu (2011).

Fu, Lu, and Zhang (2016) are the first to consider this issue in an asymmetric setting. Two potential contestants differ in both their prize valuations and exogenous entry probabilities. They characterize the conditions under which full disclosure (concealment) outperforms. It is shown that disclosure policy plays a role in moderating the balance of the playing field in the contest.

Another strand of literature views contestants' entry as their strategic decisions, which endogenizes their entry pattern. The literature dates back to Higgins, Shughart, and Tollison (1985). Morgan, Orzen, and Sefton (2012) and Fu and Lu (2010) let potential contestants enter sequentially, so they know the entry status of the contest before they make their own entry decisions. Kaplan and Sela (2010) expound on the role played by entry fees, but assume simultaneous entry. Fu and Lu allow a resource-constrained designer to either charge an entry fee and use the revenue collected to top up the prize purse or provide a subsidy, which trims the prize.

Kaplan and Sela (2010) consider an all-pay auction model in which potential contestants, who differ in their abilities, simultaneously decide whether to bear a cost and enter the competition. They show that the contest can attract more participation from high-ability players by imposing a fee on the winner. Kaplan and Sela assume that contestants know whether the others have entered in the competition stage when deciding on their effort entries. Fu, Jiao, and Lu (2015) model the competition as a Tullock contest. They also assume simultaneous entry, but in contrast to Kaplan and Sela, entrants remain uninformed. Fu, Jiao, and Lu consider a number of design problems in this setting. First, it is shown that a larger discriminatory parameter r may not be optimal when the designer can set its value. A larger r implies a more precise winner-selection mechanism. It incentivizes competitions on the one hand, while the anticipated fierce competition discourages entry on the other. Second, it allows a resource-constrained designer to charge an entry fee and use the revenue collected to top up the prize. The designer is thus allowed to commit to a prize schedule that is contingent on the number of actual participants and, therefore, the actual revenue collected through entry fees. Third, it shows that the designer can improve the performance of the contest by excluding a subset of potential contestants from the competition. In a rank-order tournament model, Morgan, Tumlinson, and Värdy (2018) also espouse the positive effect of noise in encouraging entry, despite its negative effect of stifling competition.

The studies discussed in the section so far focus on standalone contests. A handful of studies allow players to self-select into competing parallel contests. With Tullock contests and identical contestants, Azmat and Möller (2009) study how to structure contests when they compete for players' participation. It is shown that the prevailing prize structure of a contest affects not only players' incentives to bid, but also the attractiveness of the contest. Multiple prizes are more likely to be optimal—i.e., maximizing participation—when the contest is more discriminatory. Azmat and Möller (2018) allow for heterogeneous contestants and study the sorting outcome when contestants are allowed to choose which contest to enter. It is shown that high-ability contestants may enter contests with smaller prizes to avoid competition when the proportion of high-ability contestants is high. Morgan, Sisak, and Várdy (2018) demonstrate that entry into "big ponds"—i.e., contests with higher show-up fees and more rewarding prizes—is nonmonotonic in ability; entry into more discriminatory contests is also nonmonotonic in ability. Konrad and Kovenock (2012) demonstrate that coordination failure occurs when contestants play a mixed strategy in self-selecting into contests, which reduces rent dissipation. Grossmann (2018) compares two situations: In one, each contestant can participate in multiple parallel contests; in the other, each is constrained to enter one competition. Suppose that a global planner exists who governs the institutions of all the parallel contests. It is shown that prohibiting multiple participation can maximize individual performance in each specific contest at the cost of the aggregate effort.

A few studies—such as Taylor (1995), Fullerton and McAfee (1999), and Che and Gale (2003)—show that shortlisting can improve the performance of R&D contests/tournaments, and explore the optimal shortlisting mechanisms.

Asymmetric Contests and Optimal Favoritism

It is well known in the literature that the performance of a contest crucially depends on the competitive balance among contenders, and a more level playing field tends to fuel competition.[17] A substantial ability differential among contestants discourages the underdog, while allowing the favorite to slack off. Baye, Kovenock, and de Vries (1993) elegantly illustrate this logic in a multi-player complete-information all-pay auction model. When the favorite possesses an excessive advantage, the contest can, paradoxically, generate a higher revenue by excluding him, while keeping only the underdogs in the slate. Using data from professional golf tournaments, Brown (2011) demonstrates empirically that the presence of a superstar in a competition leads to lower performance.[18]

Conventional wisdom inspires research efforts to examine the incentive effects of identity-dependent contest rules and search for the mechanism that optimally exploits the heterogeneity in contestants' abilities and manipulate the balance of the playing field to induce desirable equilibrium behaviors. Konrad (2002) considers a two-player complete-information all-pay auction model. A contestant can be favored (discriminated against) in two ways: (1) his bid can be scaled up (down) by a fixed percentage (handicap); and (2) a fixed constant can be added to (subtracted from) his bid (head start). Siegel (2009, 2014a) depicts more general settings that allow for discriminatory contest rules.

The level and format of favoritism in the contest rule are exogenous in Konrad (2002) and Siegel (2009, 2014a). In a two-player complete-information all-pay auction model, Fu (2006b) allows the contest designer to scale up the weaker contestant's bid, which is equivalent to placing a handicap on the stronger contestant's bid. It is shown that nonexcessive favoritism encourages both contestants to step up efforts. The playing field is fully balanced in the optimum, which simultaneously maximizes the total effort in the contest and the expected winner's effort. Pastine and Pastine (2012) and Li and Yu (2012) let the designer choose a head start awarded to the weaker contestant. The optimal head start completely offsets the ex ante asymmetry, but it can only incentivize the stronger contestant and not the weaker.

Clark and Riis (2000) examine the effects of handicaps in two-player incomplete-information all-pay auctions. Assuming that contestants' types are drawn from uniform distributions, they show that expected revenue increases when the ex ante stronger contestant is handicapped. Kirkegaard (2012) allows for more general distributions of contestants' types and arms the designer with the flexibility to use both handicap and head start. It is shown that the optimum does not necessarily handicap the ex ante stronger contestant. However, the optimum tends to have a combination of both instruments.

Franke (2012) considers the optimal biased contest rule in lottery contests, i.e., Tullock contests with $r = 1$. He compares the outcomes under two discrete policy options—equal treatment and affirmative action—and demonstrates that equal treatment could prevail when the number of contestants exceeds two. Franke defines affirmative action as a set of biases that neutralize the disadvantage of the weak players. It remains unknown, however, whether there exists another set of biases that outperform both equal treatment and such affirmative action.

It is well known that the optimum fully balances the playing field in a two-player Tullock contest. The optimum in an n-person setting, however, remains elusive. Franke, Kanzow, Leininger, and Schwartz (2013) take a pioneering step by formally addressing this design problem, allowing the contest designer to assign an individualized weight to each contestant's effort entry. Generalizing the design problem to an n-player setting fundamentally alters the nature of the optimization problem and enormously complicates the analysis. First, the contest design in a two-player setting is a one-dimensional problem, because favoring one reciprocally and equivalently handicaps the other. In contrast, with more than two contestants, contestants interact intricately in a complex network: One's incentive depends not only on his strength relative to each of his opponents, but also the strength differential between each of his opponents and each of the others. Second, searching for the set of optimal weights assigned to contestants' effort entries gives rise to a nonsmooth optimization problem: A contestant may choose to stay inactive when he faces slim odds to win, while contestants' winning odds are endogenously determined by the chosen weights. Third, the typical optimization approach to a mathematical program with equilibrium constraints (MPEC) requires a solution to the equilibrium bidding strategies for any given set of weights, insert the equilibrium solution into the objective function, then solve for optimal weights that maximize it. This approach is technically infeasible, because an n–player asymmetric Tullock contest, in general, yields no closed-form equilibrium solution.

Franke, Kanzow, Leininger, and Schwartz (2013) provide a complete solution to the optimization of n-player asymmetric Tullock contests. It develops an algorithm to calculate the equilibrium solution for lottery contests—i.e., the case of linear impact function with $r = 1$—which allows them to obtain the optimal weights that maximize total effort in the contest. Their approach, however, does not apply to settings of nonlinear impact function and alternative objections. Fu and Wu (2018a) develop a novel approach to bypassing the technical difficulty and tackling this nagging MPEC program. They identify a correspondence among the weights, contestants' efforts, and their winning odds in equilibrium. Fu and Wu treat the distribution of contestants' equilibrium winning odds as the design variable, which renders a tractable optimization problem. They then use the solution to the alternative optimization problem—i.e., the distribution of equilibrium winning odds that corresponds to the optimum—to recover the optimal weights. This approach applies to the maximization of a wide array of objective functions.

Besides identity-dependent preferential contest rules, the literature has identified alternative mechanisms that could balance the playing field and catalyze competitions. Baye, Kovenock, and de Vries (1993) suggest efficient exclusion. Szymanski and Valletti (2005), Dahm (2018), and Dahm and Esteve-González (2018) highlight the role of extra prizes. Szymanski and Valletti assume that the second prize is open to the entire pool of contestants, while in the other two studies one prize is targeted in the sense that it can be awarded to only disadvantaged players. A handful of studies have espoused the positive incentive effect of noisy contests. With a noisier winner-selection mechanism, one's win depends more on random factors, and a higher bid is less likely to be translated into higher winning odds.

The inherent randomness mitigates the favorite's advantage in ability, which motivates the weaker and prevents the stronger from slacking off. Fang (2002), for instance, shows that (1) with asymmetric contestants, a lottery contest could outperform an all-pay auction, and (2) the exclusion principle obtained in an all-pay auction setting is obsolete in Tullock contests. Wang (2010) characterizes the optimal level of discriminatory power—i.e., the optimal size of r—of two-player Tullock contests that maximizes total effort. Morgan, Tumlinson, and Várdy (2018) demonstrate in a rank-order tournament model that perfect meritocracy can be optimal only if contestants are sufficiently heterogeneous.

The same logic underpins the observation by Franke (2012) and Franke, Kanzow, Leininger, and Schwartz (2013) that the optimal weights on contestants' effort entries in lottery contests do not fully balance the playing field, as in all-pay auctions (see, for instance, Fu, 2006b). The presence of noise offsets stronger players' advantage, thereby substituting away the need to use biased contest rules to restore balance.

Fang (2002) sparks interest in the comparison between all-pay auctions and Tullock contests. A few studies allow a contest designer to play a role, and compare the optimally designed Tullock contest with its all-pay-auction counterpart. Epstein, Mealem, and Nitzan (2013) consider a two-player asymmetric setting. Allowing the designer to set multiplicative biases, as well as the discriminatory power term r for Tullock contests, they show that Tullock contests and all-pay auctions generate the same expected total effort. Franke, Kanzow, Leininger, and Schwartz (2014) consider multi-player contests and show that an all-pay auction always outperforms a lottery contest if the contest designer can bias the contest rule by assigning individualized weights on contestants' effort entries. Franke, Leininger, and Wasser (2018) further arm the designer with headstarts in her toolkit and reaffirm the dominance of all-pay auctions.[19] These studies rank all-pay auctions and lottery contests in terms of total effort. Epstein, Mealem, and Nitzan (2011) consider a scenario in which the contest designer cares about both effort supply and contestants' welfare and confirm the superiority of the all-pay auctions.

A growing strand of literature identifies the contexts in which the conventional wisdom of leveling the playing field could lose its appeal. Drugov and Ryvkin (2017) demonstrate that in rank-order tournaments, it can be optimal to bias a contest with symmetric players when their effort cost functions are concave. Fu and Wu (2018a), based on their optimization approach, demonstrate that the optimal Tullock contest depends on the number of contestants and the level of precision in the contest. In particular, an effort-maximizing contest designer could further upset the initial balance by giving additional favoritism to stronger contestants when the contest involves three or more players and sufficient noise in choosing a winner. The biases placed on contestants' effective effort entry can be nonmonotone when the level of noise in the winner-selection mechanism is in an intermediate range. In other words, the ex ante middle-ranked contestant receives the most favoritism.

