Multi-Prize Contests with Expectation-Based Loss-Averse Players*

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Abstract

We investigate the optimal prize allocation in a multi-winner nested Tullock contest model with symmetric contestants who exhibit expectation-based loss aversion. We show that (i) multiple prizes can be optimal when contestants are sufficiently loss averse; (ii) all prizes should be equal in the optimal contest; and (iii) the number of prizes increases as the degree of loss aversion increases.

Keywords: Reference-dependent Preferences; Expectation-based Loss Aversion; Prize Allocation; Contest Design.

JEL Classification Codes: C72, D72, D81, D91.

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1 Introduction

A broad spectrum of competitive activities resemble a contest, ranging from R&D tournaments (e.g., XPRIZE Challenges), sporting events, and college admissions to internal labor markets inside firms: Contestants strive for limited prizes, while their costly efforts are nonrefundable. Numerous economics studies have examined contestants' strategic behavior and the optimal design of contests.[1]

In this paper, we explore a classical contest design problem: the optimal prize allocation. A designer allocates a fixed amount of prize money over a number of prizes to maximize the total effort of the contest. The majority of the literature has conventionally assumed risk neutrality and held that a winner-take-all structure, or a steep reward schedule, provides maximum incentives (e.g., Rosen, 1986; Fu and Lu, 2012).[2] However, investment in a contest is risky. A large prize lures the effort to strive for a win, yet it may also be discouraging, especially for contestants who are sensitive to the loss from the nonrecoverable input when losing. The asymmetric responses to upward vis-à-vis downward shocks in payoffs casts doubt on the optimality of a steep prize structure.

We reexamine optimal prize allocation, while assuming that contestants are reference-dependent loss averse à la Kőszegi and Rabin (2006, 2007). Contestants' gain or loss is evaluated against an endogenously formed reference point from rational expectations, and a loss hurts one more than a gain of the same magnitude would benefit him. Experimental and empirical studies have gathered an increasing amount of evidence for such preference.[3] A burgeoning theoretical literature has also been devoted to the strategic interactions among loss-averse agents.[4]

We first characterize the unique symmetric choice-acclimating personal Nash equilibrium (CPNE). Based on the equilibrium result, we show that multiple prizes may emerge in the optimum when contestants are sufficiently loss averse, in which case the prize money is uniformly split among several prizes. We further identify an upper bound for the number of prizes in the optimum and demonstrate that the number of prizes weakly increases as the degree of loss aversion ascends. Our results contrast with those obtained in settings with risk neutrality or risk aversion. The former typically espouses the optimality of winner-take-all structures, while the latter highlights a monotonically decreasing prize series in the optimum when multiple prizes are required.

[1] For comprehensive overviews, see, e.g., Konrad (2009) and Fu and Wu (2019).

[2] See Sisak (2009) for a survey of studies on this topic.

[3] See Chowdhury, Jeon, and Ramalingam (2018); Crawford and Meng (2011); Gill and Prowse (2012); and Müller and Schotter (2010), among many others.

[4] See, e.g., Lange and Ratan (2010); Daido and Murooka (2016); Chen, Ong, and Segev (2017); Mermer (2017); Dato, Grunewald, Müller, and Strack (2017); Dato, Grunewald, and Müller (2018); Rosato and Tymula (2019); Eisenhuth (2019); Eisenhuth and Grunewald (2020); Rosato (2021); and Balzer and Rosato (2021).

2 The Model

A contest involves $N \geq 3$ homogeneous contestants, indexed by $i \in \mathcal{N} \equiv \{1, \cdots, N\}$, with N prizes to be awarded based on contestants' ranks. The prizes are ordered in a decreasing prize series $V_1 \geq \ldots \geq V_N \geq 0$, with strict inequality holding for at least one.[5] Contestants simultaneously commit to their efforts $e_i \geq 0$—which incurs a unity marginal cost—and each contestant is eligible for at most one prize.

