Disclosure policy in Tullock contests with asymmetric stochastic entry

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Abstract. We examine how disclosure policy can be optimally designed to incentivize contestants when their participation is exogenously stochastic. In a generalized Tullock contest setting with two players who are asymmetric in both their values and entry probabilities, we fully characterize the necessary and sufficient conditions under which no disclosure dominates full disclosure. We find that the comparison depends solely on a balance effect exercised by entry probabilities on the expected total effort. The optimal disclosure policy must better balance the competition. These conditions continue to hold when the precision r of Tullock contests is endogenously chosen by the designer.

Résumé. *Politique de divulgation dans des concours à la Tullock quand l'entrée est stochastique et asymétrique.* On examine comment une politique de divulgation ou non du nombre des participants à un concours peut être optimisée pour donner des incitations aux participants quand leur participation est le résultat d'aléas exogènes. Dans le contexte généralisé d'un concours à la Tullock avec deux joueurs qui sont asymétriques tant dans leurs valeurs que dans leurs probabilités d'entrée, on définit les conditions nécessaires et suffisantes pour lesquelles la non divulgation est préférable à la pleine divulgation. On découvre que la comparaison dépend seulement de l'effet de balance exercé par les probabilités d'entrée sur l'effort total anticipé. La politique optimale de divulgation doit balancer au mieux la concurrence. Ces conditions sont maintenues quand la précision r des concours à la Tullock est choisie de manière endogène par le designer du concours.

JEL classification: C72, D72, D82

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1. Introduction

In a contest, players vie for limited prizes by exerting their costly and non-refundable efforts. Many real-world competitive events resemble a contest, ranging from patent races, political campaigns, lobbying and internal labour markets to the competitions for scientific research grants.

The contest literature has conventionally focused on contests with a fixed set of contestants. However, potential contenders can be absent from the competition for various nonstrategic reasons, and their availability is often subject to exogenous randomness. For example, a research lab interested in an innovative project may undergo a sudden budget freeze imposed by higher authorities and be forced to quit the race. An architect who eyes a design competition may be distracted by other opportunities. A firm may have to forego a procurement competition because of the difficulty of scheduling concurrent tasks when meeting fluctuating demands. In recent years, multinationals in Singapore have often failed to receive tender offers from local contractors because of the disruption in labour market caused by unforeseeable government regulations on the employment of foreign workers. In these situations, contenders' ability to participate is subject to exogenous shocks beyond their control. As a result of this randomness, a participant often has to compete against an uncertain set of opponents, with each entering the competition with certain probability.

A number of prominent studies, such as McAfee and McMillan (1987) and Matthews (1987), have investigated such situations in auction games. These studies have described several scenarios in which potential contenders' entry patterns are determined exogenously. For instance, a potential contender may have to bear an opportunity cost for participating in a competition, which is either zero or prohibitively high and follows a given distribution. In this paper, we follow this literature and focus on contests in which contenders enter the contest with fixed probabilities.[1] A model of exogenous random entry can be interpreted alternatively. There is often a mismatch between contenders' characteristics, such as qualifications and expertise, and the specific preference or demand of the contest organizer; the contest organizer thus has to conduct prescreening to shortlist qualified candidates. In this scenario, contenders face an uncertain level of competition even if participation or entry is deterministic, because not all opponents are necessarily qualified or "relevant," which is determined by the contest organizer's requirement, instead of their own willingness to participate.

Random entry poses a challenge to a contest designer who aims at inducing high productive effort from contestants. How should a designer enhance incentives in this environment? One instrument the designer can employ is a policy to either fully disclose or not disclose the actual number of entrants.[2] Both

1 We will discuss the implications of endogenously determined entry probabilities in the conclusion.
2 The literature, including McAfee and McMillan (1987), Matthews (1987), Lim and Matros (2009) and Fu et al. (2011), has focused mainly on the comparison between no and full disclosure of the number of competitors. Other policies are rarely observed in real-life competitions, which might be caused by the difficulty of commitment and implementation in practice.

disclosure policies can be widely observed in practice. For instance, in the famous General Electric CEO succession contest, Jack Welch publicized the shortlist of three candidates. In contrast, the number of participants is typically withheld by the organizer in many open innovation challenges and crowdsourcing contests. The choice of disclosure policy triggers a nontrivial trade-off. Random entry limits effort supply in different manners under alternative disclosure policy. Consider a situation with two potential contestants. With disclosure, a contestant would be allowed to exert zero effort if he turns out to be the only entrant, in which case the prize will be wasted. If a competitor shows up, however, with disclosure the contestant will be forced to exert more effort than he would under no disclosure. This is because, under no disclosure, an entrant places his bids without knowing whether his opponent is present. He must take into account the likely event that his opponent will not enter and he will win automatically; the opponent's possible absence disincentivizes a participant because any outlay is ex post wasteful in that event.

Should the designer disclose or conceal the actual number of contestants when entries are observable to him? Consider, for instance, the usual situation in which the Department of Defense (DoD) posts inducement prizes to solicit innovations from the market. To foster competition, should the DoD keep each participating firm being informed of other contractors' responses to the inducement incentives? Similarly, should a company announce the list of active candidates eligible for promotions? The answer requires a thorough analysis of contestants' behaviour under alternative disclosure policies.

Such considerations motivate several recent studies.[3] Lim and Matros (2009) explore the optimal disclosure policy while adopting a Tullock contest setting. In a Tullock contest, a contestant i wins with a probability $\rho_i = b_i^r / \Sigma_{j=1}^n b_j^r$, where b_j is the effort of contestant j and $r > 0$ indicates the precision of the contest winner-selection mechanism. A contestant's payoff is simply the value of the prize he receives minus his effort. They establish a disclosure-independence principle, which states that the expected effort of the contest is independent of the prevailing disclosure policy. Fu et al. (2011) generalize the Tullock winning probability to $\rho_i = f(b_i)/\Sigma_{j=1}^n f(b_j)$, where the impact function $f(\cdot)$ increases with one's effort and $f(0) = 0$. They define $H(\cdot) = f(\cdot)/f'(\cdot)$ as the characteristic function of the contest and find that the optimal disclosure policy depends on the property of this function: Full disclosure (no disclosure) generates a higher total effort if the characteristic function is strictly concave (convex). As a result, the disclosure-independence principle does not hold in general.