The majority of the studies on optimal biased contests focus on the objective of total effort maximization. A few studies demonstrate that the principle of leveling the playing field may fail when the designer pursues other goals. Fu and Wu (2018a) show that when maximizing the expected winner's effort, the optimal contest could favor the stronger contestant even if only two players participate. Fu, Lu, and Lu (2012) consider a two-player R&D Fullerton-McAfee-type research tournament, which is technically equivalent to a Tullock contest. A designer has a budget to allocate between a prize for the winner and subsidies provided to contenders, with the subsidy given to a firm multiplicatively amplifying the output of its effort entry. It is shown that when the designer maximizes the expected quality of the winning entry, the optimum may subsidize the ex ante stronger firm more, which further upsets the competitive balance and creates a "national champion." Seel and Wasser (2014) consider an all-pay auction with ex ante symmetric players and independently drawn private values. It is shown that a head start reduces expected total effort, but can be optimal when the designer cares about the expected winner's effort. Pérez-Castrillo and Wettstein (2016) demonstrate, in a symmetric innovation contest with private information, that the designer may prefer to discriminate among contestants by setting identity-dependent prizes when she aims to achieve the highest quality output. While the majority of studies in this research stream focus on eliciting effort, Kawamura and Moreno de Barreda (2014) focus on the selective efficiency in contests. They show that headstart can be awarded to one of two ex ante identical contestants when the designer wishes to have a more capable winner.

A few studies refute the conventional wisdom in dynamic contest settings. Denter and Sisak (2016) show that a head start can be optimal in a two-stage contest in which two symmetric contestants exert effort in both stages and one wins with a higher aggregate output. Cohen, Maor, and Sela (2018) and Fu and Wu (2018b) provide a rationale for ex post asymmetry in the context of sequential elimination contests. A total of $n \geq 3$ contestants are pooled to compete to qualify for the finale, and the designer can choose to favor the finalist who is top ranked in the first stage—i.e., by assigning weights on their second-stage effort entries based on their interim ranking. Although the weights upset the balance and reduce effort supply in the finale, contestants are motivated to vie for more favorable ranks in the first stage. In a best-of-three contest between two ex ante identical contestants, Barbieri and Serena (2018) show that the contest designer can bias the competition to improve its performance. In particular, a biased contest rule based on the outcomes of early battles mitigates the well-known discouragement effect in dynamic multi-battle contests.

In addition, Brown and Chowdhury (2017) show that handicapping the stronger contestants could incentivize destructive efforts, and provide empirical evidence for the prediction.

Group Contests

Contests often take place among groups or alliances instead of between individual players, and one strand of literature is devoted to the strategic interactions between groups. In a standard group contest framework, economic agents form groups and join forces to compete collectively for common goals. Each group member voluntarily decides on the resources he is willing to contribute, and individual contributions are aggregated at the group level. Rent is allocated among groups based on their aggregate performance. Plenty of examples are available to exemplify such situations. Consider, for instance, competitive lobbying for preferred policy options among interest groups. Alternatively, imagine the broadly observed conflicts among ethnic groups or socioeconomic classes. The class of team contests is one example of such competitions, but they focus on a multi-battle setting, requiring that each team member participate in a single disjoint battle and the outcomes of each disjoint battle be aggregated to determine the winning team. In the literature to be discussed in this section, in contrast, group members collectively produce a composite output and the winner is determined through a single battle based on the composite output.

Nitzan (1991a) was among the first to formally model such situations. Nitzan considers a setting in which n groups—with each consisting of n_i identical players—compete for a rent of value $s > 0$. The rent awarded to the winning group has a private-good feature, so it can be shared among group members. Let a contestant be indexed by ki, with $k \in \{1, \ldots, n_i\}$ denoting his identity within a group, and $i \in \{1, \ldots, n\}$ denoting his group affiliation. A group's aggregate effort is given by $x_i = \sum_{k=1}^{n_i} x_{ki}$, and the winning group is chosen through a Tullock lottery contest. That is, a group i wins the contest with a probability

$$P_i(x_i, \mathbf{x}_{-i}) = \frac{x_i}{\sum_{j=1}^{n} x_j}.$$

When a group i wins, a member ki receives a share $f_{ki} = \lambda \frac{x_{ki}}{\sum_{t=1}^{n_i} x_{ti}} + (1-\lambda)\frac{1}{n_i}$ of the rent, with $\lambda \in [0,1]$. The share is a linear combination between two sharing rules: a merit-based sharing rule, $x_{ki}/\sum_{t=1}^{n_i} x_{ti}$, and an egalitarian sharing rule, $1/n_i$.

A number of studies have adopted and extended the setting, such as Baik and Lee (1997, 2007). Skaperdas (1998) assumes that the prize is distributed among winning group members purely based on their relative contributions. Nitzan (1991b) allows groups to have different sharing rules and demonstrates that an interior equilibrium does not exist. Davis and Reilly (1999), however, verify the existence of noninterior equilibria in this setting by resorting to a nonlinear programming approach. Baik and Lee (2007) assume that the sharing rule is privately known within each group. It is shown that a higher outlay results when the sharing rule is publicly known than in the case of private knowledge. Balart, Chowdhury, and Troumpounis (2017) innovatively suggest using Nitzan's (1991a) sharing rule in collective contests to model noisy contests among individuals.

In contrast, a handful of studies view the prize to be won by a group as a group-specific public good, such as Esteban and Ray (1999), Baik (2008), Ryvkin (2011b), Kolmar and Rommeswinkel (2013), Barbieri, Malueg, and Topolyan (2014), Chowdhury and Topolyan (2016a, 2016b), Chowdhury, Lee, and Topolyan (2016), and Eliaz and Wu (2018). Esteban and Ray (2001) allow the prize to be a mix of a public component and also a private-good component, with the latter to be equally shared among members of the winning group. Nitzan and Ueda (2011), in contrast, assume that the private-good component is to be shared by a linear combination of a relative-performance-based rule and an egalitarian rule, as in Nitzan (1991a). Nitzan and Ueda (2009) consider a setting in which the prize is a club or common good, and the members of the winning group decide the levels of their usage individually; anticipating their future use of the prize, contestants in the first stage decide on their input in the contest. Esteban and Ray (1999) introduce identity-dependent externality: A player's payoff when his group loses depends on which of the other groups wins.

Studies of group contests also differ in how individual contributions are aggregated into group outlays. The majority of these studies assume that within a group, individual efforts are perfect substitutes and the group outlay is the sum of individual efforts, such as Nitzan (1991a, 1991b), Esteban and Ray (1999, 2001, 2008), and Boosey, Brookins, and Ryvkin (2018). In an incomplete-information all-pay auction framework, Eliaz and Wu (2018) assume that each contestant, by exerting an effort x_i, produces an output $h(x_i)$, where $h(\cdot)$ is contestants' individual production function. For a group with n_x members, its aggregate entry in the contest is given by the sum of individual outputs, i.e., $H_{n_x}(x_1, \ldots, x_{n_x}) = \sum_{i=1}^{n_x} h(x_i)$.

A few studies assume alternative group production technology. Chowdhury, Lee, and Topolyan (2016) assume that a group's output is determined by the minimum of the contributions made by its member, i.e., the weakest link. Chowdhury, Lee, and Sheremeta (2013) and Barbieri, Malueg, and Topolyan (2014) assume the opposite scenario, in which group output is given by the maximum of its members' contributions, i.e., the best shot. In contrast, Chowdhury and Topolyan (2016a, 2016b) consider a combination of the two situations: In a group contest, one group's outlay is the weakest link of individual efforts, while the other's is the best shot. Kolmar and Rommeswinkel (2013) and Choi, Chowdhury, and Kim (2016) assume that individual members' efforts complement each other and are converted into their group's outlay through a CES production function. Barbieri, Konrad, Malueg (2018) consider a preemption contest between groups: A group wins if one of its members takes a costly action to "grab" the prize sooner than those of the other groups.

An alternative approach to model situations in which contestants join forces is to allow for transfer of scarce resources. Kovenock and Roberson (2012) and Rietzke and Roberson (2013), for instance, consider contention between two allied players and one independent player in a series of simultaneous disjoint battles and explore the incentives of the allied players to exchange their resources.

The group contest literature sheds light on the long-lasting question about the effect of group size in collective actions. Olson (1965), for instance, presents the celebrated argument that small groups can be more efficient or competent because they are less vulnerable to free-riding, which is referred to as the famous thesis of *group size paradox*. Nitzan (1991a) demonstrates that a larger group is more likely to win when the prize is shared among contestants based on their relative performance, while the opposite holds when the sharing rule is entirely egalitarian. Nitzan and Ueda (2009) view the prize as common good and endogenize its use. Aside from free-riding, a tragedy of the commons may loom large, which also contributes to the group size paradox. Esteban and Ray (2001) assume that the prize has a public-good component and a private-good component. They show that a larger group is more effective in the competition when contestants' effort cost functions are sufficiently elastic. The contest still favors the large group even if the elasticity of the cost function is small, but the public-good component of the prize is sufficiently large. Eliaz and Wu (2018) demonstrate that the group size paradox does not loom large when individual contestants' production function is concave, which echoes the finding of Esteban and Ray.

In line with the setup of Sen (1966) for a collective production problem, Esteban and Ray (2001) assume that the private component of the prize is equally shared among members of the winning group. Nitzan and Ueda (2011) instead assume that a portion of the private component is to be shared based on members' relative performance. Further, they assume that this portion is determined endogenously by a benevolent leader of each group before the contest begins. It is shown that endogenous determination of sharing rules completely eliminate the usual group size paradox. Vázquez-Sedano (2014) shows that an endogenously determined cost-sharing rule plays a similar role. While the majority of studies in the literature assume that contestants are homogeneous within groups, Nitzan and Ueda (2018) allow for within-group heterogeneity and demonstrate that heterogeneity can exacerbate collective-action problems, because it prevents the use of selective incentives (Olson, 1982)—i.e., the incentives that selectively apply to economic agents depending on their actions. Nitzan and Ueda (2018) show that cost-sharing schemes can be a remedy.

The section has so far assumed that groups or alliances are exogenously determined with a fixed number of players. Bloch, Sánchez-Pagés, and Soubeyran (2006), Sánchez-Pagés (2007), and Baik (2016) allow for endogenous group formation. Boosey, Brookins, and Ryvkin (2018) examine group contests when the size of each group is a random variable and can even be unknown to its own group members. It is shown that the uncertainty in group size lowers contestants' investment incentives and also tends to mute the usual group size paradox. Assuming heterogeneous players, Ryvkin (2011b) explores how aggregate effort exerted in contests is affected by the sorting of players into groups. It is shown that the optimal sorting depends on the curvature of players' effort cost functions. Brookins, Lightle, and Ryvkin (2018) examine the effect of players' sorting across groups on the performance of the competition in a setting of weakest-link group contests. The theory predicts that in contrast to contests between individual players, a more uneven competition—caused by more imbalanced sorting—boosts overall output. The prediction, however, is not supported by their experimental findings.

Barbieri and Malueg (2016) introduce private information into this framework by assuming each contestant's valuation for the public good prize to be known only to himself. They show that teams' performance can go either way when the number of teams increases. The group size paradox looms large, in that contestants in the smaller group tend to bid more aggressively. Barbieri, Kovenock, Malueg, and Topolyan (2018) allow each player's marginal effort cost to be private information, and show that coordination could take place in various forms. In particular, contestants' incentives to share information within the group are also considered as a device to facilitate coordination.