2.1 Winner-selection Mechanism

We adopt the popularly studied multi-winner nested Tullock contest (Clark and Riis, 1996, 1998b) to depict the multi-prize contest. The model resembles a sequential lottery process. Fixing an effort profile $\boldsymbol{e} := (e_1, \ldots, e_N)$, a contestant i is picked as the recipient of the first prize, V_1, with a probability

$$p_1^i(\boldsymbol{e}) := \begin{cases} \frac{(e_i)^r}{\sum_{j \in \mathcal{N}} (e_j)^r}, & \text{if } \boldsymbol{e} \neq (0 \ldots, 0), \\ \frac{1}{N}, & \text{if } \boldsymbol{e} = (0 \ldots, 0), \end{cases}$$

as in a standard Tullock contest. The parameter $r \in (0, 1]$ conventionally indicates the discriminatory power of the contest and conveniently measures the level of precision in the winner-selection mechanism. A large r implies a higher marginal return for effort and that noise and luck count less in determining the winner. The recipient of this prize is removed immediately from the pool of contestants eligible for the rest of the prizes, and a similar lottery picks the recipient of the second prize. The process is repeated until all prizes have been distributed.

To put this formally, let Ω^m, $m \in \{1, \ldots, N\}$ be the set of contestants who remain eligible for the mth-draw—i.e., those who were not picked in the previous $m-1$ draws—with $\Omega^1 \equiv \mathcal{N}$. Further denote by $\boldsymbol{e}^{\Omega^m}$ the effort profile of all contestants in Ω^m, with $\boldsymbol{e}^{\Omega^1} \equiv \boldsymbol{e}$. The probability of a contestant i's receiving the mth prize *conditional* on not having been picked in the previous $m-1$ draws is given by

$$p_m^i\left(\boldsymbol{e}^{\Omega^m}; \Omega^m\right) := \begin{cases} \frac{(e_i)^r}{\sum_{j \in \Omega^m} (e_j)^r}, & \text{if } \boldsymbol{e}^{\Omega^m} \neq (0, \ldots, 0), \\ \frac{1}{N-m+1}, & \text{if } \boldsymbol{e}^{\Omega^m} = (0, \ldots, 0). \end{cases}$$

[5]The prize allocation $V_1 = \ldots = V_N$ is clearly suboptimal, because no effort can be elicited in the equilibrium.

2.2 Contestants' Preference

Players have expectation-based reference-dependent preferences à la Kőszegi and Rabin (2006, 2007). Specifically, one's utility of consuming some deterministic material payoff c consists of two components: (i) the traditional intrinsic material payoff c and (ii) the psychological gain-loss utility he derives by comparing his material payoff with some reference material payoff ℓ according to

$$\mu(c-\ell)=\begin{cases}\eta(c-\ell) & \text{if } c-\ell\geq 0,\\ \eta\lambda(c-\ell) & \text{if } c-\ell<0,\end{cases}$$

with $\lambda > 1$ and $\eta \geq 0$. Loss aversion—i.e., a loss outweighs a gain of equal size—is captured by $\lambda > 1$, and η is the weight on gain-loss utility.

Following Kőszegi and Rabin (2006, 2007), we assume that the reference point is determined by the player's rational beliefs and corresponds to a reference lottery over his potential material payoffs if the outcome is stochastic. Fixing opponents' effort profile $\boldsymbol{e}_{-i} \equiv (e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_N)$, contestant i's expected payoff of exerting e_i when he expects himself to exert effort $\hat{e}_i$, denoted by $U_i(e_i, \hat{e}_i, \boldsymbol{e}_{-i}; \boldsymbol{V})$, is given by

$$U_i(e_i,\hat{e}_i,\boldsymbol{e}_{-i};\boldsymbol{V})=\sum_{m=1}^{N}\left\{P_m^i(e_i,\boldsymbol{e}_{-i})\times\left[V_m+\sum_{j=1}^{N}\Big[P_j^i(\hat{e}_i,\boldsymbol{e}_{-i})\mu(V_m-V_j)\Big]\right]\right\}-e_i+\mu(\hat{e}_i-e_i), \tag{1}$$

where $P_m^i(e_i, \boldsymbol{e}_{-i})$ is his ex ante probability of obtaining the mth prize.

2.3 Equilibrium Concept

We follow Kőszegi and Rabin (2007); Dato, Grunewald, Müller, and Strack (2017); and Dato, Grunewald, and Müller (2018) and adopt the following equilibrium notion.

