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POSITIVE AND NEGATIVE SORTING IN TEAM CONTESTS*

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This paper investigates the formation of teams in a contest. A manager sorts four workers—who differ in their productivity—into two teams. Workers on each team join forces to produce team output, and one team wins a prize; for example, a bonus package. Two sorting patterns are possible: Positive sorting requires that each team consist of players of same caliber and negative sorting the opposite. We characterize the optimum. We further extend the model to allow the manager to set a prize schedule for the workers on each team upon a win, allocate productive resources between teams, and pick the level of competition of the contest.

I. INTRODUCTION

Teamwork is commonplace in modern firms (e.g., Che and Yoo [2001]; Hamilton *et al.* [2003]; Kambhampati and Segura-Rodriguez [2022]). According to Deloitte's 2017 Global Human Capital Trend Survey, more than 30% of survey respondents operate primarily in teams.[1] In addition, firms/organizations

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[1] See *https://www2.deloitte.com/content/dam/Deloitte/global/Documents/About-Deloitte/central-europe/ce-global-human-capital-trends.pdf https://www2.deloitte.com/content/dam/Deloitte/global/Documents/About-Deloitte/central-europe/ce-global-human-capital-trends.pdf*.

often pit teams against each other and incentivize collective performance based on relative performance evaluation (RPE).[2] Birkinshaw [2001] documented a number of salient cases of internal competitions between units or divisions. For instance, salespersons inside a firm are often organized as teams and rewarded by team-based performance relative to others (Chen and Lim [2013]; Lim and Chen [2014]), and the Houston Independent School District offered incentive pay to high schools based on their school-level value-added rank in each subject (see Imberman and Lovenheim [2015]).

In this paper, we investigate the sorting of heterogeneous workers in a setting with two teams inside a firm who will be ranked and rewarded by a RPE scheme, which can intuitively be interpreted as a contest.

We construct a team contest model for this purpose. Four workers of different levels of productivity are to be sorted into two teams. Workers' ability or strength, measured by an effort cost parameter, can be either high or low, with two of each type. The game proceeds in two stages. A manager first forms the teams—that is, sorting the workers into two teams—to maximize the total output of the contest. Two sorting patterns can arise. Under *positive sorting*, each team consists of homogeneous workers—that is, two strong (low-cost) or two weak (high-cost) workers on each team. Under *negative sorting*, each team involves workers of different types. Teams compete in the second stage. Workers simultaneously contribute their efforts, and a constant elasticity of substitution (CES) production function converts individual efforts into team-level output. Workers on the winning team equally share the prize—for example, a bonus package.[3]

The manager's choice is governed by a fundamental trade-off between (intra-team) production efficiency and (inter-team) competition. The former requires efficient conversion of individual efforts into composite output. Conventional wisdom in economics holds that a supermodular production function calls for positive sorting; positive sorting—which assigns workers of equal caliber to the same team—effectively leverages effort complementarity. The latter concern compels the manager to ensure sufficient competition under RPE. Positive sorting generates a polarized distribution of talents and team output, causing a lopsided competition in which a strong team confronts a weak one. The unlevel playing field mutes contenders' incentives, in accordance with the conventional wisdom of the contest literature: It discourages the weak team, while allowing the strong team to slack off.

[2] An intuitive rationale for internal competitions arises from the fact that the performance of teams within an organization is often subject to common shocks (see, e.g., Green and Stokey [1983]).

[3] Instead of positive versus negative sorting, Ryvkin [2011] and Brookins *et al.* [2015] used the terminology of balanced versus unbalanced sorting. Balanced sorting indicates the assignment of workers that minimizes variance in ability across groups, which is equivalent to negative sorting in our setting. Our terminology, positive versus negative, describes worker composition within a team.

The optimal sorting pattern must strike a balance between these two concerns. Our analysis fully characterizes the equilibrium under each sorting pattern, which enables us to compare the resulting equilibrium total output and obtain the optimum. We first establish a unique threshold for the degree of effort complementarity. Positive sorting emerges in the optimum when efforts are sufficiently complementary—that is, when the degree of effort complementarity exceeds that threshold; negative sorting arises otherwise. This prediction embodies the fundamental trade-off delineated above.

Our equilibrium result enables lucid comparative statics. The threshold is a function of the level of competitiveness in the contest and strictly decreases with it. The competitiveness level is measured by the parameter r of the usual Tullock contest success function. A larger r alludes to a more intense and selective competition, since additional effort can more effectively be converted into one's winning odds, which implies a higher marginal return to efforts. As a result, the strong team's advantage will be limited when the competitiveness level is lowered because winner selection is more random and depends less on efforts. This weakens effort incentives on the one hand, while leveling the playing field on the other. We formally verify that negative sorting is more likely when the level of competitiveness rises: More intense competition enlarges the loss of an unlevel playing field caused by positive sorting.

The equilibrium analysis paves the way for exercises of contest design in a broader context. In practice, a manager can be endowed with various instruments to incentivize workers and manipulate the competition in workplace. We explore three extended settings of practical relevance. The baseline model assumes that workers on each team split the prize equally upon a win. We first allow the manager—with a fixed prize purse—to set a prize schedule that specifies the reward each worker on a team would receive when the team wins. We show that positive sorting is less likely to emerge in the optimum. Further, we assume that the manager is endowed with a fixed amount of productive resources and allocates the resources between teams. For instance, a pharmaceutical company may provide laboratory equipment or funding to research labs that compete for an innovative solution, and the resources amplify a recipient team's productivity. The allocation, however, also alters teams' relative competency and reshapes the competitive balance of the contest. A lopsided competition could arise when the manager unevenly splits the resources, even if teams are ex ante symmetric, with each team consisting of heterogeneous workers; that is, under negative sorting. The baseline model is a special case in which the resources are evenly split. We demonstrate that positive sorting emerges in the optimum more often. Finally, we allow the manager to set the level of competitiveness endogenously. She would intensify the competition under negative sorting to maximize incentives, while softening it under positive sorting to level the playing field. The flexibility to set the competitiveness level may favor either positive or negative sorting.

I(i). *Link to the Literature*

Our paper primarily belongs to the literature on contests between teams/groups, which dates to Nitzan [1991a]. The majority of these studies assume no complementarities between players within a group (e.g., Nitzan [1991a, 1991b]; Esteban and Ray [1999, 2001, 2008]; Ryvkin [2011]; Eliaz and Wu [2018]). Chowdhury *et al.* [2016] assume a "weakest link" group production function, in that the minimum of the contribution within the group determines the aggregate output. Chowdhury *et al.* [2013] and Barbieri *et al.* [2014] assume the opposite, such that aggregate output is given by the maximum—that is, the "best shot."[4] Kolmar and Rommeswinkel [2013] and Choi *et al.* [2016] allow individual efforts to complement each other and to be converted into their group's outlay through a CES production function.[5] However, these studies do not consider the endogenous formation of teams.

Our paper is most closely related to work by Ryvkin [2011] and Brookins *et al.* [2015].[6] Both studies examine the optimal sorting of players with different abilities in group contests. Ryvkin [2011] assumes that efforts are perfect substitutes and demonstrate that the optimum depends on the curvature of cost functions. Brookins *et al.* [2015] assume a CES production and contend that technology plays a role. In contrast to our paper, both studies assume $n \times m$ players, with $n \geq 2$ groups and each consisting of $m \geq 2$ players. Because multiplayer asymmetric contests, in general, do not yield closed-form solutions, they resort to quadratic approximation to the equilibrium output and focus on the case of weakly heterogeneous players. Our paper adopts a four-player-two-type setting, which can be viewed as a special case of Brookins *et al.* [2015]. In contrast to them, we assume a linear cost function and a Tullock contest success function. The simplified setup enables complete characterization of the equilibrium without restrictions on the degree of heterogeneity and enables lucid comparative statics of the degree of competitiveness in the contest. Moreover, the setup further paves the way for us to examine the sorting problem in broader contexts that allow for additional design instruments. Our paper is thus complementary to theirs.

[4] Chowdhury and Topolyan [2016a, 2016b] allow for a combination of weakest links and best shot: One group's output is the weakest link of individual efforts, while the other's is the best shot.

[5] Dragon *et al.* [1996] allow a worker to not only contribute a productive effort that enhances his own contribution but also a helpful effort that increases those of his fellow teammates. In contrast, Gürtler [2008] allows each worker on a team to sabotage one of the workers on the adversarial team.

[6] Assuming a Cobb-Douglas effort aggregation function, Arbatskaya and Konishi [2023] consider a dynamic Tullock contest between two teams. They focus on the impact of the timing of players' moves and the observability of their actions on equilibrium efforts and teams' winning probabilities.

The literature on team/group contests either lets players in the winning group share the prize equally or considers a merit-based sharing scheme—that is, a portion of the prize will be distributed among the winning players based on their relative performance (efforts) within the group.[7] Our paper assumes noncontractible efforts. In one extension, we allow the manager to set identity-dependent rewards contingent on stochastic outcome (i.e., win or loss). This feature links our study to those by Franco *et al.* [2011] and Kaya and Vereshchagina [2015]: They consider principal-agent settings, in which the principal sets the optimal contract based on an independent performance evaluation (IPE) scheme, while we consider RPE. Another extension of our paper regarding resource allocation is related to work by Fu *et al.* [2012], Deng *et al.* [2021], and Gao *et al.* [2022], who allow a principal to allocate productive resources between competing parties. They focus on contests between individuals, in which the problem of freeriding is absent. In contrast, we consider competition between teams. Finally, we allow the manager to set the level of competitiveness in the contest, which places our paper in the company of Gershkov *et al.* [2009], Wang [2010], and Fu *et al.* [2015].