These results, however, are obtained in symmetric settings, in which equally competent contestants enter the contest with the same exogenously given probability. It remains unclear as to what extent these findings remain robust when the competitive environment involves asymmetry. In this paper, we adopt a generalized Tullock contest setting and investigate the optimal disclosure policy while allowing for asymmetry across contestants. Our model involves two potential

3 See also Myerson and Wärneryd (2006) and Münster (2006), among others.

contestants competing for a single prize. They are allowed to not only value the prize asymmetrically but also enter the contest with different exogenous frequencies. The contest designer decides either to reveal the actual number of participants or to conceal the information. Our analysis demonstrates the subtle interaction between the two-dimensional asymmetries and their implications for disclosure policy.

The following observations are highlighted:

1. Disclosure policies matter only when contestants enter with different probabilities. The resultant total effort remains independent of the prevailing disclosure policy when contestants enter with the same probability, even if they value the prize unequally. We then reinstate the disclosure-independence principle in an asymmetric setting.
2. When contestants enter the contest with different probabilities, the disclosure-independence principle does not hold in general, despite the linear characteristic function featured in a Tullock contest.
3. Under asymmetric entry probabilities, the optimum is determined jointly by players' entry probabilities and their valuations of prizes. No disclosure is optimal if and only if the stronger contestant, i.e., the one who values the prize more, enters moderately more often than the weaker one.
4. The optimal disclosure policy in a Tullock contest is independent of the prevailing precision of the contest, i.e., the parameter r of the impact function.

We now briefly interpret these results. Let a contestant i's valuation be v_i and his entry probability be p_i. Under full disclosure, the expected total effort of the contest equals that of an alternative contest, in which two players participate deterministically and have values $(p_1p_2)v_1$ and $(p_1p_2)v_2$, respectively. This is because players' expected efforts in a Tullock contest are homogenous of degree one in their values. We further find that the total effort of the contest under no disclosure equals that in an alternative deterministic-entry contest in which players have values $[p_1p_2(v_1+v_2)]p_2v_1/(p_2v_1+p_1v_2)$ and $[p_1p_2(v_1+v_2)]p_1v_2/(p_2v_1+p_1v_2)$, respectively.[4]

Based on these observations, the impacts of stochastic entry on expected total effort under both disclosure policies can be analyzed by studying the equivalent deterministic-entry contests. The total effort of a deterministic contest with players having values v_1 and v_2 is determined by two factors: (1) the *level* of overall rent available to players, i.e., v_1+v_2, and (2) the *balance* of contest, i.e., the ratio between their values v_1/v_2. Stochastic entry affects the expected total effort through two channels: *level effect* and *balance effect*. The level effect captures how the entry probabilities discount the (absolute) level of rent available in the contest, while the balance effect kicks in when entry probabilities change the (relative) balance between players in the competition, i.e., the ratio of their

4 Note that under the no disclosure policy, a contest with stochastic entry is strategically equivalent to an alternative contest, in which two contestants, with valuations of p_2v_1 and p_1v_2, participate deterministically.

values. Note that in the two equivalent deterministic contests, the sum of players' values amounts to the same level $p_1 p_2(v_1 + v_2)$. The level effect is the same under both disclosure policies. The balance under full disclosure remains to be v_1/v_2, which implies stochastic entry exercises no balance effect. In contrast, the balance under no disclosure is $\max\{p_2 v_1, p_1 v_2\}/\min\{p_2 v_1, p_1 v_2\}$. A balance effect looms large, which could be positive or negative.

As a result, the comparison in total expected efforts across the two disclosure policies is determined solely by the balance effect that comes into play under no disclosure policy. A no disclosure policy may either even the playing field or further upset its balance: For instance, the advantage of a high-valuation contestant can be diminished, if his incentive is discounted excessively by a very low probability of his opponent's entry. The conventional wisdom in the contest literature tells that a more even playing field creates more competitions. When asymmetric entry pattern narrows the margin between contestants caused by their asymmetric valuations, the no disclosure policy would lead to a closer competition, which triggers a positive balance effect and, in turn, incentivizes effort supply.

The no disclosure policy elicits more effort if and only if it triggers a positive balance effect. We find that the balance effect is independent of the contest precision r. The total effort in a deterministic-entry contest is always maximized when players' values are equal given the sum is fixed, i.e., when a perfectly balanced competition takes place.[5] Hence, the no disclosure policy prevails if and only if it results in a more even playing field compared to its benchmark level determined by the initial value structure v_1/v_2. One can simply compare the balance of the contest under no disclosure, i.e., $\max\{p_2 v_1, p_1 v_2\}/\min\{p_2 v_1, p_1 v_2\}$, to its benchmark level v_1/v_2 to determine which policy prevails. The four observations enumerated above can all be explained by the balance effect.

Our results provide a clear guide for the design of competition rules in practice. For instance, one straightforward implication of our analysis is that full disclosure elicits more effort whenever potential contestants have an equal valuation, regardless of their entry probabilities. Hence, when potential participants are relatively homogeneous in terms of their competence, a designer must create a more transparent contest.