Definition 1 ***(Choice-acclimating personal Nash equilibrium)*** *The effort profile* $\boldsymbol{e}^* \equiv (e_1^*, \dots, e_N^*)$ *constitutes a choice-acclimating personal Nash equilibrium (CPNE) in pure strategy if for all* $i \in \mathcal{N}$*,*

$$U_i(e_i^*, e_i^*, \boldsymbol{e}_{-i}^*; \boldsymbol{V}) \geq U_i(e_i, e_i, \boldsymbol{e}_{-i}^*; \boldsymbol{V}), \text{ for all } e_i \in [0, \infty).$$

By Definition 1, each contestant's expectations about future outcomes will have fully adapted to his actual strategic choice when the uncertainty is resolved; he then commits to a strategy that maximizes his expected utility given his opponents' strategy profile. In other words, the expectation is *choice acclimating*.[6]

2.4 Contest Design

A contest designer splits a fixed prize purse of $V > 0$ into N nonnegative prizes with $V_1 \geq \ldots \geq V_N \geq 0$ and $\sum_{m=1}^{N} V_m = V$. The prize schedule $\boldsymbol{V} \equiv (V_1, \ldots, V_N)$ is announced publicly prior to the competition. The designer aims to maximize the total effort of the contest, i.e., $\sum_{i=1}^{N} e_i^*$. Obviously, a winner-take-all contest arises when $V_2 = 0$.

3 Analysis

We first characterize the equilibrium of the contest game for an arbitrary prize schedule $\boldsymbol{V} \equiv (V_1, \ldots, V_N)$, then explore the optimal prize allocation based on the equilibrium results.

3.1 Equilibrium Fundamentals

We follow the tradition in the literature on multi-prize contests (e.g., Azmat and Möller, 2009; Fu and Lu, 2012) and focus on the symmetric equilibrium of the game.[7] In what follows, we drop the subscript in $U_i(\cdot, \cdot, \cdot; \cdot)$ and the superscript in $P_m^i(\cdot, \cdot)$. Consider an indicative contestant and suppose that the rest place the same bid e. An effort e' allows the indicative contestant to win the mth prize with a probability

$$P_m\left(e', e\right) \equiv \frac{(N-1)!}{(N-m)!} \times \left(\prod_{j=1}^{m-1} \frac{(e)^r}{(N-j)(e)^r + (e')^r} \right) \times \frac{(e')^r}{(N-m)(e)^r + (e')^r},$$

where the term $\frac{(e')^r}{(N-m)(e)^r + (e')^r}$ is the probability of being picked in the mth draw conditional on his not having been selected for the previous $m-1$ prizes. Obviously, $P_m(e, e) = 1/N$. Substituting $e_i = \hat{e}_i =: e'$ as required by Definition 1 and the symmetry condition $\boldsymbol{e}_{-i} =$

[6] Kőszegi and Rabin (2006, 2007) propose another equilibrium concept, the (preferred) personal equilibrium. It requires that a contestant's reference point be fixed and does not adjust to his choice of effort when taking action, i.e., choice unacclimating. These equilibrium concepts may depict different decision environments. A comprehensive analysis under the alternative notion of choice-unacclimating equilibrium is definitely worthwhile and should be attempted in future studies.

[7] The equilibrium existence and uniqueness of multi-prize nested Tullock contests with standard preferences have not formally been explored until very recently. Fu, Wu, and Zhu (2020) prove equilibrium existence in a generalized multi-prize lottery contest. Fu, Wang, and Wu (2021) apply their results and approach to a multi-winner nested Tullock contest with symmetric risk-averse players and verify the uniqueness of a symmetric equilibrium. Almost all previous studies in this strand of the literature have assumed homogeneous agents and focused on symmetric equilibrium for the sake of tractability.

$(e, \ldots, e)$ into (1), his expected payoff is

$$U\left(e', e; \boldsymbol{V}\right) = \underbrace{\sum_{m=1}^{N} P_m(e', e) \times V_m - e'}_{\text{material utility}} \underbrace{-k \times \sum_{m=1}^{N} \sum_{j=m}^{N} \left[P_m(e', e) \times P_j(e', e) \times (V_m - V_j)\right]}_{\text{gain-loss utility}}, \quad (2)$$

where $k := \eta(\lambda - 1) \geq 0$ is the overall weight in the expected utility attached to the net loss relative to the reference point and is a composite measure of the intensity of his reference-dependent loss aversion.