Müller and Wärneryd (2001) consider a two-layer contest inside a firm. Inside owners collectively compete for a larger share of surplus with outside owners; another conflict ensues after the struggle with outside owners, as inside owners strive to appropriate the rents they have won. They show that the presence of outside ownership can reduce the efficiency loss caused by internal conflicts. Konrad and Kovenock (2009b) consider a similar two-layer contest: Two allied players compete against a standalone player, and a conflict occurs within the alliance once they defeat the standalone player. They explore players' incentives to form an alliance in the first place, and demonstrate that resource constraints can lead to alliance formation despite free-riding within the alliance and conflict subsequent to the alliance's victory.

Contests with Non-risk-neutral Players

The modelling frameworks of contests have typically assumed that contestants are risk-neutral.[20] Because the outcome of a contest is typically uncertain, contestants' risk attitude toward the winning-probability distribution may figure prominently in their effort decisions and performance in the contest.

A natural way to introduce risk attitudes is to assume that contestants are risk averse.[21] A number of authors have investigated contenders' incentive with the presence of risk aversion since Hillman and Katz (1984). Different from models with risk-neutral players of linear utility, contestants' utility function $u(\cdot)$ is assumed to be strictly increasing and concave (i.e., $u'' < 0 < u'$). Given an endowment w, a contestant i's expected payoff in the contest game is given by

$$\pi_i(\mathbf{x}) = p_i(\mathbf{x}) \cdot u(w + v_i - x_i) + [1 - p_i(\mathbf{x})] \cdot u(w - x_i).$$

Note that the above expression degenerates to the payoff function specified in the introduction when $u(\cdot)$ is linear.

Next, the survey provides a summary of the literature on risk aversion in contests with ratio-form success functions. The discussion is confined to two main issues: (1) the existence and uniqueness of a pure-strategy equilibrium; and (2) comparison of the equilibrium effort level between a contest with risk-neutral contestants and that with risk-averse ones.[22]

Equilibrium Existence and Uniqueness

In a two-player contest, Skaperdas and Gan (1995) prove the existence of a pure-strategy Nash equilibrium if contestants exhibit constant absolute risk aversion (i.e., CARA). Cornes and Hartley (2003) generalize their results and establish the existence and uniqueness of equilibrium in an asymmetric contest. Similar to Skaperdas and Gan, their result relies on the assumption of CARA preference. Yamazaki (2009) further relaxes the CARA assumption and proves the existence and uniqueness of the equilibrium. Yamazaki requires that the Arrow-Pratt measure of relative risk aversion (i.e., $-u''/u'$) of each contestant is nonincreasing in her consumption level. Cornes and Hartley (2012) show that risk aversion may lead to multiple equilibria, and both symmetric and asymmetric equilibria may arise in a contest with homogeneous contestants. Fu and Wu (2018c) prove the existence and uniqueness of the symmetric pure-strategy equilibrium in multi-prize contests, given that the contest is not excessively discriminatory.

Incentives with Risk Aversion

Skaperdas and Gan (1995) and Konrad and Schlesinger (1997) show that the impact of risk aversion on contestants' investment in effort is ambiguous. As Skaperdas and Gan point out, this general ambiguity stems from two opposing effects caused by risk aversion. On the one hand, a more risk-averse contestant has an incentive to exert less effort in the contest, because doing so reduces his safe payment; this is called the gambling effect. On the other hand, as a contestant becomes more risk averse, he has an incentive to exert more effort, because doing so reduces the probability of losing the contest game; this is referred to as the self-protection effect.

In a model similar to Konrad and Schlesinger (1997), Treich (2010) and Sahm (2017) conclude that risk aversion always leads to less equilibrium effort level. The additional condition that Treich needs to derive this definite comparative static result is that risk-averse contestants are also prudent (i.e., $u''' < 0$). Complementarily, Jindapon and Whaley (2015) show that contestants' risk lovingness and imprudence lead to overinvestment in effort above the risk-neutral outcome.

Concluding Remarks

A vast literature has been devoted to uncovering the strategic substance of contest-like competitive activities. The literature delineates, in a wide array of contexts, how contestants respond to various environmental factors and institutional elements in their strategic decisions. This knowledge thus inspires and paves the way for studies of how to structure and design contests to influence contestants' behavior and achieve stated objectives.

This survey provides a review of the theoretical studies of contests/tournaments. A number of important lessons can be learned from these important contributions. The findings highlight the characteristics peculiar to the strategic situations of contests, and the knowledge contributes to the understanding of social and economic interactions in various areas. For instance, the literature has extensively discussed how heterogeneity among contestants affects their behavior and the roles played by various institutional instruments that manipulate the balance in competitions. The contest literature thoroughly studies the discouragement or strategic momentum effect in dynamic contests and identifies various conditions under which it could loom large or fade away, which generates useful implications for the design of electoral competitions and sporting events. A large research effort has been devoted to collective actions in contests. The findings shed light on the famous proposition of the group size paradox in contest scenarios. The literature has also demonstrated that a noisy winner-selection rule could paradoxically improve performance, which challenges the usual belief about meritocracy and provides useful insights on performance evaluation schemes in competitive situations. In addition, contestants' behavior responds sensitively to the information available regarding their opponents and the competitive environments. This knowledge lays a foundation for the design of information-disclosure rules in contests.

Our survey is unable to encompass the entire spectrum of the contest literature. Many important contributions and issues in the field deserve comprehensive discussion, but have escaped our review. Our survey, as well as the literature, has mainly focused on contestants' effort incentives and the institutional elements of a contest that affects effort supply. Aside from being an effective mechanism to elicit productive effort[23], a contest can also be used as a device to select appropriate candidates. For instance, Fang and Noe (2018) explore the effect of the precision of the winner-selection mechanism on the selection efficiency of contests. A few earlier studies, such as Ryvkin and Ortmann (2008), Ryvkin (2010b), and Clark and Riis

(2001, 2007), are also devoted to exploring this alternative role of contests. In a market-based tournament, Gürtler and Gürtler (2015) investigate how the heterogeneity among employees affects the sensitivity of ability assessment to promotion decisions. In addition, players often make strategic decisions in multiple dimensions other than effort supply. Hvide (2002), Seel and Strack (2013), and Strack (2016), for instance, consider contestants' incentives to take risks. We look forward to future contributions in this area of research, and in future survey updates we will delve into this in greater depth.

Despite the voluminous literature on contests, many open questions remain and leave ample room for future studies. First, the fundamentals of contest games in many scenarios have yet to be fully uncovered. As stated above, the literature remains limited on the equilibrium in multi-player Tullock contests with intermediate r. A solution to the nested multi-winner Tullock contest model requires symmetric players. In addition, the equilibrium behavior in Tullock contests with incomplete information deserves closer inspection. Research in these unchartered areas calls for novel techniques.

Second, a nagging observation in the contest literature is the persistent dichotomy between theoretical prediction and experimental findings (see Dechenaux, Kovenock, and Sheremeta, 2015). It is imperative that contest modelling incorporate new elements—e.g., behavioral components—to shed light on the seemingly inconsistent behavior observed in laboratory settings.

Third, many theoretical predictions can be sensitive to modelling nuances. It is important to examine more thoroughly the robustness of previous results and the logic for their robustness/fragility. Such knowledge not only helps us test the boundaries of prior results, but also further unveils the strategic nature of contest games in different forms.

Fourth, the contest, as a convenient and tractable framework to depict conflict situations, can be embedded with other game theoretical models to explore strategic behavior in broader contexts. For instance,a growing literature embeds contests in network structures, such as König, Rohner, Thoenig, and Zilibotti (2017), Franke and Öztürk (2015), Jiao, Shen, and Sun (2019), Xu and Zhou (2018), Kovenock and Roberson (2018), and Dziubiński, Goyal, and Minarsch (2018).[24,25] Alternatively, the majority of the studies on contests take conflict as given—while in reality, confrontation is a deliberate choice of players and conflicts arise endogenously (Fu, 2005; Beviá and Corchón, 2010; Corchón and Yıldızparlak, 2013). A more thorough understanding is required regarding the occurrence of conflict as the outcome of players' strategic interaction. A contest model, for instance, can complement a bargaining model to examine negotiations in the shadow of conflict and generate useful predictions on the likelihood of settlement under various circumstances.

By no means could the discussions laid out above exhaust the numerous paths for the advancement of contest theory. Plentiful opportunities lie ahead for future research in this area.

Notes

[1] A few exceptions can be found in the literature that assumes the prize is endogenously dependent on contestants' efforts, such as Chung (1996); Kaplan, Luski, Sela, and Wettstein (2002); and Clark and Riis (2007).

[2] Ratio-form success functions—e.g., Tullock contests—require that one's winning probability depend on the ratio between his output and those of others. Another popular framework to model winner-selection mechanism is the difference-form contest success function, in which one's winning odds are a function of the absolute difference between his output and that of the other. Hirshleifer (1988, 1989) proposes a logit contest success function. Consider a two-player contest with an effort profile (x_i, x_j): A contestant i wins with a probability $P_i = 1/\{1 + \exp[k(x_j - x_i)]\}$, where k is a positive constant. A two-player rank-order tournament gives rise to a probit contest success function (see the "Rank-order Tournaments with Additive Noise" subsection). Beviá and Corchón (2015) consider a mix between the two approaches and propose a relative-difference-form contest success function that factors in both the difference between two contestants' efforts and its ratio to the total effort. Mildenberger and Pietri (2018) provide empirical estimates of four popular contest success functions: Tullock, logit, probit, and relative-difference form. They show that the last one provides the closest fit to the data.

[3] Fu, Lu, and Wang (2014) propose a reverse-lottery contest. Its winner-selection mechanism stands in contrast to that of a regular ratio-form contest success function. They demonstrate that it is underpinned by a noisy ranking system in which contestants' weakest links count in determining the winner. Lu and Wang (2016) provide an axiomatic foundation for the model.

[4] Linster (1993) and Esteban and Ray (1999) consider identity-dependent externalities in imperfectly discriminatory contest models.

[5] Singh and Wittman (2001) construct a model in which one's output is jointly determined by his ability and effort, and the production function takes a general form. Further, they allow for an extension in which a higher output does not lead to a sure win, and the function that maps contestants' output into winning odds is the designer's choice.

[6] Minchuk and Sela (2014) provide a unique setup. They consider an all-pay auction with certain and uncertain prizes. All certain prizes are commonly valued, while each player has a private value for the uncertain one.

[7] We use p_i^k to denote the conditional probability of contestant i's receiving the kth prize.

[8] In addition to the studies conducted by Clark and Riis (1996b, 1998a), lottery contest models have been applied to multiple-winner settings by Amegashie (2000), Yates and Heckelman (2001), Szymanski and Valletti (2005), and Fu and Lu (2009, 2012a).

[9] Although Berry (1993) provides a convenient approach to modelling the distribution of multiple prizes, the model is equivalent to a single-prize contest in which the rest of the prizes are uniformly and nondiscriminatorily given to the losers (see Clark and Riis, 1996b).

[10] Axiom 2 of Blavatskyy (2010) requires that in a two-player contest, one's winning probability is strictly increasing in its own effort, which is not satisfied by the framework of Skaperdas (1996) and Clark and Riis (1998c). The latter study considers a zero effort as one's withdraw and the other's automatic win for any positive effort entry.

[11] Assuming unmeasurable psychological factors, this literature investigates the randomized choices of decision makers (consumers) that result from a stochastic ranking. Among others, McFadden (1973, 1974) has demonstrated that the econometric implementation of modelling reveals choice among discrete alternatives while adopting a probabilistic choice model.

[12] The framework of McFadden's (1973, 1974) discrete choice model is further introduced and studied in a variety of aspects by studies collected in Manski and McFadden (1981).

[13] Define $\log g_i(x_i) = -\infty$ if $g_i(x_i) = 0$.

[14] This property is first non-constructively proposed by Luce and Suppes (1965) as a hypothetical decision rule. It is first used in the econometrics literature by Beggs, Cardell, and Hausman (1981), and first applied in the multi-prize contest literature by Fu and Lu (2012b).