There is an extensive literature on the optimal sorting of players with heterogeneous traits into teams and how the sorting outcome is subject to various frictions. For instance, Lamberson and Page [2012], Chade and Eeckhout [2018], and Kaya and Vereshchagina [2022] focus on players of different abilities to acquire information. Our paper is particularly related to one strand of this literature—work by Franco *et al.* [2011] and Kaya and Vereshchagina [2014, 2015] —which focuses on the sorting of players with different levels of productivity in the presence of moral hazard. Franco *et al.* [2011] focus on a principal-agent setting in which workers inside a firm are organized into teams. They demonstrate that with complementary technology, the optimal sorting pattern may turn out to be negative because of the cost to provide incentive under moral hazard. Kaya and Vereshchagina [2015] examine a partnership setting. Kaya and Vereshchagina [2014] investigate how different organizational forms—a partnership versus a corporation—would endogenously emerge, depending on the different roles played by moral hazard in different organizational environments. Kambhampati and Segura-Rodriguez [2022] allow for not only moral hazard but also adverse selection, such that workers possess private information about their own productivity. They find that nonassortative matching may arise when complementarity is sufficiently weak. Glover and Kim [2021], in a repeated team production setting, demonstrate that diverse teams—which

[7] Technically, an equal sharing rule is no different from assuming a public-good prize; for example, Esteban and Ray [1999], Baik [2008], Ryvkin [2011], Kolmar and Rommeswinkel [2013], Barbieri *et al.* [2014], Chowdhury and Topolyan [2016a, 2016b], Chowdhury *et al.* [2016], and Eliaz and Wu [2018].

generate productive complementarity—may facilitate tacit cooperation within teams. Imhof and Kräkel [2023] consider a setting in which workers on a team not only join forces to produce composite output but also compete for career advancement. They contend that a more diverse distribution of competence may incentivize workers more effectively.[8] In contrast to our paper, these studies do not consider the competition between teams under RPE.[9]

II. THE BASELINE MODEL

The production in a firm requires the joint work of two workers. The firm employs four workers, and a manager of the firm splits the pool of workers into two teams; teams—with each consisting of two workers—independently execute the production tasks.

Teams are indexed by $i \in \{1, 2\}$, and workers on a team i are indexed by ik, with $k \in \{1, 2\}$. Workers simultaneously exert their efforts $e_{ik} \geq 0$. The efforts contributed by the workers on a team i are converted into a composite team output through a CES aggregation function:

$$\mathcal{Y}_i(e_{i1}, e_{i2}) = \left(\frac{1}{2}e_{i1}^{\rho} + \frac{1}{2}e_{i2}^{\rho}\right)^{1/\rho}, \text{ with } \rho < 1.$$

The function is supermodular for $\rho < 1$ and the parameter ρ measures the degree of complementarity of the team production process. When $\rho \to 1$, workers' efforts are perfect substitutes. When $\rho \to 0$, the production technology degenerates to the Cobb–Douglas function. When $\rho \to -\infty$, the Leontief production function $\mathcal{Y}_i(e_{i1}, e_{i2}) = \min\{e_{i1}, e_{i2}\}$ ensues, and workers' efforts exhibit perfect complementarity.

An effort e_{ik} incurs a linear cost of $\mathcal{C}(e_{ik}; c_{ik}) = c_{ik}e_{ik}$. The parameter $c_{ik} > 0$ refers to the worker's constant marginal effort cost and measures his ability, which can take either of two values, c_H or c_L, with $c_H > c_L > 0$. Workers differ in their marginal effort costs; there are two workers of each type and their types are publicly observable.

[8] Li *et al.* [2020] and Bergeron *et al.* [2022] examine the effect of team heterogeneity on team performance. Li *et al.* [2020] demonstrate that workers' effort responses to increasing heterogeneity within teams depend on the prevailing compensation schemes. Bergeron *et al.* [2022] espouse the merits of positive sorting and demonstrate that the benefit stems from complementarity.

[9] Chade and Eeckhout [2020] consider a matching model that allows post-match competitions between teams. Competitions create externalities, such that a team's payoff depends not only on its own members' attributes, but also those of other teams. In contrast to our model, moral hazard is abstracted away from their settings.

II(i). *Winner-Selection Mechanism and Workers' Payoffs*

The manager organizes a contest between the two teams to incentivize workers. The winning team is awarded a team prize with a normalized value of two, and its workers share the prize equally. Teams compete by their composite output: With $\boldsymbol{e}_i := (e_{i1}, e_{i2})$ and an effort profile $(\boldsymbol{e}_1, \boldsymbol{e}_2)$, a team $i \in \{1, 2\}$ wins the contest with a probability

$$(1) \quad p_i(\boldsymbol{e}_1, \boldsymbol{e}_2) = \begin{cases} \dfrac{\left[\mathcal{Y}_i(e_{i1}, e_{i2})\right]^r}{\left[\mathcal{Y}_1(e_{11}, e_{12})\right]^r + \left[\mathcal{Y}_2(e_{21}, e_{22})\right]^r} & \text{if } \left[\mathcal{Y}_1(e_{11}, e_{12})\right]^r \\ & \quad +\left[\mathcal{Y}_2(e_{21}, e_{22})\right]^r > 0, \\ \dfrac{1}{2} & \text{otherwise,} \end{cases}$$

where the parameter $r \in (0, 1]$ is conventionally called the discriminatory power of the contest. The parameter provides an intuitive measure about the level of competitiveness in the contest: A larger r implies that a larger effort can more effectively be converted into a higher probability of winning, which magnifies the marginal return to efforts and incentivizes effort supply.[10] This winning probability formulation is called a Tullock contest in the literature. Clark and Riis [1996] provide a microfoundation for this winning probability specification from a noisy-ranking perspective related to the discrete choice model of McFadden [1973, 1974]. They show that a Tullock contest is equivalent to a rank-order tournament à la Lazear and Rosen [1981] when the idiosyncratic noises are independently and identically drawn from a type I extreme-value (maximum) distribution. Further, Baye and Hoppe [2003] show that the contest is isomorphic to the research tournament model proposed by Fullerton and McAfee [1999] and the patent race model of Loury [1979] and Dasgupta and Stiglitz [1980].

Workers are risk-neutral. With an effort profile $(\boldsymbol{e}_1, \boldsymbol{e}_2)$, a worker ik receives an expected payoff

$$(2) \qquad \pi_{ik} := p_i(\boldsymbol{e}_1, \boldsymbol{e}_2) - C(e_{ik}; c_{ik}).$$

II(ii). *Manager's Sorting Decision*

Prior to the competition, the manager decides how to sort the four workers into two teams. There are two sorting patterns, which we denote by $\theta \in \{N, P\}$, with N and P to indicate *positive sorting* and *negative sorting*,

[10] The assumption $r \in (0, 1]$ ensures that workers' expected payoff is concave in their effort and thus they will adopt pure strategy in the equilibrium. As is well known in the contest literature, when $r > 1$, pure-strategy equilibrium dissolves for sufficiently large c_H/c_L (see, e.g., Alcalde and Dahm [2010]; Ewerhart [2015, 2017]; Feng and Lu [2017]).

respectively. Under positive sorting, the manager sorts workers of like types into each team; negative sorting arises if she does the opposite.

The manager forms the teams to maximize the total output $\sum_{i=1}^{2} \mathcal{Y}_i(e_{i1}, e_{i2})$ of the firm. Worker assignment is revealed publicly before they sink efforts.

III. ANALYSIS

In this section, we first characterize the equilibrium under each sorting pattern. We then compare performance in the contest under different sorting patterns.

III(i). *Equilibria under Different Sorting Patterns*

A worker ik chooses his effort $e_{ik} \geq 0$ to maximize his expected payoff (2). Given the winning probability formulation (1), it can be verified that each worker must exert a positive effort in the equilibrium. Let $\mathcal{Y}_{-i} := \sum_{j=1}^{2} \mathcal{Y}_j - \mathcal{Y}_i$. The first-order condition with respect to e_{ik} gives

$$\frac{1}{2} e_{ik}^{\rho-1} \left(\frac{1}{2} e_{i1}^{\rho} + \frac{1}{2} e_{i2}^{\rho} \right)^{\frac{1}{\rho}-1} \frac{r \mathcal{Y}_i^{r-1} \mathcal{Y}_{-i}^{r}}{\left(\mathcal{Y}_1^r + \mathcal{Y}_2^r \right)^2} = c_{ik}, \forall i \in \{1,2\}, k \in \{1,2\}. \tag{3}$$

The above condition is not only necessary but also sufficient to determine workers' payoff-maximizing efforts, since the expected payoff π_{ik} is strictly concave in e_{ik} for all $r \in (0,1]$ and $\rho < 1$.

Denote by e_H^{θ} and e_L^{θ}, respectively, the equilibrium individual efforts for the high- and low-cost types associated with a sorting pattern $\theta \in \{N, P\}$. Solving the above system of equations (3) yields the following.

Lemma 1 (Equilibrium Effort Profile). Fixing a sorting pattern $\theta \in \{N, P\}$, there exists a unique pure-strategy Nash equilibrium in the contest game. Equilibrium individual efforts $\left(e_H^{\theta}, e_L^{\theta}\right)$ are fully characterized as the following.

(i) Under positive sorting, the equilibrium individual efforts of the high- and low-cost types are, respectively,

$$e_H^P = \frac{r}{2c_H} \frac{c_H^r c_L^r}{\left(c_H^r + c_L^r\right)^2}, \text{ and } e_L^P = \frac{r}{2c_L} \frac{c_H^r c_L^r}{\left(c_H^r + c_L^r\right)^2}.$$

(ii) Under negative sorting, the equilibrium efforts of the high-cost and low-cost types are, respectively,

$$e_H^N = \frac{r}{4c_H^{\frac{1}{1-\rho}}} \frac{c_H^{\frac{\rho}{1-\rho}} c_L^{\frac{\rho}{1-\rho}}}{c_H^{\frac{\rho}{1-\rho}} + c_L^{\frac{\rho}{1-\rho}}}, \text{ and } e_L^N = \frac{r}{4c_L^{\frac{1}{1-\rho}}} \frac{c_H^{\frac{\rho}{1-\rho}} c_L^{\frac{\rho}{1-\rho}}}{c_H^{\frac{\rho}{1-\rho}} + c_L^{\frac{\rho}{1-\rho}}}.$$

Note that the equilibrium effort profile under positive sorting is independent of the degree of effort complementarity—that is, the parameter ρ—in the team production process. This observation echoes the finding of Kolmar and Rommeswinkel [2013]. However, effort complementarity affects equilibrium effort supply when workers on each team are heterogeneous—that is, under negative sorting.