The first-order condition with respect to e' leads to

$$\sum_{m=1}^{N} \left[\frac{\partial P_m\left(e', e\right)}{\partial e'} V_m\right] = 1 + k \sum_{m=1}^{N} \sum_{j=m}^{N} \left\{\left[\frac{\partial P_m(e', e)}{\partial e'} P_j(e', e) + \frac{\partial P_j(e', e)}{\partial e'} P_m(e', e)\right] (V_m - V_j)\right\}. \quad (3)$$

A symmetric equilibrium requires $e' = e$, in which case

$$\left.\frac{\partial P_m\left(e', e\right)}{\partial e'}\right|_{e'=e} = \frac{r}{Ne} \times \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g}\right) =: \mu_m \times \frac{r}{Ne}, \quad (4)$$

where μ_m, with $m \in \{1, \ldots, N\}$, is defined as $\mu_m := 1 - \sum_{g=0}^{m-1} \frac{1}{N-g}$. We establish the following.

Proposition 1 ***(Equilibrium Effort)*** *Suppose that $k \equiv \eta(\lambda - 1) \in (0, \frac{1}{3}]$ and $r \in (0, 1]$. Then there exists a unique symmetric pure-strategy equilibrium of the contest game, in which each contestant's equilibrium effort is given by*

$$e^*(\boldsymbol{V}) = \frac{r}{N} \times \sum_{m=1}^{N-1} \left[\beta_m \times m(V_m - V_{m+1})\right], \quad (5)$$

with β_m, $m \in \{1, \ldots, N-1\}$, to be given by

$$\beta_m := \frac{N-m}{m} \times \frac{(1-k)N + 2km}{N} \times (1 - \mu_m).$$

Two remarks are in order. First, Proposition 1 establishes a pure-strategy CPNE of the contest game for moderate loss aversion.[8] The requirement of $k \leq 1/3$ is a sufficient

[8] We follow the literature and focus on the case of moderate loss aversion. For example, Herweg, Müller, and Weinschenk (2010) study a principal-agent model with moral hazard and an expectation-based loss-averse agent. They assume that the degree of loss aversion is less than one; otherwise, a decision maker may choose stochastically dominated options, which is undesirable and less interesting from a theoretical perspective.

condition that ensures the concavity of the indicative player's expected payoff $U(e', e; \boldsymbol{V})$ in his effort e'. The same condition is imposed by Fu, Lyu, Wu, and Zhang (2020), who consider a single-prize Tullock contest with heterogeneous players and investigate the impact of loss aversion on players' effort incentives. They show that a CPNE may not exist if the degree of loss aversion k is greater than 1/3, even in a simple two-player single-prize contest.[9]

Second, expression (5) sheds light on the nature of the incentives provided by a contest. The equilibrium effort is a linear combination of the differentials between neighboring prizes—i.e., $V_m - V_{m+1}$—weighted by β_m. Contestants strive to vie for the prize premium that rewards higher ranks. Intuitively, the parameter β_m measures the effectiveness of the prize differential $V_m - V_{m+1}$ in eliciting effort.

3.2 Optimal Prize Allocation

The designer sets $\boldsymbol{V} \equiv (V_1, \ldots, V_N)$, with $V_1 \geq \ldots \geq V_N \geq 0$ and $\sum_{m=1}^{N} V_m = V$, to maximize $e^*(\boldsymbol{V})$ as given by (5). The budget constraint $\sum_{m=1}^{N} V_m = V$ can be rewritten as

$$\sum_{m=1}^{N-1} m(V_m - V_{m+1}) + NV_N = V. \tag{6}$$

Simple logic can verify that the optimum always requires $V_N = 0$. Then the optimization problem can intuitively be interpreted as one that manipulates $m(V_m - V_{m+1})$ to maximize (5), subject to the constraint of $\sum_{m=1}^{N-1} m(V_m - V_{m+1}) = V$. The optimum requires

$$V_1 = \ldots = V_{\mathcal{M}(k)} = \frac{V}{\mathcal{M}(k)}, \; V_{\mathcal{M}(k)+1} = \ldots = V_N = 0,$$

where

$$\mathcal{M}(k) := \arg\max_{m} \beta_m. \tag{7}$$

To see the logic, recall that β_m measures the effectiveness of the prize differential $V_m - V_{m+1}$ in incentivizing effort. The designer must maximize the prize differential $V_m - V_{m+1}$ associated with the largest β_m and set the rest to zero. As a result, the designer evenly splits her budget among $\mathcal{M}(k)$ positive prizes; contestants strive only to ascend to the top $\mathcal{M}(k)$.

Define the cutoff $\overline{k} := \min\left\{\frac{N^2}{(5N-2)(N-2)}, \frac{1}{3}\right\}$.[10] The following result can be formally stated.