[15] In addition to the five categories of dynamic contest models, Taylor (1995) and Lang, Seel, and Strack (2014) allow contestants to make a decision on stopping time in a dynamic process with deadlines.

[16] They demonstrate that a contest designer prefers a grand contest with performance bundling to a series of independent contests when contestants are sufficiently symmetric.

[17] Introducing heterogeneous contestants into imperfectly discriminatory contests/tournaments renders the model less tractable: A closed-form solution is no longer available in general. Assuming that players' heterogeneity is not too strong, Ryvkin (2007, 2009, 2011b, 2013a) uses linear/quadratic approximation around the symmetric equilibrium to study contestants' behavior and the impact of heterogeneity on aggregate effort.

[18] Connolly and Rendleman (2014) confirm the main result of Brown (2011)—i.e., the superstar effect—based on an alternative data set. However, her second and third results—on the effects of Tiger Woods' "hot" and "cool" periods and unexpected absence on other players' performance—are claimed to be sensitive to data and the specification of empirical strategies.

[19] In an earlier version of Franke, Leininger, and Wasser (2018), the authors allow the designer to choose an arbitrary form of impact functions and demonstrate that a linear one—i.e., lottery contests—outperforms all other concave impact functions.

[20] Siegel (2009, 2010, and 2014a) provides remarkable a exception in this regard by allowing for risk attitude in a general form.

[21] Another way to introduce risk attitudes is to assume that contestants are loss averse. See Chowdhury, Jeon, and Ramalingam (2018), Chen, Ong, and Segev (2017), Gill and Prowse (2012), Gill and Stone (2010), Dato, Grunewald, Müller, and Strack (2017), Dato, Grunewald, and Müller (2018), and Fu, Wu, Lyu, and Zhang (2018) for more discussion.

[22] For other research topics on contests with risk aversion, see Münster (2006), Schroyen and Treich (2016), Liu, Meyer, Rettenmaier, and Saving (2018), and Fu and Wu (2018c).

[23] Competitions in contest may also spark wasteful efforts or even unethical and destructive activities. Rent seeking and political campaigns, for instance, are widely considered to be wasteful, e.g., Klumpp and Polborn (2006). The literature has also recognized that high-power incentives could encourage sabotage. See Chowdhury and Gürtler (2015) for a thorough survey on sabotage in contests. Gilpatric (2011) and Ryvkin (2013b) explore contestants' cheating or doping behavior.

[24] Jiao, Shen, and Sun (2019), Xu and Zhou (2018), and Kovenock and Roberson (2018) allow multiple battles. In contrast, König, Rohner, Thoenig, and Zilibotti (2017) consider a single battle, in which one's performance depends on his relations to other players.

[25] The majority of the literature on conflict networks, such as Dziubiński and Goyal (2013, 2017), and Goyal and Vigier (2014) does not model conflicts as contests.

Acknowledgment

We are grateful to Stephano Barbieri, Subhasish M. Chowdhury, Luis Corchón, Dan Kovenock, Jingfeng Lu, Dmitry Ryvkin, Ron Siegel, Jun Zhang, Junjie Zhou, and two anonymous referees for very helpful comments and suggestions. Wu thanks the financial support of the National Natural Science Foundation of China (No. 71803003), and the seed fund of the School of Economics, Peking University. Any errors are our own.

Further Reading

Chowdhury, S.M. and Gürtler, O., 2015. Sabotage in contests: A survey. *Public Choice*, 164(1), pp.135-155.

Congleton, R.D. and Hillman, A.L., 2015. *Companion to the Political Economy of Rent Seeking*. Edward Elgar Publishing.

Connelly, B.L., Tihanyi, L., Crook, R.T. and Gangloff, A.K., 2014. Tournament theory: Thirty years of contests and competitions. *Journal of Management*, 40(1), pp.16-47.

Corchón, L.C., 2007. The theory of contests: A survey. *Review of Economic Design*, 11(2), pp.69-100.

Corchón, L.C. and Serena, M., 2018. Contest theory. In *Handbook of Game Theory and Industrial Organization, Volume II: Applications*, pp.125-146.

Dechenaux, E., Kovenock, D. and Sheremeta, R.M., 2015. A survey of experimental research on contests, all-pay auctions and tournaments. *Experimental Economics*, 18(4), pp.609-669.

Konrad, K.A., 2009. *Strategy and Dynamics in Contests.* Oxford, UK: Oxford University Press.

Mealem Y. and Nitzan, S., 2016. Discrimination in contests. *Review of Economic Design*, 20(2), pp.145-172.

Nitzan, S., 1994. Modelling rent-seeking contests. *European Journal of Political Economy*, 10(1), pp.41-60.Sisak, D., 2009. Multiple-prize contests – the optimal allocation of prizes. *Journal of Economic Surveys.* 23(1), pp.82-114.

Szymanski, S., 2003. The economic design of sporting contests. *Journal of Economic Literature*, 41(4), pp.1137-1187.

Vojnović, M., 2016. *Contest Theory: Incentive Mechanisms and Ranking Methods.* Cambridge University Press.

References

[1]

[2] Akerlof, R.J. and Holden, R.T., 2012. The nature of tournaments. *Economic Theory*, 51(2), pp.289-313.

[3] Alcalde, J. and Dahm, M., 2010. Rent seeking and rent dissipation: A neutrality result. *Journal of Public Economics*, 94(1-2), pp.1-7.

[4] Amann, E. and Leininger, W., 1996. Asymmetric all-pay auctions with incomplete information: The two-player case. *Games and Economic Behavior*, 14(1), pp.1-18.

[5] Amegashie, J.A., 2000. Some results on rent-seeking contests with shortlisting. *Public Choice*, 105(3-4), pp.245-253.

[6] Aoyagi, M., 2010. Information feedback in a dynamic tournament. *Games and Economic Behavior*, 70(2), pp.242-260.

[7] Arbatskaya, M. and Mialon, H.M., 2010. Multi-activity contests. *Economic Theory*, 43(1), pp.23-43.

[8] Arbatskaya, M. and Mialon, H.M., 2012. Dynamic multi-activity contests. *Scandinavian Journal of Economics*, 114(2), pp.520-538.

[9] Arce, D.G., Kovenock, D. and Roberson, B., 2012. Weakest-link attacker-defender games with multiple attack technologies. *Naval Research Logistics*, 59(6), pp.457-469.

[10] Azmat, G. and Möller, M., 2009. Competition among contests. *RAND Journal of Economics*, 40(4), pp.743-768.

[11] Azmat, G. and Möller, M., 2018. The distribution of talent across contests. *Economic Journal*, 128(609), pp.471-509.

[12] Baik, K.H., 2008. Contests with group-specific public-good prizes. *Social Choice and Welfare*, 30(1), pp.103-117.

[13] Baik, K.H., 2016. Endogenous group formation in contests: Unobservable sharing rules. *Journal of Economics & Management Strategy*, 25(2), pp.400-419.

[14] Baik, K.H. and Lee, S., 1997. Collective rent seeking with endogenous group sizes. *European Journal of Political Economy*, 13(1), pp.121-130.

[15] Baik, K.H. and Lee, S., 2007. Collective rent seeking when sharing rules are private information. *European Journal of Political Economy*, 23(3), pp.768-776.

[16] Baik, K.H. and Shogren, J.F., 1992. Strategic behavior in contests: Comment. *American Economic Review*, 82(1), pp.359-362.

[17] Balafoutas, L., Dutcher, E.G., Lindner, F. and Ryvkin, D., 2017. The optimal allocation of prizes in tournaments of heterogeneous agents. *Economic Inquiry*, 55(1), 461-478.

[18] Balart, P., Chowdhury, S.M. and Troumpounis, O., 2017. Linking individual and collective contests through noise level and sharing rules. *Economics Letters*, 155, pp.126-130.

[19] Barbieri, S. and Malueg, D.A., 2016. Private-information group contests: Best-shot competition. *Games and Economic Behavior*, 98, pp.219-234.

[20] Barbieri, S., Malueg, D.A. and Topolyan, I., 2014. The best-shot all-pay (group) auction with complete information. *Economic Theory*, 57(3), pp.603-640.

[21] Barbieri, S., Konrad, K.A. and Malueg, D.A., 2018. Preemption contests between groups. *Working Paper.*

[22] Barbieri, S., Kovenock, D., Malueg, D.A. and Topolyan, I., 2018. Group contests with private information and the "weakest link." *Working Paper.*

[23] Barbieri, S. and Serena, M., 2018. Biasing unbiased dynamic contests. *Working Paper.*

[24] Barut, Y. and Kovenock, D., 1998. The symmetric multiple prize all-pay auction with complete information. *European Journal of Political Economy*, 14(4), pp.627-644.

[25] Baye, M.R. and Hoppe, H.C., 2003. The strategic equivalence of rent-seeking, innovation, and patent-race games. *Games and Economic Behavior*, 44(2), pp.217-226.

[26] Baye, M.R., Kovenock, D. and de Vries, C.G., 1993. Rigging the lobbying process: An application of the all-pay auction. *American Economic Review*, 83(1), pp.289-294.

[27] Baye, M.R., Kovenock, D. and de Vries, C.G., 1994. The solution to the Tullock rent-seeking game when R>2: Mixed-strategy equilibria and mean dissipation rates. *Public Choice*, 81(3-4), pp.363-380.

[28] Baye, M.R., Kovenock, D. and de Vries, C.G., 1996. The all-pay auction with complete information. *Economic Theory*, 8(2), pp.291-305.

[29] Baye, M.R., Kovenock, D. and de Vries, C.G., 2005. Comparative analysis of litigation systems: An auction-theoretic approach. *Economic Journal*, 115(505), pp.583-601.

[30] Baye, M.R., Kovenock, D. and De Vries, C.G., 2012. Contests with rank-order spillovers. *Economic Theory*, 51(2), pp.315-350.

[31] Beggs, S., Cardell, S. and Hausman, J., 1981. Assessing the potential demand for electric cars. *Journal of Econometrics*, 17(1), pp.1-19.

[32] Berry, S.K., 1993. Rent-seeking with multiple winners. *Public Choice*, 77(2), pp.437-443.

[33] Beviá, C. and Corchón, L.C., 2010. Peace agreements without commitment. *Games and Economic Behavior*, 68(2), pp.469-487.

[34] Beviá, C. and Corchón, L.C., 2013. Endogenous strength in conflicts. *International Journal of Industrial Organization*, 31(3), pp.297-306.

[35] Beviá, C. and Corchón, L.C., 2015. Relative difference form contest success function. *Theory and Decision*, 78(3), pp.377-398.

[36] Blavatskyy, P.R., 2010. Contest success function with the possibility of a draw: Axiomatization. *Journal of Mathematical Economics*, 46(2), pp.267-276.

[37] Bloch, F., Sánchez-Pagés, S. and Soubeyran, R., 2006. When does universal peace prevail? Secession and group formation in conflict. *Economics of Governance*, 7(1), pp.3-29.

[38] Bodoh-Creed, A. and Hickman, B., 2018. College assignment as a large contest. *Journal of Economic Theory*, 175, pp.88-126.

[39] Boosey, L., Brookins, P. and Ryvkin, D., 2018. Contests between groups of unknown size. *Games and Economic Behavior*, forthcoming.

[40] Borel, E., 1921. La theorie du jeu les equations integrales a noyau symetrique. *Comptes Rendus del Academie* 173, pp.1304-1308; English translation by Savage L., 1953. The theory of play and integral equations with skew symmetric kernels. *Econometrica*, 21(1), pp.97-100.

[41] Borel, E. and Ville, J., 1938. *Application de la théorie des probabilités aux jeux de hasard*, Gauthier-Villars.

[42] Bozbay, I. and Vesperoni, A., 2018. A contest success function for networks. *Journal of Economic Behavior & Organization*, 150, pp.404-422.