III(ii). *Positive Versus Negative Sorting*

Lemma 1 enables us to explore the optimal sorting pattern. Recall that the manager chooses $\theta \in \{N, P\}$ to maximize the equilibrium total output:

$$\mathcal{Y} := \sum_{i=1}^{2} \mathcal{Y}_i = \left(\frac{1}{2}e_{11}^{\rho} + \frac{1}{2}e_{12}^{\rho}\right)^{1/\rho} + \left(\frac{1}{2}e_{21}^{\rho} + \frac{1}{2}e_{22}^{\rho}\right)^{1/\rho}. \tag{4}$$

By Lemma 1, the equilibrium total output under positive and negative sorting, denoted respectively by $\mathcal{Y}^P$ and $\mathcal{Y}^N$, can be obtained as

$$\mathcal{Y}^P := \mathcal{Y}_1^P + \mathcal{Y}_2^P = \frac{r\left(c_H + c_L\right) c_H^{r-1} c_L^{r-1}}{2\left(c_H^r + c_L^r\right)^2}, \quad \text{and}$$

$$\mathcal{Y}^N := \mathcal{Y}_1^N + \mathcal{Y}_2^N = \frac{r\left(c_H^{\frac{\rho}{1-\rho}} + c_L^{\frac{\rho}{1-\rho}}\right)^{\frac{1-\rho}{\rho}}}{2^{1+\frac{1}{\rho}} c_H c_L}.$$

Positive sorting is optimal if $\mathcal{Y}^P > \mathcal{Y}^N$ and negative sorting prevails otherwise.[11] The following result ensues.

Proposition 1 (Optimal Sorting Pattern). The following statements hold:

(i) Fixing $r \in (0, 1)$, there exists a threshold $\rho^*(r) \in (-\infty, 1)$ for the degree of effort complementarity—which strictly decreases with r—such that positive sorting prevails (i.e., $\mathcal{Y}^P > \mathcal{Y}^N$) if $\rho < \rho^*(r)$ and negative sorting prevails (i.e., $\mathcal{Y}^P < \mathcal{Y}^N$) if $\rho > \rho^*(r)$.
(ii) Fixing $r = 1$, negative sorting always prevails irrespective of the value of ρ.

Proposition 1 establishes a unique threshold $\rho^*(r)$, such that positive sorting emerges in the optimum if and only if ρ falls below $\rho^*(r)$—that is, when workers' efforts are sufficiently complementary. Further, the threshold

[11] It is noteworthy that the comparison between $\mathcal{Y}^P$ and $\mathcal{Y}^N$ is complicated, because the parameter ρ factors into the expression of $\mathcal{Y}^N$ nonlinearly. For instance, the term $[(c_H)^{\frac{\rho}{1-\rho}} + (c_L)^{\frac{\rho}{1-\rho}}]^{\frac{1-\rho}{\rho}}$ is in the form of a general mean, whose property had not been uncovered and formally established until Nam and Minh [2008].

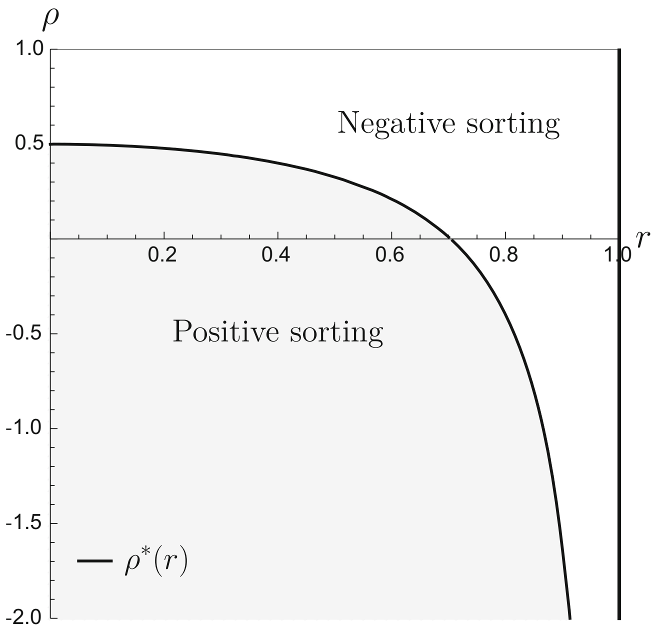

Figure 1
Optimal Sorting Pattern in the (r, ρ) Space: $c_H/c_L = 2$.

$\rho^*(r)$ strictly decreases in r, the level of competitiveness in the contest; that is, negative sorting is more likely to prevail when the competition in the contest gets more intense.

We illustrate the result in Figure 1. The vertical axis measures the level of effort complementarity—that is, ρ—and the horizontal axis measures the degree of competition r. The downward-sloping curve traces the threshold $\rho^*(r)$. Negative sorting arises in the region above and to the right of the curve and positive sorting prevails below and to the left. In what follows, we elaborate on the logic underlying this result.

III(iii). *Intuition for Proposition 1*

Recall that the manager must reconcile two fundamental concerns when forming teams: (i) ensuring efficient production within a team and (ii) promoting competition between teams. We interpret our results based on the trade-off between these concerns.

On the one hand, production technology $\mathcal{Y}_i(e_{i1}, e_{i2})$ plays an important role in determining the optimal sorting pattern. As discussed above, a supermodular production technology leverages effort complementarity and favors positive sorting. On the other hand, positive sorting limits competition, since it leads to a lopsided competition between a weak team and a strong one. The asymmetry discourages the weak team, which in turn entices the stronger to slack off. In contrast, negative sorting levels the playing field in the contest, thereby fueling competition.

The tension between these forces governs the comparison: Positive sorting prevails when efforts are sufficiently complementary—i.e., with $\rho < \rho^*(r)$—in which case the gain from intra-team production efficiency outweighs the loss from the uneven race.

By Proposition 1, a more competitive contest—that is, with a larger r—tends to favor negative sorting. Recall that a larger r implies a more significant marginal return of efforts, since a higher effort can more effectively contribute to an eventual win. This further discourages the weaker team under positive sorting and magnifies the loss from the uneven race. The trade-off is thus tilted toward negative sorting when r increases, for example, with more precise performance evaluation or less costly monitoring.

Finally, we obtain the following observation, which sheds further light on the economic forces underlying team formation.

Remark 1 (The Role of Freeriding). Suppose that the two members on each team choose their effort level jointly to maximize total team expected utility. The equilibrium effort of each worker doubles that when each worker chooses his own effort. Therefore, the optimal sorting pattern remains unchanged.[12]

This remark demonstrates that the choice of sorting patterns does not factor in the concern of freeriding. That is, freeriding arises in teamwork when workers choose their efforts independently regardless of the prevailing sorting pattern, and neither sorting pattern possesses an advantage in alleviating the problem.

IV. EXTENSIONS

The equilibrium results allow us to extend the model to allow for richer space for contest design. In this section, we examine three extended settings, which allow for additional instruments in the manager's toolkit. First, we let the manager set the prize structure. We then let the manager allocate productive resources across teams, which not only boosts recipients' productivity but also manipulates the competitive balance of the playing field. Finally, we allow her to set the level of competitiveness, r, in the contest.

IV(i). *Optimal Sorting with Endogenous Prize Allocation*

The baseline model assumes that the two workers on the winning team equally share the prize. In practice, firms' management has large flexibility

[12] We thank Editor Jidong Zhou for motivating discussions of the free-riding problem within a team and its implications for the optimal sorting pattern.

in setting compensation packages for workers, which would presumably vary workers' incentives in the contest. We now consider a natural extension of our baseline model: The manager precommits to an identity-dependent prize schedule that specifies the prize share each worker would secure upon winning. This allows the manager to exploit the heterogeneity of the workers when providing incentives.

The game proceeds in two stages. In the first stage, the manager forms teams and decides how to split the prize purse between the workers on each team when they win. Workers exert efforts and compete in the second stage. We continue to assume a total prize purse of a value two ($V = 2$). The manager specifies and publicly announces a prize schedule $\boldsymbol{v} := (\boldsymbol{v}_1, \boldsymbol{v}_2)$, with $\boldsymbol{v}_i := (v_{i1}, v_{i2})$, $i \in \{1, 2\}$, and $v_{i1} + v_{i2} \leq V = 2$: Namely, a worker ik, $k \in \{1, 2\}$, on a team i receives a prize $v_{ik} \geq 0$ if the team wins. We assume that efforts are not contractible. Thus, the prize schedule $\boldsymbol{v} \equiv (\boldsymbol{v}_1, \boldsymbol{v}_2)$ is independent of individual efforts.

Fix a sorting pattern $\theta \in \{N, P\}$ and prize schedule $\boldsymbol{v} \equiv (\boldsymbol{v}_1, \boldsymbol{v}_2)$. The expected payoff of worker ik is given by

$$\pi_{ik} = \frac{\mathcal{Y}_i^r}{\mathcal{Y}_1^r + \mathcal{Y}_2^r} v_{ik} - c_{ik} e_{ik}. \tag{5}$$

In what follows, we first derive the optimal prize schedule under an arbitrary sorting pattern, then the optimal sorting pattern with endogenously allocated prizes.

IV(i)(a). *Optimal Prize Allocation under Positive/Negative Sorting*

For the sake of brevity and expositional efficiency, we only present the expression of equilibrium total output, which facilitates the search for the optimal sorting pattern. We first consider positive sorting. Without loss of generality, the stronger team is labeled team 1—i.e., $c_{11} = c_{12} = c_L$ and $c_{21} = c_{22} = c_H$.