Our study distinguishes itself from existing work by allowing for a full-spectrum analysis of Tullock contests that imposes no restriction on the size of the precision r.[6] We find that the optimal disclosure policy is independent of the precision r of a Tullock contest. The power term r depicts how one's additional effort can be translated into increments in his likelihood of winning. Hence, it is conventionally interpreted as a measure of the precision of the winner-selection mechanism, as well as a measure of the intensity of incentives in the contest. The size of r has been widely recognized as an instrument for contest design in recent

5 The details are given in property 1 (iii).

6 Both Lim and Matros (2009) and Fu et al. (2011) focus on Tullock contests with moderate precision. The limited setting renders pure strategy in equilibrium.

literature (see Nti 2004, Gershkov et al. 2009 and Fu et al. 2012). Our equilibrium analysis lays a foundation for the optimal contest design through a proper choice of r. Our result that the optimal disclosure policy is independent of r immediately reveals that the designer's choices of the two structural elements, i.e., disclosure policy and the size of r, do not interact. We fully characterize the optimal parameter r under either disclosure policy. We show that the choice of r is subject to an interesting trade-off when contestants are asymmetric in valuations and/or entry probabilities. A noisier contest, i.e., a contest with a moderate r, may also exercise a positive balance effect despite the ex ante weaker incentive, thereby incentivizing competition.

The rest of the paper proceeds as follows. Section 2 sets up the model and characterizes the equilibria when the actual number of participants is either revealed or concealed. Section 3 carries out the main analysis of the optimal disclosure policy. Section 4 provides an extended analysis of contest design when both precision r and disclosure policy are choice variables. We also carry out some comparative static analysis on the impact of entry probabilities on the total expected effort. Section 5 provides a concluding remark, and the appendix contains technical proofs.

2. Model setup and equilibria

2.1. Model

We consider a simple contest model with random participation. Two potential contestants, indexed by $i = 1, 2$, compete for an indivisible prize. Each contestant i values the prize at v_i and participates in the contest with a fixed probability p_i. Each contestant's entry is independent of the other's. Without loss of generality, we assume that $v_1 \geqslant v_2$, i.e., contestant 1 is ex ante stronger, as he values the prize more than contestant 2. Contestants' valuations and entry probabilities are common knowledge.

Each contestant's entry is unobservable to his rival, but observed by the contest designer. The contest designer decides whether to announce to each participant whether his rival has entered. The designer commits to either of these two disclosure policies at the beginning of the game:

1. Full disclosure (F): The designer announces who has entered the contest.
2. No disclosure (N): The designer does not allow a participant to know whether his rival has entered the contest.

The timing of the game is as follows. First, the designer commits to her disclosure policy, either F or N. Second, contestants learn the policy and enter the contest randomly. Third, the designer implements the committed disclosure policy, and contestants exert their effort outlays b_i to compete for the prize simultaneously. The induced effort cost is also b_i, which is sunk regardless of winning or losing. Contestant i receives a payoff $v_i - b_i$ if he wins.

If neither contestant enters the contest, the designer retains the prize. If only one contestant enters, he receives the prize regardless of his effort. If both enter, the winner is determined in a Tullock contest. A contestant i wins with a probability $\rho_i = b_i^r/(b_1^r + b_2^r)$, with $r > 0$. The parameter r measures the precision of the winner-selection mechanism. The contestant who exerts a higher effort is more likely to win when the size of r increases. When r goes to infinity, the contest converges into an all-pay auction in which a higher bidder always wins. In the event of $b_1 = b_2 = 0$, the prize is randomly given away. Following the mainstream of contest literature (see Konrad 2009), we assume that the contest designer intends to maximize the expected total effort of the contest and strategically chooses her disclosure policy to achieve the goal.

2.2. *A review on equilibria in contests with deterministic entry*

We first review the existing equilibrium results obtained in contest models with deterministic entry. We will utilize them in our analysis of contests with stochastic entry, which substantially improves the expositional efficiency in the remainder of the paper.

Consider a deterministic-entry Tullock contest with a precision level $r > 0$. Two contestants compete for a prize, and their valuations of the prize are given, respectively, by x and y, with $x \geqslant y > 0$. Denote by $b(x, y; r)$ and $\tilde{b}(x, y; r)$ the equilibrium bidding strategies of the two contestants with values x and y, respectively, in this contest. It must be noted that both $b(x, y; r)$ and $\tilde{b}(x, y; r)$ can refer to either pure or mixed-strategy equilibrium strategies.

Let $\bar{r}(\alpha) \in (1, 2]$ be the unique solution to:

$$r = 1 + \alpha^r, \forall \alpha \in (0, 1],$$

where $\bar{r}(\alpha)$ strictly increases with α because:

$$\frac{d\bar{r}(\alpha)}{d\alpha} = \frac{\bar{r}(\alpha)\alpha^{\bar{r}(\alpha)-1}}{1 - \alpha^{\bar{r}(\alpha)} \ln \alpha} > 0.$$

With deterministic entry, the equilibrium depends crucially on the size of r. Nti (1999) shows that a pure-strategy equilibrium exists if and only if r is bounded from above by a cutoff $\bar{r}(y/x) \leqslant 2$ and provides a complete characterization of the equilibrium strategy. The mixed-strategy equilibrium in an all-pay auction, i.e., when $r \to \infty$, has been analyzed extensively in the literature. The equilibrium for $r \in (\bar{r}(y/x), \infty)$, however, remains unknown until very recently. Alcalde and Dahm (2010) analyze the case of $r > 2$. They identify an "all-pay-auction equilibrium" in mixed strategies, although a closed form solution to the equilibrium strategies remains less than explicit. Wang (2010) analyzes the case of $r \in (\bar{r}(y/x), 2]$ and obtains a closed-form solution to the equilibrium strategy.[7]

These results are summarized as follows:

7 The cutoff $\bar{r}(y/x)$ converges to 2 when x approaches y, i.e., when the two players are symmetric. In that case, the particular case analyzed by Wang (2010) vanishes.

(i) If $r \leqslant \bar{r}(y/x)$, we have $b(x,y;r) = rx(y/x)^r/[1+(y/x)^r]^2$, $\tilde{b}(x,y;r) = ry(y/x)^r/[1+(y/x)^r]^2$.