[43] Brookins, P., Lightle, J. and Ryvkin, D., 2018. Sorting and communication in weak-link group contests. *Journal of Economic Behavior & Organization*, 152, pp.64-80.

[44] Brookins, P. and Ryvkin, D., 2016. Equilibrium existence in group contests. *Economic Theory Bulletin*, 4(2), pp.265-276.

[45] Brown, A. and Chowdhury, S.M., 2017. The hidden perils of affirmative action: Sabotage in handicap. *Journal of Economic Behavior & Organization*, 133, pp.273-284.

[46] Brown, J., 2011. Quitters never win: The (adverse) incentive effects of competing with superstars. *Journal of Political Economy*, 119(5), pp.982-1013.

[47] Bulow, J. and Levin, J., 2006. Matching and price competition, *American Economic Review*, 96(3), pp.652-668.

[48] Che, Y.K. and Gale, I.L., 1998. Caps on political lobbying. *American Economic Review*, 88(3), pp.643-651.

[49] Che, Y.K. and Gale, I.L., 2003. Optimal design of research contests. *American Economic Review*, 93(3), pp.646-671.

[50] Chen, Z., Ong, D. and Segev E., 2017. Heterogeneous risk/loss aversion in complete information all-pay auctions. *European Economic Review*, 95, pp.23-37.

[51] Choi, J.P., Chowdhury, S.M. and Kim, J., 2016. Group contests with internal conflict and power asymmetry. *Scandinavian Journal of Economics*, 118(4), pp.816-840.

[52] Chowdhury, S.M. and Gürtler, O., 2015. Sabotage in contests: A survey. *Public Choice*, 164(1), pp.135-155.

[53] Chowdhury, S.M., Jeon, J.Y. and Ramalingam, A., 2018. Property rights and loss aversion in contests. *Economic Inquiry*, 56(3), pp.1492-1511.

[54] Chowdhury, S.M. and Kim, S.H., 2017. "Small, yet beautiful": Reconsidering the optimal design of multi-winner contests. *Games and Economic Behavior*, 104, pp.486-493.

[55] Chowdhury, S.M., Lee, D. and Topolyan, I., 2016. The max-min group contest: Weakest-link (group) all-pay auction. *Southern Economic Journal*, 83(1), pp.105-125.

[56] Chowdhury, S.M., Lee, D. and Sheremata, R.M., 2013. Top guns may not fire: Best-shot group contests with group-specific public good prizes. *Journal of Economic Behavior & Organization*, 92, pp.94-103.

[57] Chowdhury, S.M. and Topolyan, I., 2016a. Best-shot versus weakest-link in political lobbying: An application of group all-pay auction. *Social Choice and Welfare*, 47(4), pp.959-971.

[58] Chowdhury, S.M. and Topolyan, I., 2016b. The attack-and-defense group contests: Best shot versus weakest link. *Economic Inquiry*, 54(1), pp.548-557.

[59] Chung, T.Y., 1996. Rent-seeking contest when the prize increases with aggregate efforts. *Public Choice*, 87(1-2), pp.55-66.

[60] Clark, D.J. and Konrad, K.A., 2007. Contests with Multi-tasking. *Scandinavian Journal of Economics*, 109(2), pp.303-319.

[61] Clark, D.J. and Nilssen, T., 2013. Learning by doing in contests. *Public Choice*, 156(1-2), pp.329-343.

[62] Clark, D.J. and Nilssen, T., 2018a. Keep on fighting: The dynamics of head starts in all-pay auctions. *Games and Economic Behavior*, 110, pp.258-272.

[63] Clark, D.J. and Nilssen, T., 2018b. Beating the Matthew effect: Head starts and catching up in a dynamic all-pay auction. *Working Paper*.

[64] Clark, D.J. and Riis, C., 1996a. On the win probability in rent-seeking games, *Mimeo*.

[65] Clark, D.J. and Riis, C., 1996b. A multi-winner nested rent-seeking contest, *Public Choice*, 87(1-2), pp.177-184.

[66] Clark, D.J. and Riis, C., 1998a. Influence and the discretionary allocation of several prizes, *European Journal of Political Economy*, 14(4), pp.605-625.

[67] Clark, D.J. and Riis, C., 1998b. Competition over more than one prize, *American Economic Review*, 88(1), pp.276-289.

[68] Clark, D.J. and Riis, C., 1998c. Contest success functions: An extension. *Economic Theory*, 11(1), pp.201-204.

[69] Clark, D.J. and Riis, C., 2000. Allocation efficiency in a competitive bribery game. *Journal of Economic Behavior & Organization*, 42(1), pp.109-124.

[70] Clark, D.J. and Riis, C., 2001. Rank-order tournaments and selection. *Journal of Economics*, 73(2), pp.167-191.

[71] Clark, D.J. and Riis, C., 2007. Contingent payments in selection contests. *Review of Economic Design*, 11(2), pp.125-137.

[72] Cohen, N., Maor, G. and Sela, A., 2018. Two-stage elimination contests with optimal head starts. *Review of Economic Design*, 22(3-4), pp.177-192.

[73] Connelly, B.L., Tihanyi, L., Crook, R.T. and Gangloff, A.K., 2014. Tournament theory: Thirty years of contests and competitions. *Journal of Management*, 40(1), pp.16-47.

[74] Congleton, R.D. and Hillman, A.L., 2015. *Companion to the Political Economy of Rent Seeking*. Edward Elgar Publishing.

[75] Connolly, R.A. and Rendleman, R.J.J., 2014. The (adverse) incentive effects of competing with superstars: A reexamination of the evidence. *Working Paper*.

[76] Corchón, L.C., 2007. The theory of contests: A survey. *Review of Economic Design*, 11(2), pp.69-100.

[77] Corchón, L.C. and Dahm, M., 2010. Foundations for contest success functions. *Economic Theory*, 43(1), pp.81-98.

[78] Corchón, L.C. and Dahm, M., 2011. Welfare maximizing contest success functions when the planner cannot commit. *Journal of Mathematical Economics*, 47(3), pp.309-317.

[79] Corchón, L.C. and Serena, M., Contest theory, 2018. In *Handbook of Game Theory and Industrial Organization, Volume II: Applications*, pp.125-146.

[80] Corchón, L.C. and Yıldızparlak, A., 2013. Give peace a chance: The effect of ownership and asymmetric information on peace. *Journal of Economic Behavior & Organization*, 92, pp.116-126.

[81] Cornes, R. and Hartley, R., 2003. Risk aversion, heterogeneity and contests. *Public Choice*, 117(1-2), pp.1-25.

[82] Cornes, R. and Hartley, R., 2005. Asymmetric contests with general technologies. *Economic Theory*, 26(4), pp.923-946.

[83] Cornes, R. and Hartley, R., 2012. Risk aversion in symmetric and asymmetric contests. *Economic Theory*, 51(2), pp.247-275.

[84] Dahm, M., 2018. Semi-targeted all-pay auctions: A partial exclusion principle. *International Journal of Industrial Organization*, 61, pp.256-282.

[85] Dahm, M. and Esteve-González, P., 2018. Affirmative action through extra prizes. *Journal of Economic Behavior & Organization*, 153, pp.123-142.

[86] Dasgupta, P., 1986. The theory of technological competition. In *New Developments in the Analysis of Market Structure* (pp.519-549). Palgrave Macmillan, London.

[87] Dasgupta, P. and Stiglitz, J.E., 1980. Industrial structure and the nature of innovative activity. *Economic Journal*, 90(358), pp.266-293.

[88] Dato, S., Grunewald, A. and Müller, D., 2018. Expectation-based loss aversion and rank-order tournaments. *Economic Theory*, 66(4), pp.901-928.

[89] Dato, S., Grunewald, A., Müller, D. and Strack, P., 2017. Expectation-based loss aversion and strategic interaction. *Games and Economic Behavior*, 104, pp.681-705.

[90] Davis, D.D. and Reilly, R.J., 1999. Rent-seeking with non-identical sharing rules: An equilibrium rescued. *Public Choice*, 100(1-2), pp.31-38.

[91] Dechenaux, E., Kovenock, D. and Sheremeta, R.M., 2015. A survey of experimental research on contests, all-pay auctions and tournaments. *Experimental Economics*, 18(4), pp.609-669.

[92] Deng, S., Wang, X. and Wu, Z., 2018. Incentives in lottery contests with draws. *Economics Letters*, 163, pp.1-5.

[93] Denter, P., Morgan, J. and Sisak, D., 2018. Showing off or laying low? The economics of psych-outs. *Working Paper.*

[94] Denter, P. and Sisak, D., 2016. Head starts in dynamic tournaments?. *Economics Letters*, 149, pp.94-97.

[95] DeVaro, J. and Gürtler, O., 2016. Strategic shirking: A theoretical analysis of multitasking and specialization. *International Economic Review*, 57(2), pp.507-532.

[96] Dixit, A., 1987. Strategic behavior in contests. *American Economic Review*, 77(5), pp.891-898.

[97] Drugov, M. and Ryvkin, D., 2017. Biased contests for symmetric players. *Games and Economic Behavior*, 103, pp.116-144.

[98] Drugov, M. and Ryvkin, D., 2018. Tournament rewards and heavy tails. *Working Paper.*

[99] Dziubiński, M. and Goyal, S., 2013. Network design and defense. *Games and Economic Behavior*, 79, pp.30-43.

[100] Dziubiński, M. and Goyal, S., 2017. How do you defend a network?. *Theoretical Economics*, 12(1), pp.331-376.

[101] Dziubiński, M., and Goyal, S., and Minarsch, D.E.N., 2018. The strategy of conquest. working paper.

[102] Eden, M., 2007. Optimal ties in contests. *working paper.*

[103] Ederer, F., 2010. Feedback and motivation in dynamic tournaments. *Journal of Economics & Management Strategy*, 19(3), pp.733-769.

[104] Einy, E., Haimanko, O., Moreno, D., Sela, A. and Shitovitz, B., 2015. Equilibrium existence in Tullock contests with incomplete information. *Journal of Mathematical Economics*, 61, pp.241-245.

[105] Einy, E., Moreno, D. and Shitovitz, B., 2017. The value of public information in common-value Tullock contests. *Economic Theory*, 63(4), pp.925-942.

[106] Eliaz, K. and Wu, Q., 2018. A simple model of competition between teams. *Journal of Economic Theory*, 176, pp.372-392.

[107] Epstein, G.S., Mealem, Y. and Nitzan, S., 2011. Political culture and discrimination in contests. *Journal of Public Economics*, 95(1-2), pp.48-60.

[108] Epstein, G.S., Mealem, Y. and Nitzan, S., 2013. Lottery vs. all-pay auctions in fair and biased contests. *Economics & Politics*, 25(1), pp.88-93.

[109] Esteban, J. and Ray, D., 1999. Conflict and distribution, *Journal of Economic Theory*, 87(2), pp.379-415.

[110] Esteban, J. and Ray, D., 2001. Collective action and the group size paradox. *American Political Science Review*, 95(3), pp.663-672.

[111] Esteban, J. and Ray, D., 2008. On the salience of ethnic conflict. *American Economic Review*, 98(5), pp.2185-2202.

[112] Ewerhart, C., 2014. Unique equilibrium in rent-seeking contests with a continuum of types. *Economics Letters*, 125(1), pp.115-118.

[113] Ewerhart, C., 2015. Mixed equilibria in Tullock contests. *Economic Theory*, 60(1), pp.59-71.

[114] Ewerhart, C., 2017a. Contests with small noise and the robustness of the all-pay auction. *Games and Economic Behavior*, 105, pp.195-211.

[115] Ewerhart, C., 2017b. Revenue ranking of optimally biased contests: The case of two players. *Economics Letters*, 157, pp.167-170.