Lemma 2 (Optimal Prize Allocation and Equilibrium Total Output under Positive Sorting). Under positive sorting,

(i) for $\rho \in (-\infty, 1/2)$, the manager evenly splits the prize purse within the winning team, that is, $v_{ik} = 1, \forall i, k \in \{1, 2\}$; thus the equilibrium coincides with that in the baseline model, in which case the contest generates a total output

$$\mathcal{Y}_a^P = \frac{r c_H^{r-1} c_L^{r-1} (c_H + c_L)}{2(c_H^r + c_L^r)^2};$$

(ii) the manager otherwise allocates the entire prize to only one worker on the winning team, that is, $v_{11} = v_{21} = 2$, without loss of generality; the contest generates a total output

$$\mathcal{Y}_a^P = \frac{rc_H^{r-1}c_L^{r-1}(c_H + c_L)}{2^{\frac{1-\rho}{\rho}}(c_H^r + c_L^r)^2}.$$

Lemma 2 is intuitive. When efforts are sufficiently complementary—that is, $\rho < 1/2$—production requires that workers join force. The output depends on not only the sum of the efforts contributed by the two workers on a team, but also their distribution: Efficient production requires more balanced contributions within a team. The prize schedule must incentivize both workers, and the manager thus evenly splits the prize purse accordingly. In contrast, when efforts are relatively less complementary—that is, $\rho > 1/2$—the output maximization relies more on the sum of efforts and less on their distribution. Consider the extreme case of perfect substitutes, that is, $\rho \to 1$: Production can be completed with one worker's solo input. To induce a large sum of efforts, the manager can award the entire prize purse to one worker on the winning team to maximize incentive. The contest boils down to a head-to-head competition between two individual workers—one strong and one weak—from rival teams.

We then consider negative sorting, in which case teams are symmetric and each consists of a low-cost worker and a high-cost one. Without loss of generality, the low-cost worker on each team $i \in \{1, 2\}$ is labeled $i1$, which yields $c_{11} = c_{21} = c_L$ and $c_{12} = c_{22} = c_H$.

Lemma 3 (Optimal Prize Allocation and Equilibrium Total Output under Negative Sorting). Under negative sorting,

(i) for $\rho \in (-\infty, 1/2)$, the manager splits the prize purse between workers such that

$$v_{11} = v_{21} = \frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}, \text{ and } v_{12} = v_{22} = \frac{2c_H^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}};$$

the contest generates a total output

$$\mathcal{Y}_a^N = \frac{r}{2^{\frac{1}{\rho}}\left(c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}\right)^{\frac{2\rho-1}{\rho}}};$$

(ii) the manager otherwise allocates the entire prize to the low-cost worker on the winning team, that is, $v_{11} = v_{21} = 2$; the contest generates a total output

$$\mathcal{Y}_a^N = \frac{r}{2^{\frac{1}{\rho}}c_L}.$$

Analogous to Lemma 2(ii), Lemma 3(ii) requires that the manager reward only one worker when effort complementarity is weak—that is, when ρ exceeds the cutoff $1/2$. Under negative sorting, the prize should be entirely awarded to the strong worker on the winning team, which maximally incentivizes his contribution and thus maximizes the output within a team. The contest, again, reduces to a head-to-head competition between two individual workers. In contrast to the case of positive sorting, a symmetric contest arises between two low-cost workers. When efforts are sufficiently complementary—that is, when ρ falls below the cutoff $1/2$—efficient production requires that the prize schedule incentivize both workers, since efficient production requires more even distribution of efforts within a team. As a result, the prize purse will be shared but not evenly.

IV(i)(b). *Optimal Sorting with Endogenous Prize Allocation*

Lemmas 2 and 3 enable us to identify the optimal sorting pattern with endogenous prize allocation. Positive sorting prevails if $\mathcal{Y}_a^P > \mathcal{Y}_a^N$ and negative sorting arises otherwise. The following result ensues.

Proposition 2 (Optimal Sorting Pattern with Endogenous Prize Allocation). There exists a cutoff $\bar{r} \in (0, 1]$ for the competitiveness level such that:

(i) If $r \in (0, \bar{r})$, there exists a threshold $\rho_a^*(r) \in (-\infty, 1)$ for the degree of effort complementarity such that positive sorting prevails—that is, $\mathcal{Y}_a^P > \mathcal{Y}_a^N$ —if $\rho < \rho_a^*(r)$ and negative sorting prevails—that is, $\mathcal{Y}_a^P < \mathcal{Y}_a^N$— if $\rho > \rho_a^*(r)$. Moreover, $\rho_a^*(r) \leq \rho^*(r)$.

(ii) If $r \in [\bar{r}, 1]$, negative sorting always prevails irrespective of the value of ρ.

The prediction of Proposition 2 qualitatively resembles that of Proposition 1: Positive sorting prevails in the presence of strong effort complementarity and negative sorting prevails otherwise. Notably, endogenous prize allocation favors negative sorting. First, negative sorting arises regardless of ρ whenever r exceeds $\bar{r}$; second, $\rho_a^*(r) \leq \rho^*(r)$ when r falls below the cutoff: Negative sorting can prevail under a more complementary production process. This observation is illustrated in Figure 2. The solid curve depicts the threshold $\rho^*(r)$ in the baseline model and the dashed curve traces the threshold $\rho_a^*(r)$ under endogenous prize allocation. Negative sorting is more likely to emerge in the optimum. That is, when the manager is endowed with more freedom to reward workers, she should facilitate a more balanced distribution of talents across teams while tolerating more dispersed compensation structures inside teams.

The freedom to set the prize schedule allows the manager to incentivize workers more effectively in team production. By Lemmas 2(ii) and 3(ii), the manager can effectively abandon team structure and reward only one worker

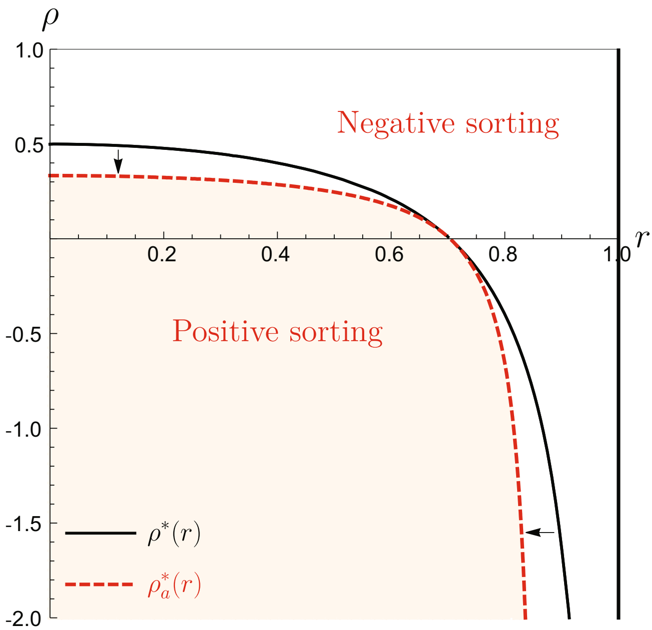

Figure 2
Optimal Sorting Pattern with Endogenous Prize Allocation: $c_H/c_L = 2$

Notes: [Colour figure can be viewed at wileyonlinelibrary.com]

when effort complementarity is sufficiently weak, regardless of the prevailing sorting pattern. Negative sorting obviously outperforms positive sorting under this circumstance, because the former gives rise to a symmetric contest. By Lemma 2(i), when efforts are sufficiently complementary, the endogenized prize schedule under positive sorting coincides with that in the baseline model and thus does not improve the performance of the contest. By Lemma 3(i), in contrast, a prize differential emerges between the strong and weak workers on each team under negative sorting. The performance of the contest thus improves when the prize schedule can be set flexibly under negative sorting.

IV(ii). *Optimal Sorting with Endogenous Resource Allocation*

We now let the manager not only sort workers into teams but also allocate productive resources between teams (Fu *et al.* [2012]; Gao *et al.* [2022]; Deng *et al.* [2021]), subject to a budget constraint. For instance, a pharmaceutical company can provide research funding, laboratory equipment, or a computing facility to selected research task forces. Alternatively, a firm may ration administrative support to sales teams. The resources improve recipients' productivity; however, uneven allocation also varies the balance of the playing field, which would affect the performance of the contest.

We assume that the production function takes the form of

$$\delta_i \mathcal{Y}_i(e_{i1}, e_{i2}) \equiv \delta_i \left(\frac{1}{2} e_{i1}^{\rho} + \frac{1}{2} e_{i2}^{\rho} \right)^{1/\rho}, \text{ with } \rho < 1,$$

where $\delta_i > 0$ is the amount of resource allocated to a team $i \in \{1, 2\}$. The resources enter the production function in a multiplicative form, which implicitly assumes that the resources are complementary to workers' efforts. They can be interpreted as "capital input"—physical or intellectual—in a production process that improves workers' productivity and scale up their output for given labor input. For instance, a more diligent research team can make better use of their access to a computing facility or laboratory equipment.

We normalize the amount of resources available for allocation to two; thus the budget constraint can be written as $\delta_1 + \delta_2 \leq 2$. The baseline model is a special case in which the manager equally splits the resources between teams.

The game proceeds in two stages. In the first stage, the manager chooses and announces team formation $\theta \in \{N, P\}$ and her resource allocation plan, $\boldsymbol{\delta} := (\delta_1, \delta_2)$, to maximize the total output $\sum_{i=1}^{2} \delta_i \mathcal{Y}_i(e_{i1}, e_{i2})$. In the second stage, workers simultaneously choose their effort input. Similar to the baseline model, a team $i \in \{1, 2\}$ wins the contest with a probability

$$p_i(\boldsymbol{e}_1, \boldsymbol{e}_2) = \begin{cases} \frac{[\delta_i \mathcal{Y}_i(e_{i1}, e_{i2})]^r}{[\delta_1 \mathcal{Y}_1(e_{11}, e_{12})]^r + [\delta_2 \mathcal{Y}_2(e_{21}, e_{22})]^r} & \text{if } [\delta_1 \mathcal{Y}_1(e_{11}, e_{12})]^r \\ & \quad + [\delta_2 \mathcal{Y}_2(e_{21}, e_{22})]^r > 0, \\ \frac{\delta_i^r}{\delta_1^r + \delta_2^r} & \text{otherwise.} \end{cases}$$

We first derive the optimal resource plan and the associated equilibrium outcome under each sorting pattern, then the optimal sorting pattern.