(ii) If $r \in (\bar{r}(y/x), 2]$, we have $b(x,y;r) = (1/r-1)^{1/r}(1-1/r)y$ and:

$$\tilde{b}(x,y;r) = \begin{cases} \left(1-\frac{1}{r}\right)y, & \text{with probability } \frac{y}{x}\left(\frac{1}{r-1}\right)^{\frac{1}{r}}; \\ 0, & \text{with probability } 1-\frac{y}{x}\left(\frac{1}{r-1}\right)^{\frac{1}{r}}. \end{cases}$$

(iii) If $r > 2$, we have $b(x,y;r) = \mu^*$ and:

$$\tilde{b}(x,y;r) = \begin{cases} \mu^*, & \text{with probability } \frac{y}{x}, \\ 0, & \text{with probability } 1-\frac{y}{x}, \end{cases}$$

where μ^* is the (symmetric) equilibrium mixed strategy identified by Baye et al. (1994) in a two-player Tullock contest with $r > 2$, which fully dissipates the rent in a symmetric contest with both contestants having a valuation y. It should be noted that a closed-form expression of the mixed strategy μ^* has yet to be identified in the literature, although Baye et al. verify its existence.

Denote by $B(x,y;r)$ and $\tilde{B}(x,y;r)$ the expected efforts of the two contestants with values x and y, respectively. Based on the equilibrium bidding strategies $b(x,y;r)$ and $\tilde{b}(x,y;r)$, their expected efforts are obtained as follows.

Lemma 1 *In a deterministic-entry Tullock contest with precision $r > 0$ between two contestants whose valuations are $x \geqslant y > 0$, the contestants' expected efforts $B(x,y;r)$ and $\tilde{B}(x,y;r)$ are:*

(i) If $r \leqslant \bar{r}(y/x)$, we have $B(x,y;r) = rx(y/x)^r/[1+(y/x)^r]^2$, $\tilde{B}(x,y;r) = ry(y/x)^r/[1+(y/x)^r]^2$.

(ii) If $r \in (\bar{r}(y/x), 2]$, we have $B(x,y;r) = (1/r-1)^{1/r}(1-1/r)y$, $\tilde{B}(x,y;r) = (1-1/r)(1/r-1)^{1/r}y^2/x$.

(iii) If $r > 2$, we have $B(x,y;r) = y/2$, $\tilde{B}(x,y;r) = y^2/2x$.

In all three cases, $b(x,y;r)$ is a more aggressive strategy than $\tilde{b}(x,y;r)$ as long as $x > y$, i.e., contestant with value x tends to exert more effort because of his higher valuation. Further, it is straightforward to verify the following properties of $B(x,y;r)$ and $\tilde{B}(x,y;r)$.

Property 1 Assume $x \geqslant y > 0$.

(i) Both $B(x,y;r)$ and $\tilde{B}(x,y;r)$ are homogenous of degree one in x and y.

(ii) $yB(x,y;r) = x\tilde{B}(x,y;r)$.

(iii) Given a fixed sum of x and y, the sum of $B(x,y;r)$ and $\tilde{B}(x,y;r)$ decreases with ratio $\frac{x}{y}$ and reaches its maximum when $x = y$.

The second property implies that the rent dissipation rate is the same across the players, i.e., $yB(x,y;r) = x\tilde{B}(x,y;r) \Leftrightarrow B(x,y;r)/x = \tilde{B}(x,y;r)/y$. The third property shows that a more balanced contest tends to elicit more efforts.[8]

2.3. Equilibria with stochastic entry

2.3.1. Full disclosure

Under full disclosure, a participant knows exactly whether the other contestant has entered before exerting effort. He would exert zero effort and receive the prize automatically if his rival were absent. A complete-information two-player contest occurs otherwise, in which both participants simultaneously sink their efforts.

A contestant's equilibrium strategy, when both contestants enter, would depend on his and his rival's valuations of prizes, as well as the precision r of the contest. Denote by $b_{Fi}(v_1, v_2; r)$ a contestant i's equilibrium strategy when both i and j have entered the contest, with $i,j \in \{1,2\}$, $i \neq j$, when a policy of full disclosure (F) is in place. Recall that the equilibrium strategies $b(\cdot,\cdot;\cdot)$ and $\tilde{b}(\cdot,\cdot;\cdot)$ described in section 2.2. We obtain the following:

Lemma 2 *Suppose that a policy of full disclosure* (F) *is adopted. In the complete-information Tullock contest that follows both contestants' entries, the two participants' equilibrium strategies are given by* $b_{F1}(v_1, v_2; r) = b(v_1, v_2; r)$ *and* $b_{F2}(v_1, v_2; r) = \tilde{b}(v_1, v_2; r)$*, respectively.*

Lemma 1 allows us to compute explicitly the ex ante expected total effort of the contest when a policy of full disclosure (F) is adopted. Recall that each contestant i enters the contest with a probability p_i. We obtain the following:

Proposition 1 *Suppose that a policy of full disclosure* (F) *is adopted. The ex ante expected effort of the contest is given by:*

$$
\begin{aligned}
R_F &= p_1p_2[B(v_1,v_2;r) + \tilde{B}(v_1,v_2;r)] = B((p_1p_2)v_1,(p_1p_2)v_2;r) \\
&\quad + \tilde{B}((p_1p_2)v_1,(p_1p_2)v_2;r) \\
&= \begin{cases}
R_{F1} = \dfrac{p_1p_2 r {v_1}^r {v_2}^r (v_1+v_2)}{({v_1}^r + {v_2}^r)^2}, & \textit{if } r \leqslant \bar{r}\left(\dfrac{v_2}{v_1}\right); \\[2ex]
R_{F2} = \dfrac{p_1p_2 v_2 (v_2+v_1)(r-1)^{1-\frac{1}{r}}}{r v_1}, & \textit{if } \bar{r}\left(\dfrac{v_2}{v_1}\right) < r \leqslant 2; \\[2ex]
R_{F3} = \dfrac{p_1p_2 v_2 (v_2+v_1)}{2v_1}, & \textit{if } r > 2.
\end{cases}
\end{aligned}
$$

Under full disclosure, the contest generates positive effort if and only if both players enter. Hence, the expected total effort is simply the effort generated in that event multiplied by the probability that they both enter, i.e., $p_1p_2[B(v_1,v_2;r) + \tilde{B}(v_1,v_2;r)]$. Note that both $B(\cdot,\cdot;r)$ and $\tilde{B}(\cdot,\cdot;r)$ are homogenous of degree one in (v_1, v_2) by property 1(i). Hence, the expected total effort in this case is the same

8 It has been demonstrated by many studies, such as Fu (2006), that a fully balanced contest maximizes total effort.

as that in an alternative deterministic-entry contest in which two players have values $(p_1p_2)v_1$ and $(p_1p_2)v_2$, respectively.