Proposition 2 ***(Optimal Prize Allocation)*** *Suppose that $r \in (0, 1]$ and contestants are expectation-based loss averse, with $k \in (0, \frac{1}{3}]$. An effort-maximizing contest designer awards*

[9]Relatedly, in a two-player rank-order tournament setting, Gill and Stone (2010) show by example that a pure-strategy CPNE ceases to exist when the degree of loss aversion is above a critical value that is strictly smaller than one.

[10]It can be verified that $\overline{k} = \frac{1}{3}$ for $N \leq 5$ and $\overline{k} < \frac{1}{3}$ for $N \geq 6$.

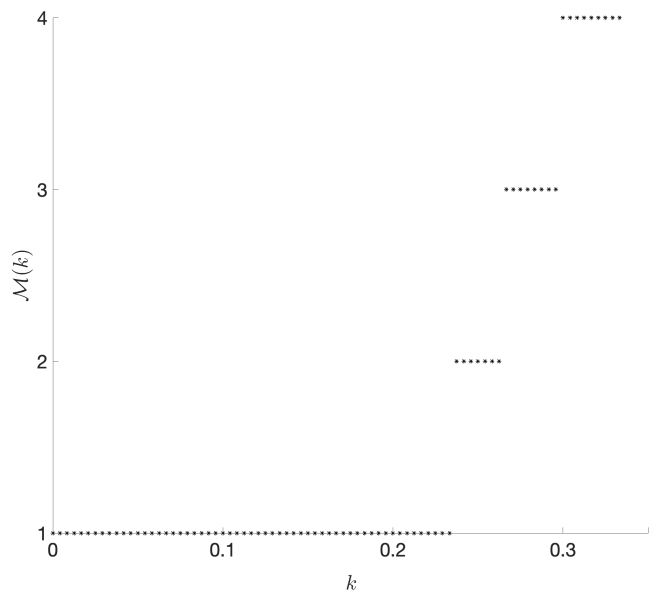

Figure 1: Optimal Number of Prizes under Different Levels of Loss Aversion: $N = 15$.

a single positive prize—i.e., $\mathcal{M}(k) = 1$—if $k < \overline{k}$ and multiple positive prizes—i.e., $\mathcal{M}(k) \geq 2$—instead if $k > \overline{k}$. Whenever multiple prizes are awarded, all $\mathcal{M}(k)$ prizes must be of the same size, i.e., $V_1 = \ldots = V_{\mathcal{M}(k)} = V/\mathcal{M}(k)$.

Multiple prizes emerge in the optimum if and only if contestants' loss aversion is sufficiently strong, i.e., $k > \overline{k}$. A closer look at (7) further yields the following.

Proposition 3 ***(Optimal Number of Prizes)*** *Suppose that $\frac{1}{3} \geq k_1 > k_2 > 0$. Then $\frac{N-1}{2} \geq \mathcal{M}(k_1) \geq \mathcal{M}(k_2)$.*

Recall that $k \equiv \eta(\lambda - 1)$ measures the intensity of loss aversion. Propositions 2 and 3 imply that more significant loss aversion favors a more dispersed prize structure. By Proposition 2, multiple prizes arise in the optimum if and only if k exceeds the cutoff $\overline{k}$. By Proposition 3, the number of prizes increases as k rises, while the size of each prize declines in response. Proposition 3 also specifies an upper limit for the number of prizes in the optimum, which would never exceed half of the number of the contestants. Figure 1 illustrates how the optimal number of prizes, $\mathcal{M}(k)$, varies with the intensity of loss aversion, k.

We now briefly interpret the results. Recall by (5) that contestants are lured by the prize premium $V_m - V_{m+1}$ to strive for higher ranks. However, a loss-averse agent strongly dislikes the uncertainty associated with the realized payoffs: A closer look at the expected payoff function (2) reveals that in the presence of loss aversion (a positive k), a larger prize premium enlarges the variance in realized payoffs and thus amplifies the utility loss when a contestant compares his payoffs across different ranks. This tension generates a fundamental trade-off for the designer: A steeper prize series entices one to step up effort, which compounds the pain when he falls short and, in turn, forces him to hold back. By (2) and the first-order condition (3), the disincentive caused by loss aversion is minimized if and only if $V_1 = \ldots = V_N$, which nevertheless defuses the competitive incentive entirely. The designer must strike a balance between minimizing the variation in realized payoffs and maintaining incentive provision, which leads her to award uniform prizes but only to fewer than half of the contestants. This echoes the result of Herweg, Müller, and Weinschenk (2010), which espouses the merit of a coarse reward structure for expectation-based loss-averse agents, in that the principal provides a (fixed) bonus whenever performance exceeds a threshold.