IV(ii)(a). *Optimal Resource Allocation under Positive/Negative Sorting*

First, consider the case of negative sorting, that is, $c_{11} = c_{21} = c_L$ and $c_{12} = c_{22} = c_H$. The following result ensues.

Lemma 4 (Optimal Resource Allocation and Equilibrium Total Output under Negative Sorting). Under negative sorting, the manager evenly splits the resources between the two teams—that is, $\delta_1^N = \delta_2^N = 1$—and in the equilibrium, the contest generates a total output

$$\mathcal{Y}_b^N = \frac{r\left(c_H^{\frac{\rho}{1-\rho}} + c_L^{\frac{\rho}{1-\rho}}\right)^{\frac{1-\rho}{\rho}}}{2^{1+\frac{1}{\rho}} c_H c_L} = \mathcal{Y}^N. \tag{6}$$

The resulting contest coincides with that in the baseline model: Teams remain symmetric, and the total output, which we denote by $\mathcal{Y}_b^N$, equals $\mathcal{Y}^N$ in the baseline setting. Teams are ex ante identical and the manager evenly splits the resources, which maintains an even race without upsetting the competitive balance of the playing field.

The manager faces a trade-off between production efficiency and competitive balance under positive sorting. The teams are ex ante asymmetric, with one strong and one weak—that is, $c_{11} = c_{12} = c_L$ and $c_{21} = c_{22} = c_H$. On the one hand, allocating more resources to the ex ante stronger team enables more efficient production: They presumably contribute a higher level of effort, so an additional amount of resources—that is, a larger δ —scales up the productivity gain. On the other hand, this further upsets the playing field, and thereby discourages competition and weakens workers' incentives.

For notational convenience, denote the degree of worker heterogeneity c_H/c_L by x. Our analysis yields the following.

Lemma 5 (Optimal Resource Allocation and Equilibrium Total Output under Positive Sorting). Fix $x \equiv c_H/c_L > 1$. Under positive sorting, the optimal resource allocation plan is $(\delta_1^P, 2-\delta_1^P)$, where δ_1^P solves

$$\frac{\partial}{\partial \delta_1}\left(\frac{\delta_1^r \delta_2^r(\delta_1 x + \delta_2)}{(\delta_1^r x^r + \delta_2^r)^2}\right)\Bigg|_{\delta_2 = 2-\delta_1} = \frac{\partial}{\partial \delta_2}\left(\frac{\delta_1^r \delta_2^r(\delta_1 x + \delta_2)}{(\delta_1^r x^r + \delta_2^r)^2}\right)\Bigg|_{\delta_2 = 2-\delta_1}.$$

Further, there exists a unique threshold $\tilde{r} \in (0,1)$ for competitiveness level—which depends on the degree of worker heterogeneity $x \equiv c_H/c_L$—such that $\delta_1^P > 1$ if and only if $r < \tilde{r}$. Let $\delta_2^P := 2 - \delta_1^P$. The total output in the equilibrium amounts to

$$\mathcal{Y}_b^P = \frac{r\left(\delta_1^P \delta_2^P\right)^r c_H^{r-1} c_L^{r-1}\left(\delta_1^P c_H + \delta_2^P c_L\right)}{2\left[\left(\delta_1^P\right)^r c_H^r + \left(\delta_2^P\right)^r c_L^r\right]^2} \geq \mathcal{Y}^P, \tag{7}$$

with the equality holding if and only if $r = \tilde{r}$.

By Lemma 5, the ex ante stronger team under positive sorting receives more (less) resources from the manager when the level of competitiveness is low (high), which further enlarges (reduces) the asymmetry between the teams. To benefit from a level playing field, the additional resources awarded to the ex ante weaker team must effectively incentivize its effort. A less-competitive contest—that is, a smaller r —implies that it is harder to convert additional input into higher output, which implies a higher cost to elicit efforts. This limits the benefit of leveling the playing field. As a result, resource allocation prioritizes the ex ante stronger team for a larger productivity gain when r falls below $\tilde{r}$. The opposite takes place when the competitiveness level is higher—that is, when $r > \tilde{r}$—in which case incentivizing the ex ante weaker team comes at a lower cost, and thereby generates a larger gain from leveling the playing field.

IV(ii)(b). *Optimal Sorting with Endogenous Resource Allocation*

We can simply compare $\mathcal{Y}_b^P$ with $\mathcal{Y}_b^N$ to obtain the optimal sorting, with positive sorting to prevail if $\mathcal{Y}_b^P > \mathcal{Y}_b^N$ and negative sorting to prevail otherwise.

Proposition 3 (Optimal Sorting with Endogenous Resource Allocation). There exists a threshold $\rho_b^*(r) \in (-\infty, 1)$ such that positive sorting prevails—that is, $\mathcal{Y}_b^P > \mathcal{Y}_b^N$—for $\rho < \rho_b^*(r)$ and negative sorting arises otherwise. Moreover, $\rho_b^*(r) \geq \rho^*(r)$.

The prediction of $\rho_b^*(r) \geq \rho^*(r)$ suggests that endogenous resource allocation favors positive sorting. The result is illustrated in Figure 3. Compared with that for $\rho^*(r)$, the curve for $\rho_b^*(r)$ is stretched upward, which enlarges the parametric space below the curve—that is, the set of parameterizations necessary for positive sorting to prevail. Recall by Lemma 4 that under negative sorting, the contest remains the same as in the baseline model; the freedom to allocate resources does not affect the performance of the contest. However, resource allocation allows the manager to exploit the asymmetry between teams under positive sorting: She may prioritize the ex ante stronger team for larger productivity gain at the cost of an imbalanced competition; she may also handicap the stronger team to stimulate competition at the cost of inefficient resource allocation. This flexibility favors positive sorting.

Analogous to $\rho^*(r)$, the cutoff $\rho_b^*(r)$ decreases with r; so a smaller r favors positive sorting. Further, by Lemma 5, resource allocation prioritizes the ex ante stronger team under positive sorting when r falls below $\tilde{r}$. In summary, imbalanced talent distribution across teams and polarized resource allocations are more likely when r is small, for example, when the performance evaluation is coarse or monitoring is costly. The parameter r can also be interpreted as a measure of the difficulty of the task. A smaller r implies less effective conversion of efforts into perceivable output—for example, a more challenging or risky research project that could yield a major discovery—in which case the manager may choose to champion a dominant team. Conversely, she may evenly distribute talents and resources when pursuing routine tasks—for example, process optimization for cost reduction in manufacturing—or when performance evaluation is more precise.

IV(iii). *Sorting with Endogenous Competitiveness Level*

We now allow the manager to not only form teams but also set the degree of competition, which is measured by the parameter r.[13] A larger r implies

[13] We thank Editor Jidong Zhou and an anonymous referee for suggesting this extension.

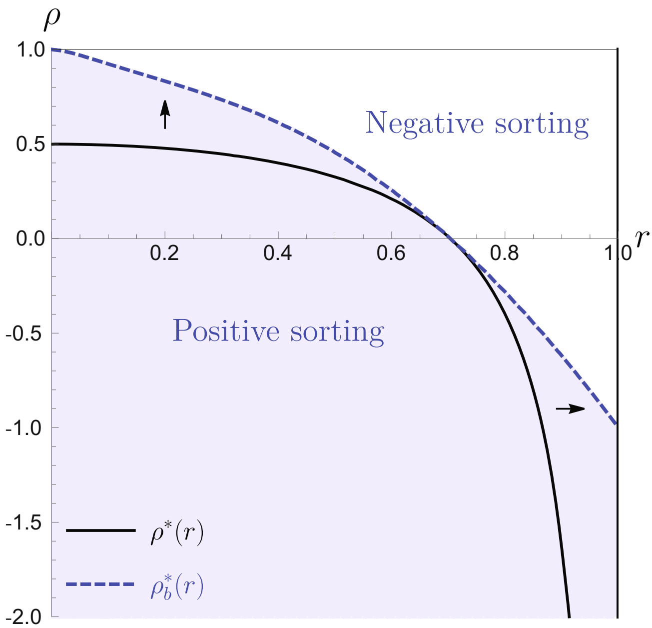

Figure 3
Optimal Sorting Pattern with Endogenous Resource Allocation: $c_H/c_L = 2$

Notes: [Colour figure can be viewed at wileyonlinelibrary.com]

a more significant role played by efforts in determining the winner vis-à-vis random factors, which magnifies the marginal return to efforts since a greater effort is more likely to translate into a win. In practice, a manager may have numerous tools at her disposal to influence the level of competitiveness in a contest. For instance, she can adjust the weight of subjective components in performance evaluation, which varies the level of randomness and the role of efforts in determining the winner. Furthermore, the manager can strategically change the composition of the judging committee, balancing between expert and nonexpert evaluators. Lastly, the intensity of monitoring effort is subject to the manager's choice, which directly impacts the accuracy of performance measurement (Gershkov *et al.* [2009]).

The game proceeds in two stages. In the first stage, the manager chooses the competitiveness level $r \in (0, 1]$ in addition to team formation $\theta \in \{N, P\}$. In the second stage, workers exert effort simultaneously. As in the baseline setting, the manager aims to maximize the total output $\sum_{i=1}^{2} \mathcal{Y}_i(e_{i1}, e_{i2})$.

IV(iii)(a). *Optimal Competitiveness Level under Positive/Negative Sorting*

We first pin down the optimal competitiveness level, r, for a given sorting pattern $\theta \in \{N, P\}$.