2.3.2. No disclosure

We now investigate the other case, in which the designer commits to the no disclosure policy (N). Under this policy, the designer does not tell either participant whether the other contestant has entered the contest. Hence, a participant i does not know whether he will encounter competition from the other contestant when he exerts his effort. He maintains his prior, i.e., believing that he may meet contestant j with a probability p_j. A contestant must form a strategy that specifies his effort upon his own entry, and the strategy cannot be conditioned on his rival's entry. Let us denote by $b_{Ni}(v_1, v_2, r)$ a contestant i's equilibrium strategy upon his entry. Again, we allow each contestant to randomize over his effort b_i.

Suppose that a contestant i enters the contest. His expected payoff from an effort b_i can be written as follows:

$$\begin{aligned}\pi_{Ni}(b_i) &= \left[(1-p_j)+p_j E_{b_{Nj}(v_1,v_2,r)}\left[\frac{b_i^r}{b_i^r+b_{Nj}(v_1,v_2,r)^r}\right]\right]v_i - b_i \\ &= (1-p_j)v_i + E_{b_{Nj}(v_1,v_2,r)}\left[\frac{b_i^r}{b_i^r+b_{Nj}(v_1,v_2,r)^r}\right](p_j v_i) - b_i.\end{aligned} \tag{1}$$

The expression yields interesting implications. The contestant wins regardless of his effort if the other stays out of the contest, which occurs with a probability $(1-p_j)$. As a result, his effort contributes to his payoff only in the event that the other contestant also enters, which occurs with a probability p_j. The possibility of an automatic win reduces the incentives provided by the reward of v_i. As a result, the expression of $\pi_{Ni}(b_i)$ alludes to a strategic equivalence between the current contest with random entry and an alternative contest with deterministic participation. In the alternative contest, two players compete for a prize, with one to value it at p_2v_1 and the other to value it at p_1v_2. Recall the equilibrium strategies $b(\cdot,\cdot;\cdot)$ and $\tilde{b}(\cdot,\cdot;\cdot)$ described in section 2.2. We obtain the following:

LEMMA 3 *Suppose that the no disclosure policy (N) is adopted. The equilibrium strategies of the entrants are given as follows:*

If $p_2v_1 \geqslant p_1v_2$, *then* $b_{N1}(v_1, v_2; r) = b(p_2v_1, p_1v_2; r)$ *and* $b_{N2}(v_1, v_2; r) = \tilde{b}(p_2v_1, p_1v_2; r)$.

If $p_2v_1 \leqslant p_1v_2$, *then* $b_{N1}(v_1, v_2; r) = \tilde{b}(p_1v_2, p_2v_1; r)$ *and* $b_{N2}(v_1, v_2; r) = b(p_1v_2, p_2v_1; r)$.

It should be noted that contestant 1, who values the prize more, may or may not adopt the more aggressive strategy $b(\cdot,\cdot;\cdot)$. A contestant i, upon his entry, tends to exert more effort if the other enters more often. By lemma 3, he exerts more effort in the contest if and only if $p_jv_i \geqslant p_iv_j$. The logic is straightforward. When p_j increases, contestant i is less likely to take advantage of his rival's accidental absence and has to step up his effort to meet the competition.

Denote by R_N the ex ante expected total effort of this contest. We compute it explicitly based on the result of lemma 3.

PROPOSITION 2 *The ex ante expected total effort of the contest R_N is given as follows. If $p_j v_i \geqslant p_i v_j$, i.e., $p_i/p_j \leqslant v_i/v_j$, where $i \neq j$, and $i,j \in \{1,2\}$, then:*

$$R_N = p_i B(p_j v_i, p_i v_j; r) + p_j \tilde{B}(p_j v_i, p_i v_j; r)$$
$$= \begin{cases} R_{N1} = \dfrac{r p_j^{r+1} v_i^r p_i^{r+1} v_j^r (v_i + v_j)}{(p_j^r v_i^r + p_i^r v_j^r)^j}, & \text{if } r \leqslant \bar{r}\left(\dfrac{p_i v_j}{p_j v_i}\right); \\ R_{N2} = \dfrac{p_i^j v_j (v_j + v_i)(r-1)^{1-\frac{i}{r}}}{r v_i}, & \text{if } \bar{r}\left(\dfrac{p_i v_j}{p_j v_i}\right) \leqslant r \leqslant 2; \\ R_{N3} = \dfrac{p_i^j v_j (v_j + v_i)}{2 v_i}, & \text{if } r > 2. \end{cases}$$

Under no disclosure, the expected total effort is the same as that in an alternative deterministic contest in which two players have values $p_1 v_2$ and $p_2 v_1$, respectively.

3. Contest design: Optimal disclosure policy

In this section, we explore the optimal disclosure policy that maximizes the expected total effort of the contest. Our analysis is based on the equilibrium results obtained in section 2. For the moment, we identify the optimal disclosure policy when the precision level r is exogenously given. In section 4, we consider an extended setting in which the contest designer chooses both her disclosure policy and the precision of the contest mechanism.

For a given r, the optimal disclosure policy can be obtained by simply comparing R_F and R_N, which are given by propositions 1 and 2, respectively. The following lemma provides a useful transformation for R_N based on proposition 2 and properties 1(i) and 1(ii).