[116] Ewerhart, C. and Quartieri, F., 2016. Unique equilibrium in contests with incomplete information. *Working Paper.*

[117] Fang, D. and Noe, T., 2018. Lowering the bar and limiting the field: The effect of strategic risk-taking on selection contests. *Working Paper.*

[118] Fang, D., Noe, T. and Strack, P., 2018. Turning up the heat: The demoralizing effect of competition in contests. *Working Paper.*

[119] Fang, H., 2002. Lottery versus all-pay auction models of lobbying. *Public Choice*, 112(3-4), pp.351-371.

[120] Feng, X. and Lu, J., 2016. The optimal disclosure policy in contests with stochastic entry: A Bayesian persuasion perspective. *Economics Letters*, 147, pp.103-107.

[121] Feng, X. and Lu, J., 2017. Uniqueness of equilibrium in two-player asymmetric Tullock contests with intermediate discriminatory power. *Economics Letters*, 159, pp.61-64.

[122] Feng, X. and Lu, J., 2018. How to split the pie: Optimal rewards in dynamic multi-battle competitions. *Journal of Public Economics*, 160, pp.82-95.

[123] Fey, M., 2008. Rent-seeking contests with incomplete information. *Public Choice*, 135(3-4), pp.225-236.

[124] Franke, J., 2012. Affirmative action in contest games. *European Journal of Political Economy*, 28(1), pp.105-118.

[125] Franke, J., Kanzow, C., Leininger, W. and Schwartz, A., 2013. Effort maximization in asymmetric contest games with heterogeneous contestants. *Economic Theory*, 52(2), pp.589-630.

[126] Franke, J., Kanzow, C., Leininger, W. and Schwartz, A., 2014. Lottery versus all-pay auction contests: A revenue dominance theorem. *Games and Economic Behavior*, 83, pp.116-126.

[127] Franke, J., Leininger, W. and Wasser, C., 2018. Optimal favoritism in all-pay auctions and lottery contests. *European Economic Review*, 104, pp.22-37.

[128] Franke, J. and Öztürk, T., 2018. Conflict networks. *Journal of Public Economics*, 126, pp.104-113.

[129] Friedman, L., 1958. Game-theory models in the allocation of advertising expenditures. *Operations Research*, 6(5), pp.699-709.

[130] Fu, Q., 2005. Litigation, settlement and legal presumptions. *Working Paper.*

[131] Fu, Q., 2006a. Endogenous timing of contest with asymmetric information. *Public Choice*, 129(1-2), pp.1-23.

[132] Fu, Q., 2006b. A theory of affirmative action in college admissions. *Economic Inquiry*, 44(3), pp.420-428.

[133] Fu, Q., 2008. Litigation, settlement and legal presumption. *Working Paper.*

[134] Fu, Q. and Iyer, G., 2019. Multimarket value creation and competition. *Marketing Science*, 38(1), pp.129-149.

[135] Fu, Q., Jiao, Q. and Lu, J., 2011. On disclosure policy in contests with stochastic entry. *Public Choice*, 148(3-4), pp.419-434.

[136] Fu, Q., Jiao, Q. and Lu, J., 2014. Disclosure policy in a multi-prize all-pay auction with stochastic abilities. *Economics Letters*, 125(3), pp.376-380.

[137] Fu, Q., Jiao, Q. and Lu, J., 2015. Contests with endogenous entry. *International Journal of Game Theory*, 44(2), pp.387-424.

[138] Fu, Q. and Lu, J., 2009. The beauty of "bigness": On optimal design of multi-winner contests. *Games and Economic Behavior*, 66(1), pp.146-161.

[139] Fu, Q. and Lu, J., 2010. Contest design and optimal endogenous entry. *Economic Inquiry*, 48(1), pp.80-88.

[140] Fu, Q. and Lu, J., 2012a. The optimal multi-stage contest. *Economic Theory*, 51(2), pp.351-382.

[141] Fu, Q. and Lu, J., 2012b. Micro foundations of multi-prize lottery contests: A perspective of noisy performance ranking. *Social Choice and Welfare*, 38(3), pp.497-517.

[142] Fu, Q. and Lu, J., 2013. Competitive effects of cross-shareholdings in all-pay auctions with complete information. *International Journal of Industrial Organization*, 31(3), pp.267-277.

[143] Fu, Q. and Lu, J., 2018. On equilibrium player ordering in dynamic team contests. *Working Paper.*

[144] Fu, Q., Lu, J. and Lu, Y., 2012. Incentivizing R&D: Prize or subsidies?. *International Journal of Industrial Organization*, 30(1), pp.67-79.

[145] Fu, Q., Lu, J. and Pan, Y., 2015. Team contests with multiple pairwise battles. *American Economic Review*, 105(7), pp.2120-2140.

[146] Fu, Q., Lu, J. and Wang, Z., 2014. “Reverse” nested lottery contests. *Journal of Mathematical Economics*, 50, pp.128-140.

[147] Fu, Q., Lu, J. and Zhang, J., 2016. Disclosure policy in Tullock contests with asymmetric stochastic entry. *Canadian Journal of Economics*, 49(1), pp.52-75.

[148] Fu, Q., Gürtler, O. and Münster, J., 2013. Communication and commitment in contests. *Journal of Economic Behavior & Organization*, 95, pp.1-19.

[149] Fu, Q. and Wu, Z., 2018a. On the optimal design of lottery contests. *Working Paper.*

[150] Fu, Q. and Wu, Z., 2018b. Feedback and favoritism in sequential elimination contests. *Working Paper.*

[151] Fu, Q. and Wu, Z., 2018c. Prize allocation in symmetric Tullock contests with risk-averse contestants. *Working Paper.*

[152] Fu, Q., Wu, Z., Lyu Y. and Zhang Y., 2018. Incentives and effort maximization in asymmetric contests under expectation-based loss-aversions. *Working Paper.*

[153] Fullerton, R.L. and McAfee, R.P., 1999. Auctioning entry into tournaments. *Journal of Political Economy*, 107(3), pp.573-605.

[154] Gelder, A., 2014. From Custer to Thermopylae: Last stand behavior in multi-stage contests. *Games and Economic Behavior*, 87, pp.442-466.

[155] Gelder, A., Kovenock, D. and Roberson, B., 2016. All-pay auctions with ties. *Working Paper.*

[156] Gershkov, A. and Perry, M., 2009. Tournaments with midterm reviews. *Games and Economic Behavior*, 66(1), pp.162-190.

[157] Gill, D. and Prowse, V., 2012. A structural analysis of disappointment aversion in a real effort competition. *American Economic Review*, 102(1), pp.469-503.

[158] Gill, D. and Stone, R., 2010. Fairness and desert in tournaments. *Games and Economic Behavior*, 69(2), pp.346-364.

[159] Gilpatric, S.M., 2011. Cheating in contests. *Economic Inquiry*, 49(4), pp.1042-1053.

[160] Glazer, A. and Hassin, R., 1988. Optimal contests. *Economic Inquiry*, 26(1), pp.133-143.

[161] Goltsman, M. and Mukherjee, A., 2011. Interim performance feedback in multistage tournaments: The optimality of partial disclosure. *Journal of Labor Economics*, 29(2), pp.229-265.

[162] Goyal, S. and Vigier, A., 2014. Attack, defense, and contagion in networks. *Review of Economic Studies*, 81(4), 1518-1542.

[163] Gradstein, M. and Konrad, K.A., 1999. Orchestrating rent seeking contests. *Economic Journal*, 109(458), pp.536-545.

[164] Green, J.R. and Stokey, N.L., 1983. A comparison of tournaments and contracts. *Journal of Political Economy*, 91(3), pp.349-364.

[165] Groh, C., Moldovanu, B., Sela, A. and Sunde, U., 2012. Optimal seedings in elimination tournaments. *Economic Theory*, 49(1), pp.59-80.

[166] Gross, O. and Wagner, R., 1950. A continuous Colonel Blotto game. RM-408, RAND Corporation, Santa Monica.

[167] Grossmann, M., 2018. Participation, contribution and distribution in parallel contests. *Working Paper*.

[168] Gürtler, M. and Gürtler, O., 2015. The optimality of heterogeneous tournaments. *Journal of Labor Economics*, 33(4), pp.1007-1042.

[169] Häfner, S., 2017. A tug-of-war team contest. *Games and Economic Behavior*, 104, pp.372-391.

[170] Häfner, S. and Konrad, K.A., 2016. Eternal peace in the tug-of-war?. *Working Paper*.

[171] Harbaugh, R. and Klumpp, T., 2005. Early round upsets and championship blowouts. *Economic Inquiry*, 43(2), pp.316-329.

[172] Harris, C. and Vickers, J., 1987. Racing with uncertainty. *Review of Economic Studies*, 54(1), pp.1-21.

[173] Higgins, R.S., Shughart, W.F. and Tollison, R.D., 1985. Free entry and efficient rent seeking. *Public Choice*, 46(3), pp.247-258.

[174] Hillman, A.L. and Katz, E., 1984. Risk-averse rent seekers and the social cost of monopoly power. *Economic Journal*, 94(373), pp.104-110.

[175] Hillman, A.L. and Riley, J.G., 1989. Politically contestable rents and transfers. *Economics & Politics*, 1(1), pp.17-39.

[176] Hillman, A.L. and Samet, D., 1987. Dissipation of contestable rents by small numbers of contenders. *Public Choice*, 54(1), pp.63-82.

[177] Hinnosaar, T., 2018. Optimal sequential contests. *Working Paper.*

[178] Hirata, D., 2014. A model of a two-stage all-pay auction. *Mathematical Social Sciences*, 68, pp.5-13.

[179] Hirshleifer, J., 1988. The analytics of continuing conflict. *Synthese*, 76(2), pp.201-233.

[180] Hirshleifer, J., 1989. Conflicting and rent-seeking success functions: Ratio vs. difference models of relative success. *Public Choice*, 63(2), pp.101-112.

[181] Holmström, B., 1982. Managerial incentive schemes: A dynamic perspective, in *Essays in Economics and Management in Honour of Lars Wahlbeck.*

[182] Holmström, B., 1999. Managerial incentive problems: A dynamic perspective. *Review of Economic Studies*, 66(1), pp.169-182.

[183] Hurley, T.M. and Shogren, J.F., 1998a. Asymmetric information contests. *European Journal of Political Economy*, 14(4), pp.645-665.

[184] Hurley, T.M. and Shogren, J.F., 1998b. Effort levels in a Cournot Nash contest with asymmetric information. *Journal of Public Economics*, 69(2), pp.195-210.

[185] Hvide, H.K., 2002. Tournament rewards and risk taking. *Journal of Labor Economics*, 20(4), pp.877-898.

[186] Imhof, L. and Kräkel, M., 2014. Tournaments with gaps. *Economics Letters*, 122(2), pp.211-214.

[187] Imhof, L. and Kräkel, M., 2016. Ex post unbalanced tournaments. *RAND Journal of Economics*, 47(1), pp.73-98.

[188] Jia, H., 2008. A stochastic derivation of the ratio form of contest success functions. *Public Choice*, 135(3-4), pp.125-130.

[189] Jia, H., 2012. Contests with the probability of a draw: A stochastic foundation. *Economic Record*, 88(282), pp.391-406.

[190] Jia, H., Skaperdas, S. and Vaidya, S., 2013. Contest functions: Theoretical foundations and issues in estimation. *International Journal of Industrial Organization*, 31(3), pp.211-222.

[191] Jiao, Q., Shen, B. and Sun, X., 2019. Bipartite conflict networks with returns to scale technology. *Working Paper.*

[192] Joffrion, J. and Parreiras, S.O., 2013. A tortoise and a hare race. *Working Paper.*

[193] Jindapon, P. and Whaley, C.A., 2015. Risk lovers and the rent over-investment puzzle. *Public Choice*, 164(1-2), pp.87-101.