Lemma 6 (Optimal Competitiveness Level Fixing a Sorting Pattern). The following statements hold:

(i) Under negative sorting, the optimal competitiveness level is $r = 1$.
(ii) Under positive sorting, there exists a threshold $\underline{x} > 1$ such that:[14] if $c_H/c_L > \underline{x}$, the optimal competitiveness level is given by some $\hat{r} \in (0, 1)$, which depends on the degree of worker heterogeneity c_H/c_L; otherwise, the optimal competitiveness level is $r = 1$.

More intense competition amplifies the marginal return of effort, so a larger r tends to incentivize efforts. As a result, the manager must set $r = 1$ under negative sorting, in which case the total effort strictly increases with r. In contrast, a larger r catalyzes an indirect effect under positive sorting: As mentioned above, more intense competition amplifies the stronger team's advantage, which further discourages the weaker team and allows the former to slack off. As a result, under positive sorting, the manager may prefer an r of an intermediate size—$r = \hat{r} \in (0, 1)$—when the stronger and weaker workers are sufficiently heterogeneous, that is, $c_H/c_L > \underline{x}$: The randomness introduced by an intermediate r limits the excessive advantage possessed by the stronger team and levels the playing field. This indirect effect is less than significant when workers of different types are not excessively heterogeneous—that is, $c_H/c_L \leq \underline{x}$—in which case the benefit of a level playing field is insufficient to offset the loss of incentive caused by a smaller r; so the optimal competitiveness level remains at 1.

IV(iii)(b). *Optimal Sorting with Endogenous Competitiveness Level*

We are ready to characterize the optimal contest with endogenous competitiveness level.

Proposition 4 (Optimal Sorting with Endogenous Competitiveness Level). The optimal sorting pattern is characterized below.

(i) Fixing $c_H/c_L \in (\underline{x}, \infty)$, there exists a threshold $\hat{\rho} \in (-\infty, 1)$—which depends on c_H/c_L—such that positive sorting prevails if $\rho < \hat{\rho}$ and negative sorting prevails if $\rho > \hat{\rho}$.
(ii) Fixing $c_H/c_L \in (1, \underline{x}]$, negative sorting always prevails irrespective of the value of ρ.

Recall by Proposition 1(ii) that negative sorting generates more total output than positive sorting when holding fixed $r = 1$. Further, Lemma 6 states that output is maximized under positive sorting by setting r to 1 when the degree of worker heterogeneity is small—that is, $c_H/c_L \leq \underline{x}$. It is thus straightforward to conclude that the manager prefers negative sorting whenever the condition

[14] The threshold $\underline{x}$ is the solution to $1 + \underline{x} - (\underline{x} - 1) \log \underline{x} = 0$.

$c_H/c_L \leq \underline{x}$ is met, in which case the manager sets a competitiveness level $r = 1$ regardless of the sorting pattern. When $c_H/c_L > \underline{x}$, we need to compare $\mathcal{Y}^P$ with $r = \hat{r}$ to $\mathcal{Y}^N$ with $r = 1$. Consistent with Proposition 1, positive sorting arises in the optimum when efforts are sufficiently complementary, that is, when ρ falls below the cutoff $\hat{\rho}$.

It remains ambiguous whether the ability to set the competitiveness level will favor positive or negative sorting. We construct two examples to illustrate the subtlety.

Example 1. Set $(c_H, c_L) = (10, 1)$ and $\rho = -5$. We can obtain the following:

Sorting pattern	r	High-cost type effort	Low-cost type effort	Total output
Negative sorting (N)	$r = 1$	$e_H^N \approx 2.18 \times 10^{-2}$	$e_L^N \approx 3.20 \times 10^{-2}$	$\mathcal{Y}^N \approx 4.87 \times 10^{-2}$
Positive sorting (P)	$r = 1$	$e_H^P \approx 4.13 \times 10^{-3}$	$e_L^P \approx 4.13 \times 10^{-2}$	$\mathcal{Y}^P \approx 4.55 \times 10^{-2}$
Positive sorting (P)	$\hat{r} \approx 0.67$	$e_H^P \approx 4.86 \times 10^{-3}$	$e_L^P \approx 4.86 \times 10^{-2}$	$\mathcal{Y}^P \approx 5.35 \times 10^{-2}$

With r fixed at 1, the manager must prefer negative sorting to positive sorting. However, when she can flexibly pick the level of competitiveness, she can reduce r to level the playing field and thus improve the performance of the contest under positive sorting. As the table shows, the performance under positive sorting, $\mathcal{Y}^P$, rises above $\mathcal{Y}^N$ when r is set to $r = \hat{r} \approx 0.67$.

Example 1 shows that positive sorting may overtake negative sorting when the manager can set the competitiveness level. The next example demonstrates the opposite.

Example 2. Set $(c_H, c_L) = (10, 1)$ and $\rho = -1$. We can obtain the following:

Sorting pattern	r	High-cost type effort	Low-cost type effort	Total output
Positive sorting (P)	$\hat{r} \approx 0.67$	$e_H^P \approx 4.86 \times 10^{-3}$	$e_L^P \approx 4.86 \times 10^{-2}$	$\mathcal{Y}^P \approx 5.35 \times 10^{-2}$
Negative sorting (N)	$\hat{r} \approx 0.67$	$e_H^N \approx 1.27 \times 10^{-2}$	$e_L^N \approx 4.02 \times 10^{-2}$	$\mathcal{Y}^N \approx 3.87 \times 10^{-2}$
Negative sorting (N)	$r = 1$	$e_N^P \approx 1.90 \times 10^{-2}$	$e_L^N \approx 6.01 \times 10^{-2}$	$\mathcal{Y}^N \approx 5.77 \times 10^{-2}$

With r fixed at $r = \hat{r} \approx 0.67$, the manager prefers positive sorting over negative sorting. However, when she can set r, she can increase r to 1 under negative sorting to further improve the performance of the contest; meanwhile, she cannot further improve the performance under positive sorting, since the current r coincides with optimal level $\hat{r}$. As the table shows, the performance under negative sorting, $\mathcal{Y}^N$, rises above $\mathcal{Y}^P$ once r rises to 1.

V. CONCLUDING REMARKS

In this paper, we analyze the sorting of heterogeneous workers in a team contest. We identify a fundamental trade-off faced by an output-maximizing manager between intra-team production efficiency and inter-team competition. We fully characterize the optimal sorting pattern and analyze how it varies with environmental factors. Further, we consider extended settings in which the manager can set the prize schedule, allocate productive resources across teams, or manipulate the level of competitiveness of the contest.

Our results yield ample implications. However, they should also be interpreted with caution in practical context. For instance, we show that when the contest is less competitive—that is, with a smaller r—the manager may prefer positive sorting, which leads to a contest between a strong team and a weak one. Further, when the manager can allocate productive resources, she may prioritize the ex ante stronger team, which enlarges the asymmetry. This improves output within our context, but also increases unfairness in the workplace because the weaker team stands a smaller chance of winning. Although workers' placement and resource allocation are subject to the management's decision, the increased asymmetry could backfire by demoralizing workers and undermining the efficacy of the contest as an incentive device. There is evidence that economic agents value fairness or resist inequity (Fehr and Schmidt [1999]), which should be taken into account when interpreting our predictions in practice. This also suggests that our results tend to be more plausible when worker heterogeneity is mild, in which case the inter-team inequality would be less severe. Further, our analysis shows that the manager may concentrate the entire prize purse on one worker when the degree of complementarity is low. One can imagine that one worker on a team is awarded only a base pay while the other is rewarded with bonus for outstanding rank-based performance. In this scenario, however, teamwork is less significant because of lesser complementarity. This could also allude to a boundary for our study: Our analysis is more relevant for scenarios in which effort complementarity is relatively more significant.

In this paper, we assume that the manager employs RPE—that is, contests—to incentivize workers. Relatedly, Franco *et al.* [2011] consider a standard model of moral hazard with team production, in which a profit-maximizing principal decides how to sort agents into teams, as well as a wage scheme with independent performance evaluation (IPE). Their model focuses on within-team moral hazard problems and thus abstracts away the use of inter-team competition to further save agency costs. It would be interesting to reexamine the optimal sorting of workers while endogenizing the choice of RPE vis-à-vis IPE.

In this paper, we assume a contest in which team performance is ranked and workers are rewarded by collective output. Such incentive mechanisms are prevalent in practice, but the literature has yet to comprehensively address the fundamental question of why team-based incentives are adopted. Chen and Lim [2013] and Lim and Chen [2014] provide rationales from behavioral economics perspectives and experimental evidence. This question is beyond the scope of this paper, but definitely warrants future research.

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APPENDIX A

PROOFS

Proof of Lemma 1

Proof. See main text. ■

Proof of Proposition 1

Proof. Recall that we use x to denote c_H/c_L, so $x > 1$. For $\rho \neq 0$, let us define

(A1)

$$\begin{aligned}\mathcal{W}(\rho, r; x) &:= \log\left(\mathcal{Y}^P/\mathcal{Y}^N\right) \\ &= r\log x + \log(1+x) + \frac{1}{\rho}\log 2 - 2\log(1+x^r) - \frac{1-\rho}{\rho}\log\left(1 + x^{\frac{\rho}{1-\rho}}\right),\end{aligned}$$

and let $\mathcal{W}(0, r; x) := \lim_{\rho\to 0}\log\left(\mathcal{Y}^P/\mathcal{Y}^N\right)$. It can be verified that $\mathcal{W}$ is continuous and differentiable with respect to both $\rho < 1$ and $r \in (0, 1]$. Further, simple algebra verifies that $\mathcal{Y}^P > \mathcal{Y}^N$ is equivalent to $\mathcal{W}(\rho, r; x) > 0$. It is useful to prove an intermediate result.

Lemma A1. $\mathcal{W}(\rho, r; x)$ is strictly decreasing in ρ. Moreover, fixing $r \in (0,1)$, there exists a unique solution to $\mathcal{W}(\rho^*, r; x) = 0$, which we denote by $\rho^*(r)$, with $\lim_{r\to 1}\rho^*(r) = -\infty$. For $r = 1$, $\mathcal{W}(\rho, 1; x) < 0$ for all $\rho > -\infty$.