In light of Propositions 2 and 3, we can inspect Proposition 1 more closely and obtain further insight. By Proposition 2, the equilibrium effort in the optimum boils down to $e^*(\boldsymbol{V}) = \frac{r}{N} \times \beta_{\mathcal{M}(k)} \times V$, where $\beta_{\mathcal{M}(k)} = \frac{N-\mathcal{M}(k)}{\mathcal{M}(k)} \times \frac{(1-k)N+2k\mathcal{M}(k)}{N} \times (1-\mu_{\mathcal{M}(k)})$. Imagine a marginal increase in k—i.e., slightly more loss-averse contestants—such that the optimal number of prizes remains the same. Simple math can verify that $\beta_{\mathcal{M}(k)}$ strictly decreases with k because $\mathcal{M}(k) \leq (N-1)/2$ by Proposition 3. This implies that equilibrium effort strictly decreases with the degree of loss aversion k.[11]

4 Discussion: Comparison with Models of Risk Aversion

We assume reference-dependent loss aversion when modeling contestants' risk attitudes. The economics literature has also often described agents' sensitivity to uncertainty in terms of risk aversion. In Table 1, we compare our results with those under risk neutrality and risk aversion, respectively.

Risk Attitude	Paper	Optimal Prize Allocation
Risk Neutral	Clark and Riis (1998b); Fu and Lu (2009)	Single prize
Risk Averse	Fu, Wang, and Wu (2021)	Single prize or multiple decreasing prize series
Loss Averse	This paper	Single prize or multiple uniform prize series

Table 1: Risk Attitude and Optimal Prize Allocation.

Assuming risk-averse contestants, Fu, Wang, and Wu (2021), in a multi-prize nested

[11]It can similarly be verified that such comparative statics continue to hold even if a sufficiently large increase in k occurs, such that $\mathcal{M}(k)$ increases in response.

Tullock contest model, demonstrate that whenever awarding multiple prizes is optimal, the prizes must be of descending sizes. In contrast, with loss-averse contestants, we show that prizes are of equal sizes in the optimum. The contrasting observations between these two settings mirror those between Hölmstrom (1979) and Herweg, Müller, and Weinschenk (2010) in a principal-agent setting. Assuming a risk-averse agent, the former shows that the principal pays different wages for different levels of output. In contrast, the latter assume an expectation-based loss-averse agent; they demonstrate that the optimal contract is a binary payment scheme by which the agent receives a base salary and may be awarded a bonus when her performance exceeds a threshold.

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Appendix: Proofs

Proof of Proposition 1

Proof. To prove the proposition, it suffices to show that $U(e', e; \boldsymbol{V})$ is concave in $e' > 0$. It is evident that $U(e', e; \boldsymbol{V})$ is linear in e' for $e = 0$, and it suffices to prove the result for the case of $e > 0$. For notational convenience, define

$$q_m(e', e) := \frac{(N-1)!}{(N-m-1)!} \times \prod_{g=1}^{m} \frac{(e)^r}{(N-g)(e)^r + (e')^r}, \; m \in \{1, \ldots, N-1\}.$$

It is straightforward to verify that $q_m(e', e) \leq 1$. Carrying out the algebra, we can obtain that

$$\begin{aligned} U(e', e; \boldsymbol{V}) =& V_1 - \sum_{m=1}^{N-1} \left[(V_m - V_{m+1}) q_m(e', e) \right] - k \sum_{m=1}^{N-1} \left\{ (V_m - V_{m+1}) q_m(e', e) \left[1 - q_m(e', e) \right] \right\} - e' \\ =& V_1 - e' - \sum_{m=1}^{N-1} \left\{ (V_m - V_{m+1}) \left[(1+k) q_m(e', e) - k \left[q_m(e', e) \right]^2 \right] \right\}. \end{aligned}$$

It remains to show that $(1+k)q_m(e',e) - k\left[q_m(e',e)\right]^2$ is convex in e' for $m \in \{1,\dots,N-1\}$.