[194] Kahana, N. and Klunover, D., 2018. Sequential lottery contests with multiple participants. *Economics Letters*, 163, pp.126-129.

[195] Kawamura, K. and Moreno de Barreda, I., 2014. Biasing selection contests with ex-ante identical agents. *Economics Letters*, 123(2), pp.240-243.

[196] Kaplan, T.R., 2012. Communication of preferences in contests for contracts. *Economic Theory*, 51(2), pp.487-503.

[197] Kaplan, T.R., Luski, I., Sela, A. and Wettstein, D., 2002. All-pay auctions with variable rewards. *Journal of Industrial Economics*, 50(4), pp.417-430.

[198] Kaplan, T.R. and Sela, A., 2010. Effective contests. *Economics Letters*, 106(1), pp.38-41.

[199] Kaplan, T.R. and Wettstein, D., 2006. Caps on political lobbying: Comment. *American Economic Review*, 96(4), pp.1351-1354.

[200] Kirkegaard, R., 2012. Favoritism in asymmetric contests: Head starts and handicaps. *Games and Economic Behavior*, 76(1), pp.226-248.

[201] Klose, B. and Kovenock, D., 2015a. The all-pay auction with complete information and identity-dependent externalities. *Economic Theory*, 59(1), pp.1-19.

[202] Klose, B. and Kovenock, D., 2015b. Extremism drives out moderation. *Social Choice and Welfare*, 44(4), pp.861-887.

[203] Klumpp, T. and Polborn, M.K., 2006. Primaries and the New Hampshire effect. *Journal of Public Economics*, 90(6-7), pp.1073-1114.

[204] Klumpp, T. and Konrad, K.A., 2018. Sequential Majoritarian Blotto Games. *Working Paper.*

[205] Klunover, D., 2018. A note on rent dissipation in lottery contests. *Economics Letters*, 168, pp.90-93.

[206] Kolmar, M. and Rommeswinkel, H., 2013. Contests with group-specific public goods and complementarities in efforts. *Journal of Economic Behavior & Organization*, 89, pp.9-22.

[207] König, M.D., Rohner, D., Thoenig, M. and Zilibotti, F., 2017. Networks in conflict: Theory and evidence from the great war of Africa. Econometrica, 85(4), pp.1093-1132.

[208] Konrad, K.A., 2002. Investment in the absence of property rights; the role of incumbency advantages. *European Economic Review*, 46(8), pp.1521-1537.

[209] Konrad, K., 2006. Silent interest and all-pay auctions. *International Journal of Industrial Organization,* 24(4), pp.701–713.

[210] Konrad, K.A., 2009. *Strategy and Dynamics in Contests.* Oxford, UK: Oxford University Press.

[211] Konrad, K.A., 2018. Budget and effort choice in sequential Colonel Blotto campaigns. *CESifo Economic Studies*, 64(4), pp.555-576.

[212] Konrad, K.A. and Kovenock, D., 2005. Equilibrium and efficiency in the tug-of-war. *Working Paper.*

[213] Konrad, K.A. and Kovenock, D., 2009a. Multi-battle contests. *Games and Economic Behavior*, 66(1), pp.256-274.

[214] Konrad, K.A. and Kovenock, D., 2009b. The alliance formation puzzle and capacity constraints. *Economics Letters*, 103(2), pp.84-86.

[215] Konrad, K.A. and Kovenock, D., 2010. Contests with stochastic abilities. *Economic Inquiry*, 48(1), pp.89-103.

[216] Konrad, K.A. and Kovenock, D., 2012. The lifeboat problem. *European Economic Review*, 56(3), pp.552-559.

[217] Konrad, K.A. and Leininger, W., 2007. The generalized Stackelberg equilibrium of the all-pay auction with complete information. *Review of Economic Design*, 11(2), pp.165-174.

[218] Konrad, K.A. and Schlesinger, H., 1997. Risk aversion in rent-seeking and rent-augmenting games. *Economic Journal*, 107(445), pp.1671-1683.

[219] Kotowski, M.H. and Li, F., 2014. On the continuous equilibria of affiliated-value, all-pay auctions with private budget constraints. *Games and Economic Behavior*, 85, pp.84-108.

[220] Kovenock, D., Morath, F. and Münster, J., 2015. Information sharing in contests. *Journal of Economics & Management Strategy*, 24(3), pp.570-596.

[221] Kovenock, D. and Roberson, B., 2009. Is the 50-state strategy optimal?. *Journal of Theoretical Politics*, 21(2), pp.213-236.

[222] Kovenock, D. and Roberson, B., 2012. Coalitional Colonel Blotto games with application to the economics of alliances. *Journal of Public Economic Theory*, 14(4), pp.653-676.

[223] Kovenock, D. and Roberson, B., 2018. The optimal defense of networks of targets. *Economic Inquiry*, 56(4), pp.2195-2211.

[224] Kovenock, D., Sarangi, S. and Wiser, M., 2015. All-pay 2×2 Hex: A multibattle contest with complementarities. *International Journal of Game Theory*, 44(3), pp.571-597.

[225] Krishna, V. and Morgan, J., 1997. An analysis of the war of attrition and the all-pay auction. *Journal of Economic Theory*, 72(2), pp.343-362.

[226] Krishna, V. and Morgan, J., 1998. The winner-take-all principle in small tournaments. *Advances in Applied Microeconomics*, 7, pp.61-74.

[227] Kvasov, D., 2007. Contests with limited resources. *Journal of Economic Theory*, 136(1), pp.738-748.

[228] Lake, M., 1979. A new campaign resource allocation model. In *Applied Game Theory* (pp.118-132). Physica, Heidelberg.

[229] Lang, M., Seel, C. and Strack, P., 2014. Deadlines in stochastic contests. *Journal of Mathematical Economics*, 52, pp.134-142.

[230] Laslier, J.F., 2002. How two-party competition treats minorities. *Review of Economic Design*, 7(3), pp.297-307.

[231] Laslier, J.F. and Picard, N., 2002. Distributive politics and electoral competition. *Journal of Economic Theory*, 103(1), pp.106-130.

[232] Lazear, E.P. and Rosen, S., 1981. Rank-order tournaments as optimum labor contracts. *Journal of Political Economy*, 89(5), pp.841-864.

[233] Leininger, W., 1993. More efficient rent-seeking – a Münchhausen solution. *Public Choice*, 75(1), pp.43-62.

[234] Li, S. and Yu, J., 2012. Contests with endogenous discrimination. *Economics Letters*, 117(3), pp.834-836.

[235] Liu, B., Lu, J., Wang, R. and Zhang, J., 2018. Optimal prize allocation in contests: The role of negative prizes. *Journal of Economic Theory*, 175, pp.291-317.

[236] Lim, W. and Matros, A., 2009. Contests with a stochastic number of players. *Games and Economic Behavior*, 67(2), pp.584-597.

[237] Linster, B.G., 1993. A generalized model of rent-seeking behavior. *Public Choice*, 77(2), pp.421-435.

[238] Liu, L, Meyer J., Rettenmaier A.J. and Saving T.R., 2018. Risk and risk aversion effects in contests with contingent payments. *Journal of Risk and Uncertainty*, 56(3), pp.289-305.

[239] Loury, G.C., 1979. Market structure and innovation. *Quarterly Journal of Economics*, 93(3), pp.395-410.

[240] Lu, J., Ma, H. and Wang, Z., 2018. Ranking disclosure policies in all-pay auctions. *Economic Inquiry*, 56(3), pp.1464-1485.

[241] Lu, J. and Parreiras, S.O., 2017. Monotone equilibrium of two-bidder all-pay auctions Redux. *Games and Economic Behavior*, 104, pp.78-91.

[242] Lu, J., Shen, B. and Wang, Z., 2019. Performance bundling across multiple competitions. *Working Paper.*

[243] Lu, J. and Wang, Z., 2015. Axiomatizing multi-prize nested lottery contests: A complete and strict ranking perspective. *Journal of Economic Behavior & Organization*, 116, pp.127-141.

[244] Lu, J. and Wang, Z., 2016. Axiomatization of reverse nested lottery contests. *Social Choice and Welfare*, 47(4), pp.939-957.

[245] Luce, R.D. and Suppes, P., 1965. Preference, utility, and subjective probability In Luce R.D., Bush R.R., & Galanter E. (eds.), *Handbook of Mathematical Psychology*, Vol. III.

[246] Ludwig, S., 2012. Contests – a comparison of timing and information structures. *Public Choice*, 153(3-4), pp.341-355.

[247] Malueg, D.A. and Yates, A.J., 2004. Sent seeking with private values. *Public Choice*, 119(1-2), pp.161-178.

[248] Malueg, D.A. and Yates, A.J., 2010. Testing contest theory: Evidence from best-of-three tennis matches. *Review of Economics and Statistics*, 92(3), pp.689-692.

[249] Manski, C. and McFadden, D., *Structural Analysis of Discrete Data with Econometric Applications*, Cambridge: The MIT Press, 1981.

[250] McAfee R.P., 2000. Continuing wars of attrition. *Working Paper.*

[251] McFadden, D., 1973. Conditional logit analysis of qualitative choice behavior, In Zarembka P. (ed.), *Frontier in Econometrics.*

[252] McFadden, D., 1974. The measurement of urban travel demand. *Journal of Public Economics*, 3(4), pp.303-328.

[253] Mealem Y. and Nitzan, S., 2016. Discrimination in contests. *Review of Economic Design*, 20(2), pp.145-172.

[254] Megidish, R. and Sela, A., 2014. Sequential contests with synergy and budget constraints. *Social Choice and Welfare*, 42(1), pp.215-243.

[255] Mendel, M., Pieroth F. and Seel C., 2018. Multi-stage intragroup elimination contests. *Working Paper.*

[256] Mildenberger, C.D. and Pietri, A., 2018. How does size matter for military success? Evidence from virtual worlds. *Journal of Economic Behavior & Organization*, 154, pp.137-156.

[257] Minchuk, Y., and Sela, A., 2014. All-pay auctions with certain and uncertain prizes. *Games and Economic Behavior* 88, pp.130–134.

[258] Miklós-Thal, J. and Ullrich, H., 2015. Belief precision and effort incentives in promotion contests. *Economic Journal*, 125(589), pp.1952-1963.

[259] Moldovanu, B. and Sela, A., 2001. The optimal allocation of prizes in contests. *American Economic Review*, 91(3), pp.542-558.

[260] Moldovanu, B. and Sela, A., 2006. Contest architecture. *Journal of Economic Theory*, 126(1), pp.70-96.

[261] Moldovanu, B., Sela, A. and Shi, X., 2007. Contests for status. *Journal of Political Economy*, 115(2), pp.338-363.

[262] Moldovanu, B., Sela, A. and Shi, X., 2012. Carrots and sticks: Prizes and punishments in contests. *Economic Inquiry*, 50(2), pp.453-462.

[263] Morath, F. and Münster, J., 2013. Information acquisition in conflicts. *Economic Theory*, 54(1), pp.99-129.

[264] Morgan, J., 2003. Sequential contests. *Public Choice*, 116(1-2), pp.1-18.

[265] Morgan, J., Orzen, H. and Sefton, M., 2012. Endogenous entry in contests. *Economic Theory*, 51(2), pp.435-463.

[266] Morgan, J., Sisak, D. and Várdy, F., 2018. The ponds dilemma. *Economic Journal*, 128(611), pp.1634-1682.

[267] Morgan, J., Tumlinson J. and Várdy, F., 2018. The limits of meritocracy. *Working Paper.*

[268] Morgan, J. and Várdy, F., 2007. The value of commitment in contests and tournaments when observation is costly. *Games and Economic Behavior*, 60(2), pp.326-338.

[269] Müller, H.M. and Wärneryd, K., 2001. Inside versus outside ownership: A political theory of the firm. *RAND Journal of Economics*, 32(3), pp.527-541.