LEMMA 4 *Suppose $p_j v_i \geqslant p_i v_j$, where $i \neq j$ and $i,j \in \{1,2\}$, then:*

$$R_N = B\left(\frac{p_1 p_2 (v_1 + v_2) p_j v_i}{p_2 v_1 + p_1 v_2}, \frac{p_1 p_2 (v_1 + v_2) p_i v_j}{p_2 v_1 + p_1 v_2}; r\right) + \tilde{B}\left(\frac{p_1 p_2 (v_1 + v_2) p_j v_i}{p_2 v_1 + p_1 v_2}, \frac{p_1 p_2 (v_1 + v_2) p_i v_j}{p_2 v_1 + p_1 v_2}; r\right).$$

Proof. Consider first the case where $p_2 v_1 \geqslant p_1 v_2$. By proposition 2, we have:

$$\frac{R_N}{p_1 p_2} = \frac{1}{p_2} B(p_2 v_1, p_1 v_2; r) + \frac{1}{p_1} \tilde{B}(p_2 v_1, p_1 v_2; r).$$

Denote $w_1 = (v_1 + v_2) p_2 v_1 / (p_2 v_2 + p_1 v_2)$ and $w_2 = (v_1 + v_2) p_1 v_2 / (p_2 v_2 + p_1 v_2)$. By properties 1(i), we have:

$$
\begin{aligned}
\frac{R_N}{p_1p_2} &= \frac{p_2v_1+p_1v_2}{v_1+v_2}\left[\frac{1}{p_2}B(w_1,w_2;r)+\frac{1}{p_1}\tilde{B}(w_1,w_2;r)\right] \\
&= \left[1+\frac{(p_1-p_2)v_2}{p_2(v_1+v_2)}\right]B(w_1,w_2;r)+\left[1-\frac{(p_1-p_2)v_1}{p_1(v_1+v_2)}\right]\tilde{B}(w_1,w_2;r) \\
&= [B(w_1,w_2;r)+\tilde{B}(w_1,w_2;r)]+\frac{p_1p_2(p_1-p_2)}{p_2v_1+p_1v_2}[p_1v_2B(p_2v_1,p_1v_2;r) \\
&\quad -p_2v_1\tilde{B}(p_2v_1,p_1v_2;r)].
\end{aligned}
$$

By property 1(ii), we have $p_1v_2B(p_2v_1,p_1v_2;r)-p_2v_1\tilde{B}(p_2v_1,p_1v_2;r)=0$. We thus obtain the result. The proof is similar for the case of where $p_2v_1 \leqslant p_1v_2$. ■

The lemma shows that under no disclosure, the contest generates the same amount of expected total effort as an alternative contest in which two contestants enter deterministically and have valuations $[p_1p_2(v_1+v_2)]p_2v_1/(p_2v_1+p_1v_2)$ and $[p_1p_2(v_1+v_2)]p_1v_2/(p_2v_1+p_1v_2)$, respectively. Recall by proposition 1 that the total effort under full disclosure is given by:

$$R_F = B(p_1p_2v_1,p_1p_2v_2;r)+\tilde{B}(p_1p_2v_1,p_1p_2v_2;r),$$

i.e., it equals the expected total effort generated by a deterministic-entry contest with values $(p_1p_2)v_1$ and $(p_1p_2)v_2$. The impacts of stochastic entry on total expected efforts under both disclosure policies can be revealed by studying these two deterministic entry contests.

As mentioned in the introduction, in a deterministic contest between two players with values $x \geqslant y$, the expected total effort is determined by $x+y$ (the level, i.e., the sum of players' values) and x/y (the balance). In our context, the level and balance of contest, without stochastic entry, would be v_1+v_2 and v_1/v_2, respectively. The impact of stochastic entry on expected total effort under either disclosure policy can be decomposed into two components: level effect and balance effect. Recall that comparing the expected total efforts across different disclosure policies is equivalent to comparing those in the two alternative deterministic contests. The sum of players' values is the same: it amounts to $p_1p_2(v_1+v_2)$ in both alternative contests, which implies that stochastic entry exercises the same level effect under the two disclosure policies. Under full disclosure, the balance effect is clearly zero because $(p_1p_2)v_1/(p_1p_2)v_2 = v_1/v_2$. Therefore, the comparison of the total expected efforts across the two disclosure policies is determined solely by the balance effect under no disclosure policy. The balance under no disclosure is $\max\{p_2v_1,p_1v_2\}/\min\{p_2v_1,p_1v_2\}$. According to property 1(iii), the balance effect under no disclosure policy is positive, i.e., creating a more even playing field if and only if $\max\{p_2v_1,p_1v_2\}/\min\{p_2v_1,p_1v_2\} < v_1/v_2$, which is equivalent to $1 < p_1/p_2 < (v_1/v_2)^2$. In particular, the balance effect under no disclosure policy is zero if and only if $p_1/p_2 = 1$ or $p_1/p_2 = (v_1/v_2)^2$. We thus have the following:[9]

9 We thank the editor, Hao Li, for his great insight, which leads to a much simpler proof of the proposition.

Proposition 3

(i) A no disclosure policy elicits strictly more effort than a full disclosure policy, i.e., $R_N > R_F$ if and only if $1 < p_1/p_2 < (v_1/v_2)^2$.
(ii) The two disclosure policies elicit the same amount of effort, i.e., $R_N = R_F$ if and only if $p_1/p_2 = 1$ or $p_1/p_2 = (v_1/v_2)^2$.
(iii) A full disclosure policy elicits strictly more effort than a no disclosure policy, i.e., $R_F > R_N$ if and only if either $p_1/p_2 < 1$ or $p_1/p_2 > (v_1/v_2)^2$.

Proof. Let $t = p_2v_1/(p_2v_1 + p_1v_2)$. Suppose first $t > 1/2$, i.e., $p_2v_1 > p_1v_2$. By lemma 4 and property 1(i), the difference $R_N - R_F$ has the same sign as $(v_1 + v_2)(B(t, 1-t; r) + \tilde{B}(t, 1-t; r)) - (B(v_1, v_2; r) + \tilde{B}(v_1, v_2; r))$. Consider the effect on $R_N - R_F$ when t increases. $B(t, 1-t; r) + \tilde{B}(t, 1-t; r)$ decreases in t by property 1(iii). When $t = v_1/(v_1 + v_2)$, or equivalently, when $p_1/p_2 = 1$, clearly we have $R_N = R_F$ due to property 1(i). So $R_N < R_F$ if $t > v_1/(v_1 + v_2)$ and $R_N > R_F$ if t is between $1/2$ and $v_1/(v_1 + v_2)$.