Proof. The proof consists of two steps.

Step I. We show that $\frac{\partial \mathcal{W}}{\partial \rho}(\rho, r; x) < 0$ for $\rho \in (-\infty, 1)$. Note that $\lim_{\rho\to 0}\partial\mathcal{W}/\partial\rho = -\log^2(x)/8 < 0$, and it suffices to show that $\partial\mathcal{W}/\partial\rho < 0$ for $\rho \neq 0$. Taking the derivative of $\mathcal{W}(\rho, r; x)$ with respect to ρ, we can obtain that

$$\frac{\partial \mathcal{W}(\rho, r; x)}{\partial \rho} = \frac{1}{\rho^2}\left[\log\left(\frac{1}{2}\left(x^{\frac{\rho}{1-\rho}} + 1\right)\right) - \frac{\rho\log(x)x^{\frac{\rho}{1-\rho}}}{(1-\rho)\left(x^{\frac{\rho}{1-\rho}} + 1\right)}\right].$$

Let $y := x^{\frac{\rho}{1-\rho}} > 0$. It follows immediately from $\rho \neq 0$ and $x > 1$ that $y \neq 1$. Now we have that

$$\frac{\partial \mathcal{W}(\rho, r; x)}{\partial \rho}\Big|_{x=y^{\frac{1-\rho}{\rho}}} = \frac{1}{\rho^2}\left[\log\left(\frac{y+1}{2}\right) - \frac{y\log(y)}{y+1}\right].$$

It suffices to show that $\log\left(\frac{y+1}{2}\right) - \frac{y\log(y)}{y+1} < 0, \forall\, y > 0, y \neq 1$. Note that

$$\frac{d}{dy}\left[\log\left(\frac{y+1}{2}\right) - \frac{y\log(y)}{y+1}\right] = -\frac{\log(y)}{(y+1)^2},$$

and the right-hand side of the above equation is strictly positive if and only if $y < 1$. Therefore, the term $\log\left(\frac{y+1}{2}\right) - \frac{y\log(y)}{y+1}$ achieves its maximum at $y = 1$, which is equal to zero and in turn implies that $\partial\mathcal{W}/\partial\rho < 0$.

Step II. We show that (i) $\mathcal{W}(1, r; x) < 0$ for all $r \in (0,1]$ and (ii) $\mathcal{W}(-\infty, r; x) := \lim_{\rho\to-\infty}\mathcal{W}(\rho, r; x) > 0$ for $r \in (0,1)$.

(i) We first show that $\mathcal{W}(1, r; x) < 0$ for all $r \in (0,1]$. Simple algebra would verify that

$$\mathcal{W}(1, r; x) = \log\left(\mathcal{Y}^P/\mathcal{Y}^N\right)\Big|_{\rho=1} = \log\left(\frac{2x^{r-1}(1+x)}{(1+x^r)^2}\right).$$

It is straightforward to verify that $2x^{r-1}(1+x) < (1+x^r)^2$ for all $x > 1$. Therefore, $\mathcal{W}(1, r; x) < 0$ for all $x > 1$ and $r \in (0,1]$.

(ii) Next, we show that $\mathcal{W}(-\infty, r; x) > 0$ for $r \in (0,1)$. Note that

$$\mathcal{W}(-\infty, r; x) = \log\left(\mathcal{Y}^P/\mathcal{Y}^N\right)\Big|_{\rho\to-\infty} = \log\left(\frac{x^{r-1}(1+x)^2}{(1+x^r)^2}\right),$$

and it is equivalent to show $p(x) := x^{r-1}(1+x)^2 - (1+x^r)^2 > 0$ for all $x > 1$, which holds due to the fact that $p(1) = 0$ and $p'(x) > 0$ for all $x > 1$ and $r \in (0,1)$.

In summary, $\partial \mathcal{W}/\partial \rho < 0$, $\mathcal{W}(1, r; x) < 0$, and $\mathcal{W}(-\infty, r; x) > 0$ for $r \in (0, 1)$. Therefore, fixing $r \in (0, 1)$, there exists a unique solution to $\mathcal{W}(\rho^*, r; x) = 0$.

For the case of $r = 1$, it is straightforward to verify that $\mathcal{W}(-\infty, 1; x) = 0$. This implies that for any constant $C > -\infty$, $\mathcal{W}(C, 1; x) < 0$, because $\mathcal{W}$ is decreasing in its first argument ρ. Therefore, negative sorting always prevails regardless of ρ when $r = 1$. Moreover, because of $\mathcal{W}$'s continuity in r, there exists a $\underline{r} < 1$ such that $\mathcal{W}(C, r; x) < 0$ for all $r > \underline{r}$. In turn, this implies that for any $r > \underline{r}$, $\rho^*(r) < C$. This holds for any $C > -\infty$, and therefore $\lim_{r \to 1} \rho^*(r) = -\infty$. This concludes the proof. ■

Now we are ready to prove the proposition. It remains to show that $\rho^*(r)$ strictly decreases with r. By the implicit function theorem, we can obtain that

$$\frac{d\rho^*(r)}{dr} = -\frac{\partial \mathcal{W}/\partial r}{\partial \mathcal{W}/\partial \rho}\Big|_{\rho=\rho^*(r)}.$$

By Lemma 1, $\frac{\partial \mathcal{W}}{\partial \rho} < 0$; together with $\frac{\partial \mathcal{W}}{\partial r} = -\frac{\log(x)(x^r-1)}{1+x^r} < 0$, we conclude that $\frac{d\rho^*(r)}{dr} < 0$. ■

A.3 Proof of Lemmas 2 and 3

Proof. Fixing an arbitrary cost profile $\boldsymbol{c} := \langle (c_{11}, c_{12}), (c_{21}, c_{22}) \rangle$ and a prize schedule $\boldsymbol{v} := \langle (v_{11}, v_{12}), (v_{21}, v_{22}) \rangle$, the first-order conditions that govern workers' equilibrium effort are

$$\text{(A2)} \qquad \frac{r}{2} e_{ik}^{\rho-1} \left(\frac{1}{2} e_{i1}^{\rho} + \frac{1}{2} e_{i2}^{\rho} \right)^{\frac{1}{\rho}-1} \frac{\mathcal{Y}_i^{r-1} \mathcal{Y}_{-i}^{r}}{(\mathcal{Y}_1^r + \mathcal{Y}_2^r)^2} = \frac{c_{ik}}{v_{ik}}, i \in \{1, 2\}, k \in \{1, 2\}.$$

The equilibrium total output—which we denote by $\mathcal{Y}$ with slight abuse of notation—can be derived as

$$\mathcal{Y} = r \left(\frac{1}{2} \right)^{\frac{1}{\rho}} \frac{\mathcal{K}_1^r \mathcal{K}_2^r (\mathcal{K}_1 + \mathcal{K}_2)}{(\mathcal{K}_1^r + \mathcal{K}_2^r)^2},$$

where

$$\mathcal{K}_i := \left[\left(\frac{v_{i1}}{c_{i1}} \right)^{\frac{\rho}{1-\rho}} + \left(\frac{v_{i2}}{c_{i2}} \right)^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}} \geq 0, i \in \{1, 2\}.$$

It can be verified that $\mathcal{Y}$ is increasing in $\mathcal{K}_i$ for $i \in \{1, 2\}$. Further, $\mathcal{K}_i$ is increasing in both v_{i1} and v_{i2}. Therefore, we must have $v_{i1} + v_{i2} = 2$ in the optimum, regardless of the sorting pattern.

A.3.1 Positive Sorting

With $c_{11} = c_{12} = c_L =: c_1, c_{21} = c_{22} = c_H =: c_2$, and $v_{i2} = 2 - v_{i1}$, we have that

$$\mathcal{K}_i = \frac{1}{c_i} \left[v_{i1}^{\frac{\rho}{1-\rho}} + (2 - v_{i1})^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}} =: \Lambda_i(v_{i1}), i \in \{1, 2\}.$$

It can be verified that Λ_i is maximized at $v_{i1} \in \{0,1,2\}$, depending on the value of ρ. For $\rho > 0$, we have

$$\Lambda_i(1) = 2^{\frac{1-\rho}{\rho}}/c_i, \text{ and } \Lambda_i(0) = \Lambda_i(2) = 2/c_i.$$

Simple algebra would verify that $\Lambda_i(1) > \Lambda_i(0) = \Lambda_i(2)$ if and only if $0 < \rho < 1/2$. For $\rho \leq 0$, we can obtain that $\Lambda_i(0) = \Lambda_i(2) = 0 < \Lambda_i(1)$.

In summary, for $\rho \in (-\infty, 1/2)$, it is optimal for the manager to set $v_{11} = v_{21} = 1$; otherwise, she should set $v_{11} = v_{21} \in \{0,2\}$. The expression of total output under positive sorting can be derived accordingly, as provided in Lemma 2.

A.3.2 Negative Sorting

With $c_{11} = c_{21} = c_L, c_{12} = c_{22} = c_H$, and $v_{i2} = 2 - v_{i1}$, we have that

$$\mathcal{K}_i = \left[\left(\frac{v_{i1}}{c_L}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{2-v_{i1}}{c_H}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}} =: \Omega_i(v_{i1}).$$

Simple algebra would verify that Ω_i is maximized at $v_{i1} = \frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}$ or at $v_{i1} = 2$, depending on the value of ρ.

For $\rho > 0$, we have that

$$\Omega_i\left(\frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}\right) = \frac{2}{\left(c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}\right)^{\frac{2\rho-1}{\rho}}}, \text{ and } \Omega_i(2) = 2/c_L.$$

It can be verified that $\Omega_i\left(\frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}\right) > \Omega_i(2)$ if $0 < \rho < 1/2$ and $\Omega_i\left(\frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}\right) < \Omega_i(2)$ if $1/2 < \rho < 1$. Therefore, the manager would set $v_{11} = v_{21} = \frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}$ if $0 < \rho < 1/2$ and $v_{11} = v_{21} = 2$ if $1/2 < \rho < 1$.