It is straightforward to verify that $q_m(e',e)$ is log-convex in e', which implies that

$$q_m(e',e)\frac{\partial^2 q_m(e',e)}{\partial e'^2} \geq \left(\frac{\partial q_m(e',e)}{\partial e'}\right)^2, \ m \in \{1,\dots,N-1\}. \tag{8}$$

Therefore, for $m \in \{1,\dots,N-1\}$, we have that

$$\begin{aligned}
&\frac{\partial^2}{\partial e'^2}\left[(1+k)q_m(e',e) - k\left[q_m(e',e)\right]^2\right] \\
=&\left[1+k-2kq_m(e',e)\right]\frac{\partial^2 q_m(e',e)}{\partial e'^2} - 2k\left(\frac{\partial q_m(e',e)}{\partial e'}\right)^2 \\
\geq&(1-k)q_m(e',e)\frac{\partial^2 q_m(e',e)}{\partial e'^2} - 2k\left(\frac{\partial q_m(e',e)}{\partial e'}\right)^2 \\
\geq&(1-3k)\left(\frac{\partial q_m(e',e)}{\partial e'}\right)^2 \geq 0,
\end{aligned}$$

where the first inequality follows from the fact that $q_m(e',e) \leq 1$; the second inequality follows from (8); and the last inequality follows from $k \leq \frac{1}{3}$. This concludes the proof. ■

Proof of Proposition 2

Proof. Note that the difference between β_m and β_{m+1} is

$$\beta_m - \beta_{m+1} = \frac{1}{N}\left[\frac{(1-k)N}{m(m+1)} \times \left(\sum_{g=1}^{m}\frac{g}{N-g}\right) + 2k\sum_{g=0}^{m}\frac{1}{N-g} - 2k\right],$$

which increases with m. Therefore, $\mathcal{M}(k)$ is the smallest integer $m \in \{1,\dots,N-1\}$ such that $\beta_m - \beta_{m+1} > 0$, i.e.,

$$\mathcal{M}(k) = \min_{\{m\}}\left\{m \,\middle|\, \frac{(1-k)N}{m(m+1)} \times \left(\sum_{g=1}^{m}\frac{g}{N-g}\right) + 2k\sum_{g=0}^{m}\frac{1}{N-g} - 2k > 0, m \in \{1,\dots,N-1\}\right\}. \tag{9}$$

Organizing a single-prize contest is optimal if $\beta_1 - \beta_2 > 0$, which is equivalent to

$$k < \frac{N^2}{(5N-2)(N-2)}.$$

Therefore, awarding a single prize is optimal if $k < \overline{k} \equiv \min\left\{\frac{N^2}{(5N-2)(N-2)}, \frac{1}{3}\right\}$, and awarding multiple uniform prizes is optimal if $k > \overline{k}$. This concludes the proof. ■

Proof of Proposition 3

Proof. For notational convenience, denote $\mathcal{M}(k_1)$ and $\mathcal{M}(k_2)$ by m_1 and m_2, respectively. By (9), we have that

$$m_1 = \min\left\{ m \,\middle|\, \frac{(1-k_1)N}{m(m+1)} \times \left(\sum_{g=1}^{m} \frac{g}{N-g}\right) + 2k_1 \sum_{g=0}^{m} \frac{1}{N-g} - 2k_1 > 0, m \in \{1, \dots, N-1\} \right\},$$

and

$$m_2 = \min\left\{ m \,\middle|\, \frac{(1-k_2)N}{m(m+1)} \times \left(\sum_{g=1}^{m} \frac{g}{N-g}\right) + 2k_2 \sum_{g=0}^{m} \frac{1}{N-g} - 2k_2 > 0, m \in \{1, \dots, N-1\} \right\}.$$

To prove $m_1 \geq m_2$, it suffices to show that

$$\frac{(1-k_2)N}{m_1(m_1+1)} \times \left(\sum_{g=1}^{m_1} \frac{g}{N-g}\right) + 2k_2 \sum_{g=0}^{m_1} \frac{1}{N-g} - 2k_2 > 0.$$