Now, suppose $t < 1/2$. Then $R_N - R_F$ has the same sign as $(v_1 + v_2)(B(1-t, t; r) + \tilde{B}(1-t, t; r)) - (B(v_1, v_2; r) + \tilde{B}(v_1, v_2; r))$ by lemma 4 and property 1(i). In this case, $B(1-t, t; r) + \tilde{B}(1-t, t; r)$ is increasing in t by property 1(iii). When $t = v_2/(v_1 + v_2)$, or equivalently, when $p_1/p_2 = (v_1/v_2)^2$, clearly we have $R_N = R_F$. So $R_N < R_F$ if $t < v_2/(v_1 + v_2)$ and $R_N > R_F$ if t is between $v_2/(v_1 + v_2)$ and $1/2$. Combining the two cases yields the proposition. ■

Proposition 3 is the main result of this paper. Under full disclosure, the balance of the contest is measured by the ratio v_1/v_2. In contrast, under no disclosure, it boils down to $\max\{p_2v_1, p_1v_2\}/\min\{p_2v_1, p_1v_2\}$: the balance is tilted by players' entry probabilities. Since the level effects are identical across the two policies, a policy outperforms the other if and only if it creates a more balanced playing field.

Suppose $v_1 > v_2$. Let us begin with $p_1 = p_2$. Then, the contest under no disclosure is equally balanced compared to the contest under full disclosure. We must have $R_N = R_F$. The prevailing disclosure policy thus does not affect ex ante effort supply. The disclosure-independence principle continues to hold even if contestants value their prizes differently, which is emphasized in the following corollary.

Corollary 1 *With identical entry probability, full disclosure and no disclosure elicit the same amount of effort.*

Now let p_1 increase. The balance under full disclosure is unchanged. However, the balance under no disclosure is tilted toward contestant 2 and narrows the margin between the two contestants. Contestant 2 is better incentivized, which further urges contestant 1 to increase his effort. By property 1(iii), the expected total effort of the contest would increase until $p_1v_2 = p_2v_1$, i.e., $p_1/p_2 = v_1/v_2$, which perfectly levels the playing field. When p_1 continues to increase, the balance is further tilted toward contestant 2, and contestant 1 becomes the one who exerts less effort. The margin between contestants is widened as p_1 increases. The contest under no disclosure remains more balanced than the contest under full disclosure until p_1/p_2 reaches $(v_1/v_2)^2$. A positive balance effect can thus be exercised under no disclosure whenever $1<p_1/p_2<(v_1/v_2)^2$, because it results in a more even playing field. The designer thus prefers the no disclosure policy in order to leverage this positive effect, whenever $1<p_1/p_2<(v_1/v_2)^2$. In the knife edge case of $p_1/p_2=(v_1/v_2)^2 \Leftrightarrow p_2v_1/p_1v_2=v_2/v_1$, the initial balance is restored, which again mutes the balance effect and equates the two policies. In contrast, when p_1/p_2 falls below 1 or when it exceeds $(v_1/v_2)^2$, a more imbalanced competition results under no disclosure, which disincentivizes contestants and reduces effort. The designer thus prefers a full disclosure policy in order to avoid the backfire of the balance effect under no disclosure.

Furthermore, let us consider the case in which the two contestants have the same valuation of the prize, i.e., $v_1=v_2=v$.

Corollary 2 *With identical valuation, full disclosure elicits strictly more effort than no disclosure unless* $p_1=p_2$.

When $p_1=p_2$, the perfectly symmetric contest, which is studied by Lim and Matros (2009) and Fu et al. (2011), is restored, in which the disclosure-independence principle holds. However, full disclosure is always optimal whenever their entry probabilities are different: The no disclosure policy upsets the balance of the otherwise perfectly even battle. It must adversely affect its efficiency and is never optimal.

It is not difficult to understand why the comparison is independent of the precision r. As revealed by our discussion, whether to conceal entry information depends solely on the balance of the competition. That is, the optimum depends on the comparison between $\max\{p_2v_1,p_1v_2\}/\min\{p_2v_1,p_1v_2\}$ and v_1/v_2, which is independent of the size of r.

4. Extensions and discussions

In this section, we discuss several extensions. First, we allow the designer to commit to not only the disclosure policy of the contest but also the precision of the winner-selection mechanism. Second, we investigate how the variation in contestants' entry probabilities affects effort supply.

4.1. Optimal contest with endogenous precision

As revealed by proposition 3, the comparison between the two disclosure policies is independent of the precision r. We need only to find out the optimal precision r^* associated with the dominant disclosure policy. As previously mentioned, the precision of a contest can largely be manipulated by the designer and has long been viewed as an important instrument for contest design. For instance, the designer can modify the judging criteria of the contest to accommodate various objectives, e.g., moderating the weights assigned to subjective components and objective components in contenders' overall ratings. She may also vary the ratio of expert judges to non-expert judges in evaluation committees. These practices magnify the noise involved in the winner-selection mechanism.

A larger r implies that one's win likelihood depends more on his superior effort and less on other noisy factors. Hence, a larger r tends to incentivize effort supply. An additional trade-off, however, would arise when contestants are asymmetric. A smaller r diminishes the advantage possessed by the stronger contestant. This effect favors the weaker contestant, thereby balancing the contest and creating more competition.[10]

4.1.1. Optimal precision under a given disclosure policy

We now proceed to pin down the optimal precision r. For this purpose, we characterize the optimal r under each disclosure policy. As we will demonstrate, the expected total effort would not monotonically increase with the precision level r unless the contest is even. A more discriminatory contest (with a greater r) may reduce effort supply when contestants are not equally motivated, i.e., either when they unequally value the prize ($v_1 \neq v_2$) under full disclosure or when they face different "discounted incentives" ($p_2 v_1 \neq p_1 v_2$) under no disclosure. As mentioned above, this also triggers a balance effect, which varies contestants' efforts.