For $\rho \leq 0$, we have that $\Omega_i\left(\frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}\right) > 0 = \Omega_i(2)$ and thus the manager would set $v_{11} = v_{21} = \frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}$.

In summary, for $\rho \in (-\infty, 1/2)$, it is optimal for the manager to set $v_{11} = v_{21} = \frac{2c_L^{\frac{\rho}{2\rho-1}}}{c_H^{\frac{\rho}{2\rho-1}} + c_L^{\frac{\rho}{2\rho-1}}}$; otherwise, the manager should set $v_{11} = v_{21} = 2$. The expression of total output under negative sorting can be derived accordingly, as provided in Lemma 3. ■

A.4 Proof of Proposition 2

Proof. The proof is similar to that of Proposition 1, and is omitted for brevity. ■

A.5 Proof of Lemmas 4 and 5

Proof. Fixing an arbitrary cost profile $\boldsymbol{c} := \langle (c_{11}, c_{12}), (c_{21}, c_{22}) \rangle$ and a resource allocation plan $\boldsymbol{\delta} := (\delta_1, \delta_2)$, the first-order conditions that govern workers' equilibrium effort are

$$\text{(A3)} \qquad \frac{r\delta_1^r \delta_2^r}{2} e_{ik}^{\rho-1} \left(\frac{1}{2} e_{i1}^{\rho} + \frac{1}{2} e_{i2}^{\rho} \right)^{\frac{1}{\rho}-1} \frac{\mathcal{Y}_i^{r-1} \mathcal{Y}_{-i}^r}{(\delta_1^r \mathcal{Y}_1^r + \delta_2^r \mathcal{Y}_2^r)^2} = c_{ik}, i \in \{1,2\}, k \in \{1,2\},$$

from which we can derive equilibrium individual efforts under positive sorting ($c_{11} = c_{12} = c_L$ and $c_{21} = c_{22} = c_H$) and those under negative sorting ($c_{11} = c_{21} = c_L$ and $c_{12} = c_{22} = c_H$), respectively. Note that δ_1 refers to the amount of resource allocated to the strong team under positive sorting. The associated equilibrium total output can then be derived as

$$\text{(A4)} \qquad \mathcal{Y}_b(\delta_1, \delta_2) = r \left(\frac{1}{2} \right)^{\frac{1}{\rho}} \delta_1^r \delta_2^r \frac{\mathcal{M}_1^r \mathcal{M}_2^r (\delta_1 \mathcal{M}_1 + \delta_2 \mathcal{M}_2)}{(\delta_1^r \mathcal{M}_1^r + \delta_2^r \mathcal{M}_2^r)^2},$$

where

$$\mathcal{M}_i := \left[(1/c_{i1})^{\frac{\rho}{1-\rho}} + (1/c_{i2})^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}}.$$

It can be verified that $\mathcal{Y}_b$ is increasing in δ_i, with $i \in \{1,2\}$. Therefore, $\delta_1 + \delta_2 = 2$ in the optimum. Further, the optimal amount of resource allocated to the first team under positive sorting, denoted by δ_1^P, solves

$$\text{(A5)} \qquad \frac{\partial}{\partial \delta_1} \left(\frac{\delta_1^r \delta_2^r (\delta_1 x + \delta_2)}{(\delta_1^r x^r + \delta_2^r)^2} \right) \Big|_{\delta_2 = 2-\delta_1} = \frac{\partial}{\partial \delta_2} \left(\frac{\delta_1^r \delta_2^r (\delta_1 x + \delta_2)}{(\delta_1^r x^r + \delta_2^r)^2} \right) \Big|_{\delta_2 = 2-\delta_1},$$

and that under negative sorting, denoted by δ_1^N, is $\delta_1^N = 1$. Plugging δ_i^P and δ_i^N into (A4), respectively, gives the expression of total output as provided in Lemma 4 and Lemma 5.

It remains to show that there exists $\tilde{r}(x) \in (0,1)$ such that $\delta_1^P > 1$ if and only if $r < \tilde{r}(x)$. It can be verified that the left-hand side of Equation (A5) ($\mathcal{LHS}$) is decreasing in δ_1 while the right-hand side ($\mathcal{RHS}$) is increasing in δ_1. Therefore, $\delta_1^P > 1$ if and only if $\mathcal{LHS} > \mathcal{RHS}$ at $\delta_1 = 1$, which is equivalent to

$$\mathcal{Q}(\delta_1, r, x) := \frac{(x-1)\delta_1(2-\delta_1)[\delta_1^r x^r + (2-\delta_1)^r]}{2r[\delta_1 x + (2-\delta_1)][\delta_1^r x^r - (2-\delta_1)^r]} > 1 \quad \text{at } \delta_1 = 1.$$

Note that

$$\mathcal{Q}(1, r, x) = \frac{(x-1)(x^r+1)}{2r(x+1)(x^r-1)},$$

is decreasing in r, with $Q(1,0,x)=+\infty$ and $Q(1,1,x)=1/2$. Therefore, there exists a cutoff $\tilde{r}\in(0,1)$—which is the unique solution to $Q(1,r,x)=1$—such that $\delta_1^P>1$ if and only if $r<\tilde{r}$. ■

A.6 Proof of Proposition 3

Proof. The proof is similar to that of Proposition 1, and is omitted for brevity. ■

A.7 Proof of Lemma 6

Proof. Part (i) of the lemma follows immediately from Lemma 1 and it remains to prove part (ii). Taking the derivative of $\mathcal{Y}^P$ with respect to r yields

$$\frac{d\mathcal{Y}^P}{dr}=\frac{(x+1)x^{r-1}\left[1+x^r-(x^r-1)\log x^r\right]}{2c_L(1+x^r)^3}.$$

Note that $f(r;x):=1+x^r-(x^r-1)\log x^r$ is concave in r, with $f(0;x)=2$ and $f(1;x)=1+x-(x-1)\log x$. Further, it can be verified that there exists a unique solution $\underline{x}>1$ to $1+x-(x-1)\log x=0$ and $f(1;x)<0$ if and only if $x>\underline{x}$.

Therefore, for $x\le\underline{x}$, we have $d\mathcal{Y}^P/dr\ge 0$ and thus it is optimal for the manager to set $r=1$. For $x>\underline{x}$, there exists a cutoff $\hat{r}$—which is the unique solution to $f(r;x)=0$—such that $f(r;x)>0$ if and only if $r<\hat{r}$. Therefore, for $x>\underline{x}$, $\mathcal{Y}^P$ is single-peaked with respect to r and is maximized at $r=\hat{r}$; and for $x\le\underline{x}$, $\mathcal{Y}^P$ is increasing in r and is maximized at $r=1$. ■

A.8 Proof of Proposition 4

Proof. Part (ii) of the proposition follows immediately from Proposition 1 and Lemma 6 and it remains to prove part (i). Fix $x>\underline{x}$. For $\rho\neq 0$, let us define

$$\mathcal{U}(\rho;x):=\log\left(\mathcal{Y}^P|_{r=\hat{r}}\Big/\mathcal{Y}^N|_{r=1}\right).$$

and $\mathcal{U}(0;x):=\lim_{\rho\to 0}\log\left(\mathcal{Y}^P|_{r=\hat{r}}/\mathcal{Y}^N|_{r=1}\right)$. Note that $\log\mathcal{Y}^P$ is independent of ρ and $\log\mathcal{Y}^N-\log r$ is independent of r. Therefore, we have $d\mathcal{U}/d\rho=-\partial\log\mathcal{Y}^N/\partial\rho=\partial\mathcal{W}/\partial\rho<0$, where the strict inequality follows from Lemma 1. Therefore, it suffices to show $\mathcal{U}(1;x)<0$ and $\mathcal{U}(-\infty;x)>0$.

It can be verified that $2\hat{r}(x+1)x^{\hat{r}-1}\le 2(x+1)x^{\hat{r}-1}<(x^{\hat{r}}+1)^2$ for all $x>1$, which in turn implies that

$$\mathcal{U}(1;x)=\log\left(\frac{2\hat{r}(x+1)x^{\hat{r}-1}}{(x^{\hat{r}}+1)^2}\right)<0.$$

Further, we have that

$$\mathcal{U}(-\infty;x)=\log\left(\frac{\hat{r}(x+1)^2x^{\hat{r}-1}}{(x^{\hat{r}}+1)^2}\right)=:g(x).$$

Recall from the proof of Lemma 6 that $\hat{r}$ is the unique solution to $f(r;x) = 0$. That is, $1 + x^{\hat{r}} - (x^{\hat{r}} - 1)\log x^{\hat{r}} = 0$, from which we can obtain that $d\hat{r}/dx = -\hat{r}/(x \log x)$ and

$$g'(x) = \left(\frac{1}{\hat{r}} + \log x - \frac{2x^{\hat{r}}\log x}{x^{\hat{r}} + 1}\right)\left(-\frac{\hat{r}}{x\log x}\right) + \left(\frac{2}{1+x} + \frac{\hat{r}-1}{x} - \frac{2\hat{r}x^{\hat{r}-1}}{x^{\hat{r}}+1}\right)$$
$$= -\frac{1 + x - (x-1)\log x}{x(x+1)\log x}.$$

Note that $1 + x - (x-1)\log x = 0$ has a unique solution for $x > 1$, which is $\underline{x}$ as defined in the proof of Lemma 6; further, $1 + x - (x-1)\log x$ is negative if $x > \underline{x}$. Therefore, $g(x)$ is increasing in x if $x > \underline{x}$. Recall again from the proof of Lemma 6 that $\hat{r} = 1$ at $x = \underline{x}$. Therefore, fixing $x > \underline{x}$, we have that

$$g(x) > g(\underline{x}) = \left[\log \hat{r} + 2\log(x+1) + (\hat{r}-1)\log x - 2\log(x^{\hat{r}} + 1)\right]\Big|_{x=\underline{x}} = 0.$$

This concludes the proof. ■