First, consider the case of $m_1 = 1$. We can obtain that

$$\begin{aligned}
&\frac{(1-k_2)N}{m_1(m_1+1)} \times \left(\sum_{g=1}^{m_1} \frac{g}{N-g}\right) + 2k_2 \sum_{g=0}^{m_1} \frac{1}{N-g} - 2k_2 \\
=&\frac{N}{2(N-1)} - k_2 \left[\frac{N}{2(N-1)} + 2 - \frac{2}{N} - \frac{2}{N-1}\right] \\
\geq&\frac{N}{2(N-1)} - k_1 \underbrace{\left[\frac{N}{2(N-1)} + 2 - \frac{2}{N} - \frac{2}{N-1}\right]}_{>0} \\
=&\frac{(1-k_1)N}{m_1(m_1+1)} \times \left(\sum_{g=1}^{m_1} \frac{g}{N-g}\right) + 2k_1 \sum_{g=0}^{m_1} \frac{1}{N-g} - 2k_1 > 0,
\end{aligned}$$

where the first equality follows from $m_1 = 1$; the first inequality follows from the postulated $k_1 > k_2$ and the second inequality follows from the definition of m_1.

Next, consider the case of $m_1 \geq 2$. From the definition of m_1, we have that

$$\frac{(1-k_1)N}{m_1(m_1+1)} \times \left(\sum_{g=1}^{m_1} \frac{g}{N-g}\right) + 2k_1 \sum_{g=0}^{m_1} \frac{1}{N-g} - 2k_1 > 0, \tag{10}$$

and

$$\frac{(1-k_1)N}{m_1(m_1-1)} \times \left(\sum_{g=1}^{m_1-1} \frac{g}{N-g}\right) + 2k_1 \sum_{g=0}^{m_1-1} \frac{1}{N-g} - 2k_1 \leq 0. \tag{11}$$

It follows from (11) that

$$\begin{aligned}
0 \geq & \frac{(1-k_1)N}{m_1(m_1-1)} \times \left(\sum_{g=1}^{m_1-1} \frac{g}{N-g}\right) + 2k_1 \sum_{g=0}^{m_1-1} \frac{1}{N-g} - 2k_1 \\
> & \frac{(1-k_1)N}{m_1(m_1-1)} \times \left(\sum_{g=1}^{m_1-1} \frac{g}{N}\right) + 2k_1 \sum_{g=0}^{m_1-1} \frac{1}{N} - 2k_1 \\
= & 2k_1 \frac{m_1}{N} + \frac{1-k_1}{2} - 2k_1 \geq \frac{2m_1}{3N} - \frac{1}{3},
\end{aligned}$$

which in turn implies that $m_1 < \frac{N}{2}$ and thus $2m_1 + 1 \leq N$. Therefore, we can obtain that

$$\begin{aligned}
\frac{N}{m_1(m_1+1)} \sum_{g=1}^{m_1} \frac{g}{N-g} + 2 - 2\sum_{g=0}^{m_1} \frac{1}{N-g} \geq & \frac{N}{m_1(m_1+1)} \sum_{g=1}^{m_1} \frac{g}{N-g} + 2 - 2 \times \frac{m_1+1}{N-m_1} \\
\geq & \frac{N}{m_1(m_1+1)} \sum_{g=1}^{m_1} \frac{g}{N-g} + 2 - 2 > 0,
\end{aligned} \tag{12}$$

where the last inequality follows from $2m_1 + 1 \leq N$. Moreover, we have that

$$\begin{aligned}
& \frac{(1-k_2)N}{m_1(m_1+1)} \times \left(\sum_{g=1}^{m_1} \frac{g}{N-g}\right) + 2k_2 \sum_{g=0}^{m_1} \frac{1}{N-g} - 2k_2 \\
= & N \times \frac{1}{m_1(m_1+1)} \times \left(\sum_{g=1}^{m_1} \frac{g}{N-g}\right) - k_2 \left(\frac{N}{m_1(m_1+1)} \sum_{g=1}^{m_1} \frac{g}{N-g} + 2 - 2\sum_{g=0}^{m_1} \frac{1}{N-g}\right) \\
\geq & N \times \frac{1}{m_1(m_1+1)} \times \left(\sum_{g=1}^{m_1} \frac{g}{N-g}\right) - k_1 \left(\frac{N}{m_1(m_1+1)} \sum_{g=1}^{m_1} \frac{g}{N-g} + 2 - 2\sum_{g=0}^{m_1} \frac{1}{N-g}\right) \\
= & \frac{(1-k_1)N}{m_1(m_1+1)} \times \left(\sum_{g=1}^{m_1} \frac{g}{N-g}\right) + 2k_1 \sum_{g=0}^{m_1} \frac{1}{N-g} - 2k_1 > 0,
\end{aligned}$$

where the first inequality follows from (12) and the second inequality follows from (10). This concludes the proof. ■