[270] Münster, J., 2006. Contests with an unknown number of contestants. *Public Choice*, 129(3-4), pp.353-368.

[271] Münster, J., 2009. Group contest success functions. *Economic Theory*, 41(2), pp.345-357.

[272] Myerson, R.B. and Wärneryd, K., 2006. Population uncertainty in contests. *Economic Theory*, 27(2), pp.469-474.

[273] Nalebuff, B.J. and Stiglitz, J.E., 1983. Prizes and incentives: Towards a general theory of compensation and competition. *The Bell Journal of Economics*, 14(1), pp.21-43.

[274] Nitzan, S., 1991a. Collective rent dissipation. *Economic Journal*, 101(409), pp.1522-1534.

[275] Nitzan, S., 1991b. Rent-seeking with non-identical sharing rules. *Public Choice*, 71(1-2), pp.43-50.

[276] Nitzan, S., 1994. Modelling rent-seeking contests. *European Journal of Political Economy*, 10(1), pp.41-60.

[277] Nitzan, S. and Ueda, K., 2009. Collective contests for commons and club goods. *Journal of Public Economics*, 93(1-2), pp.48-55.

[278] Nitzan, S. and Ueda, K., 2011. Prize sharing in collective contests. *European Economic Review*, 55(5), pp.678-687.

[279] Nitzan, S. and Ueda, K., 2018. Selective incentives and intragroup heterogeneity in collective contests. *Journal of Public Economic Theory*, 20(4), pp.477-498.

[280] Nti, K.O., 1997. Comparative statics of contests and rent-seeking games. *International Economic Review*, 38(1), pp.43-59.

[281] Olson, M.C., 1965. *The Logic of Collective Action: Public Goods and the Theory of Groups*, Vol. 124. *Harvard Economic Studies.* Cambridge: Harvard University Press.

[282] Olson, M.C., 1982. *The Rise and Decline of Nations: Economic Growth, Stagflation, and Economic Rigidities.* Yale University Press, New Haven and London.

[283] Olszewski, W. and Siegel, R., 2016. Large contests. *Econometrica*, 84(2), pp.835-854.

[284] Olszewski, W. and Siegel, R., 2018a. Performance-maximizing contests. *Working Paper.*

[285] Olszewski, W. and Siegel, R., 2018b. Large contests without single crossing. *Working Paper.*

[286] Olszewski, W. and Siegel, R., 2018c. Bid caps in large contests. *Working Paper.*

[287] Olszewski, W. and Siegel, R., 2018d. Pareto improvements in the contest for college admissions. *Working Paper.*

[288] Pastine, I. and Pastine, T., 2012. Incumbency advantage and political campaign spending limits. *Journal of Public Economics*, 96(1-2), pp.20-32.

[289] Pérez-Castrillo, D. and Wettstein, D., 2016. Discrimination in a model of contests with incomplete information about ability. *International Economic Review*, 57(3), pp.881-914.

[290] Rai, B.K. and Sarin, R., 2009. Generalized contest success functions. *Economic Theory*, 40(1), pp.139-149.

[291] Rietzke, D. and Roberson, B., 2013. The robustness of "enemy-of-my-enemy-is-my-friend" alliances. *Social Choice and Welfare*, 40(4), pp.937-956.

[292] Roberson, B., 2006. The Colonel Blotto game. *Economic Theory*, 29(1), pp.1-24.

[293] Roberson, B., 2008. Pork-barrel politics, targetable policies, and fiscal federalism. *Journal of the European Economic Association*, 6(4), pp.819-844.

[294] Roberson, B. and Kvasov, D., 2012. The non-constant-sum Colonel Blotto game. *Economic Theory*, 51(2), pp.397-433.

[295] Rosen, S., 1986. Prizes and incentives in elimination tournaments. *American Economic Review*, 76(4), pp.701-715.

[296] Ryvkin, D., 2007. Tullock contests of weakly heterogeneous players. *Public Choice*, 132(1-2), pp.49-64.

[297] Ryvkin, D., 2009. Tournaments of weakly heterogeneous players. *Journal of Public Economic Theory*, 11(5), pp.819-855.

[298] Ryvkin, D., 2010a. Contests with private costs: Beyond two players. *European Journal of Political Economy*, 26(4), pp.558-567.

[299] Ryvkin, D., 2010b. The selection efficiency of tournaments. *European Journal of Operational Research*, 206(3), pp.667-675.

[300] Ryvkin, D., 2011a. Fatigue in dynamic tournaments. *Journal of Economics & Management Strategy*, 20(4), pp.1011-1041.

[301] Ryvkin, D., 2011b. The optimal sorting of players in contests between groups. *Games and Economic Behavior*, 73(2), pp.564-572.

[302] Ryvkin, D., 2013a. Heterogeneity of players and aggregate effort in contests. *Journal of Economics & Management Strategy*, 22(4), pp.728-743.

[303] Ryvkink, D., 2013b. Contests with doping. *Journal of Sports Economics*, 14(3), pp.253-275.

[304] Ryvkin, D. and Ortmann, A., 2008. The predictive power of three prominent tournament formats. Management Science, 54(3), pp.492-504.

[305] Sahm, M., 2017. Risk aversion and prudence in contests. *Working Paper.*

[306] Sánchez-Pagés, S., 2007. Endogenous coalition formation in contests. *Review of Economic Design*, 11(2), pp.139-163.

[307] Schroyen, F. and Treich, N., 2016. The power of money: Wealth effects in contests. *Games and Economic Behavior*, 100, pp.46-68.

[308] Schweinzer, P. and Segev, E., 2012. The optimal prize structure of symmetric Tullock contests. *Public Choice*, 153(1-2), pp.69-82.

[309] Seel, C. and Strack, S., 2013. Gambling in contests. *Journal of Economic Theory*, 148(5), pp.2033-2048.

[310] Seel, C. and Wasser, C., 2014. On optimal head starts in all-pay auctions. *Economics Letters*, 124(2), pp.211-214.

[311] Segev, E. and Sela, A., 2014. Multi-stage sequential all-pay auctions. *European Economic Review*, 70, pp.371-382.

[312] Sela, A., 2012. Sequential two-prize contests. *Economic Theory*, 51(2), pp.383-395.

[313] Sela, A., 2017. Two-stage contests with effort-dependent values of winning. *Review of Economic Design*, 21(4), pp.253-272.

[314] Sen, A.K., 1966. Labour allocation in a cooperative enterprise. *Review of Economic Studies*, 33(4), pp.361-371.

[315] Serena, M., 2017. Harnessing beliefs to stimulate efforts. *Working Paper.*

[316] Siegel, R., 2009. All-pay contests. *Econometrica*, 77(1), pp.71-92.

[317] Siegel, R., 2010. Asymmetric contests with conditional investments. *American Economic Review*, 100(5), pp.2230–2260.

[318] Siegel, R., 2014a. Asymmetric contests with head starts and nonmonotonic costs. *American Economic Journal: Microeconomics*, 6(3), pp.59-105.

[319] Siegel, R., 2014b. Asymmetric all-pay auctions with interdependent valuations. *Journal of Economic Theory*, 153, pp.684-702.

[320] Singh, N. and Wittman, D., 2001. Contests where there is a variation in the marginal productivity of effort. *Economic Theory*, 18(3), pp.711-744.

[321] Sisak, D., 2009. Multiple-prize contests – the optimal allocation of prizes. *Journal of Economic Surveys*. 23(1), pp.82-114.

[322] Skaperdas, S., 1996. Contest success functions. *Economic Theory*, 7(2), pp.283-290.

[323] Skaperdas, S., 1998. On the formation of alliances in conflict and contests. *Public Choice*, 96(1-2), pp.25-42.

[324] Skaperdas, S. and Gan, L., 1995. Risk aversion in contests. *Economic Journal*, 105(431), pp.951-962.

[325] Snyder, J.M., 1989. Election goals and the allocation of campaign resources. *Econometrica*, 57(3), pp.637-660.

[326] Strack, P., 2016. Risk-taking in contests: The impact of fund manager compensation on investor welfare. *Working Paper.*

[327] Strumpf, K.S., 2002. Strategic competition in sequential election contests. *Public Choice*, 111(3-4), pp.377-397.

[328] Suppes, P., Krantz, D.H., Luce, R.D. and Tversky, A., 1989. Foundations of Measurement, Vol. II: Geometrical, Threshold, and Probability Representations.

[329] Szidarovszky, F. and Okuguchi, K., 1997. On the existence and uniqueness of pure Nash equilibrium in rent-seeking games. *Games and Economic Behavior*, 18(1), pp.135-140.

[330] Szymanski, S., 2003. The economic design of sporting contests. *Journal of Economic Literature*, 41(4), pp.1137-1187.

[331] Szymanski, S. and Valletti, T.M., 2005. Incentive effects of second prizes. *European Journal of Political Economy*, 21(2), pp.467-481.

[332] Taylor, C.R., 1995. Digging for golden carrots: An analysis of research tournaments. *American Economic Review*, 85(4), pp.872-890.

[333] Thomas, C., 2018. N-dimensional Blotto game with heterogeneous battlefield values. *Economic Theory*, 65(3), pp.509-544.

[334] Thomas, J., and Wang, Z., 2013. Optimal punishment in contests with endogenous entry. *Journal of Economic Behavior and Organization*, 91, pp.34-50.

[335] Treich, N., 2010. Risk-aversion and prudence in rent-seeking games. *Public Choice*, 145(3-4), pp.339-349.

[336] Tullock, G., 1967. The welfare costs of tariffs, monopolies, and theft. *Economic Inquiry*, 5(3), pp.224-232.

[337] Tullock G., 1980. Efficient rent seeking. In *Toward a Theory of the Rent-Seeking Society.*

[338] Vázquez-Sedano, A., 2014. Sharing the effort costs in group contests. *The B.E. Journal of Theoretical Economics*, 18(1).

[339] Vesperoni, A. and Yıldızparlak, A., 2019. Contests with draws: Axiomatization and equilibrium. *Economic Inquiry*, forthcoming.

[340] Vojnović, M., 2016. *Contest Theory: Incentive Mechanisms and Ranking Methods.* Cambridge University Press.

[341] Wang, Z., 2010. The optimal accuracy level in asymmetric contests. *The B.E. Journal of Theoretical Economics*, 10(1).

[342] Wärneryd, K., 2003. Information in conflicts. *Journal of Economic Theory*, 110(1), pp.121-136.

[343] Wärneryd, K., 2012. Multi-player contests with asymmetric information. *Economic Theory*, 51(2), pp.277-287.

[344] Wasser, C., 2013. Incomplete information in rent-seeking contests. *Economic Theory*, 53(1), pp.239-268.

[345] Wu, Z. and Zheng, J., 2017. Information sharing in private value lottery contest. *Economics Letters*, 157, pp.36-40.

[346] Xiao, J., 2016. Asymmetric all-pay contests with heterogeneous prizes. *Journal of Economic Theory*, 163, pp.178-221.

[347] Xu, J. and Zhou, J., 2018. Nexus of conflicts: A variational inequality approach. *Working Paper.*

[348] Yates, A.J. and Heckelman, J.C., 2001. Rent-setting in multiple winner rent-seeking contests. *European Journal of Political Economy*, 17(4), pp.835-852.

[349] Yamazaki, T., 2009. The uniqueness of pure-strategy Nash equilibrium in rent-seeking games with risk-averse players. *Public Choice*, 139(3-4), pp.335-342.

[350] Yildirim, H., 2005. Contests with multiple rounds. *Games and Economic Behavior*, 51(1), pp.213-227.

[351] Zhang, J. and Wang, R., 2009. The role of information revelation in elimination contests. *Economic Journal*, 119(536), pp.613-641.

[352] Zhang, J. and Zhou, J., 2016. Information disclosure in contests: A Bayesian persuasion approach. *Economic Journal*, 126(597), pp.2197-2217.