Optimal precision under full disclosure Proposition 1 provides a formula for the expected total effort of the contest when policy F is in place. Define $\alpha = v_2/v_1 \leqslant 1$. We take first-order derivative with respect to r and obtain:

$$\frac{dR_F}{dr} = \begin{cases} \dfrac{dR_{F1}}{dr} = \dfrac{\alpha^r p_2 p_1 (v_1 + v_2)}{(1+\alpha^r)^3}(1 + \alpha^r + rln\alpha - r\alpha^r ln\alpha) & \text{if } r \leqslant \bar{r}\left(\dfrac{v_2}{v_1}\right); \\ \dfrac{dR_{F2}}{dr} = \dfrac{p_2 p_1 v_2 (v_2 + v_1)}{v_1} \dfrac{\frac{1}{r}(r-1)^{1-\frac{1}{r}} ln(r-1)}{r^2} & \text{if } \bar{r}\left(\dfrac{v_2}{v_1}\right) < r \leqslant 2; \\ \dfrac{dR_{F3}}{dr} = 0 & \text{if } r > 2. \end{cases}$$

With symmetric contestants, i.e., $v_1 = v_2$, $\ln\alpha = 0$, and the case of $\bar{r}(v_2/v_1) < r \leqslant 2$ vanishes because $\bar{r}(v_2/v_1) = 2$. We then see without surprise that R_F strictly increases with r until it reaches 2 and remains flat thereafter. Hence, expected total effort can be maximized by any $r \geqslant \bar{r}(v_2/v_1)$. However, when contestants are asymmetric, a unique $r_F^* < \bar{r}(v_2/v_1) = 2$ exists, such that R_F increases until it reaches r_F^* and then decreases.

Let $z_0 \in (0,1)$ be the unique solution to $f(z) = 1 + z + lnz - zlnz = 0$.[11] Let $\hat{\alpha} = z_0^{1/1+z_0}$and $\hat{r}(\alpha) = \ln z_0 / \ln\alpha, \forall \alpha \in (0,1)$. We obtain the following:

10 This rationale has been elaborated on by Che and Gale (1997, 2000), Fang (2002), Nti (2004), Amegashie (2009) and Wang (2010). We explore this effect in a context that involves stochastic participation.

11 The details are given in the proof of proposition 4.

PROPOSITION 4 *Suppose that a full disclosure policy* (F) *is implemented.*

(i) When contestants equally value the prize, i.e., $v_1 = v_2$*, the expected total effort of the contest can be maximized by any* $r \geqslant 2$*.*
(ii) When contestants have unequal valuations of the prize, i.e., $v_1 \neq v_2$*, there exists a unique optimal (effort-maximizing) precision level* r_F^**, with:*

$$r_F^* = \begin{cases} \bar{r}\left(\dfrac{v_2}{v_1}\right) & \textit{if } \dfrac{v_2}{v_1} \in [\hat{\alpha}, 1); \\ \hat{r}\left(\dfrac{v_2}{v_1}\right) < \bar{r}\left(\dfrac{v_2}{v_1}\right) & \textit{if } \dfrac{v_2}{v_1} \in (0, \hat{\alpha}). \end{cases}$$

Note that both $\bar{r}$ and $\hat{r}$ are increasing functions, and they take the same value at $v_2/v_1 = \hat{\alpha}$. Proposition 4 confirms the logic of the balance effect. When the contest is less even, i.e., a lower ratio of v_2/v_1 is in place, the optimum tends to require a noisier winner-selection mechanism, i.e., a smaller r. Recall that $\bar{r}(v_2/v_1)$ is the unique solution to the equation of $r = 1 + (v_2/v_1)^r$. When v_2/v_1 decreases, $\bar{r}(v_2/v_1)$ decreases in response. Further, when the ratio falls below the cutoff $\hat{\alpha}$, the optimum requires that $\bar{r}$ fall further below $\bar{r}(v_2/v_1)$.

OPTIMAL PRECISION UNDER NO DISCLOSURE Consider next the case of the no disclosure policy. We use the formula for the expected total effort R_N stated in proposition 2. Recall the discussion in section 2.2. The balance of the contest under no disclosure depends on not only contestants' valuations of the prize but also their entry probabilities. The following proposition depicts the optimum. We use the same cutoff $\hat{\alpha}$ and function $\hat{r}(\cdot)$ as in definition 3. The proof is similar to that of proposition 4 and thus is omitted.

PROPOSITION 5 *Suppose that the no disclosure policy* (N) *is implemented.*

(i) If the contest is even, i.e., $p_2 v_1 = p_1 v_2$*, the expected total effort of the contest* R_N *can be maximized by any* $r \geqslant 2$*.*
(ii) If the contest is uneven, i.e., $p_j v_i > p_i v_j$*,* $\forall i, j \in \{1, 2\}$ *and* $i \neq j$*, there is a unique optimal precision level* r_N^* *that maximizes* R_N*, with:*

$$r_N^* = \begin{cases} \bar{r}\left(\dfrac{p_i v_j}{p_j v_i}\right) & \textit{if } \dfrac{p_i v_j}{p_j v_i} \in [\hat{\alpha}, 1); \\ \hat{r}\left(\dfrac{p_i v_j}{p_j v_i}\right) < \bar{r}\left(\dfrac{p_i v_j}{p_j v_i}\right) & \textit{if } \dfrac{p_i v_j}{p_j v_i} \in (0, \hat{\alpha}). \end{cases}$$

The result mimics that of proposition 4, as they reflect the same underlying logic. A more uneven contest requires a noisier winner-selection mechanism that could balance the playing field to better incentivize contestants.

4.1.2. Optimal contest

Aggregating the results of propositions 3, 4 and 5, we characterize the designer optimal bundle (d^*, r^*)—i.e., the combination of disclosure policy and precision level—as follows.

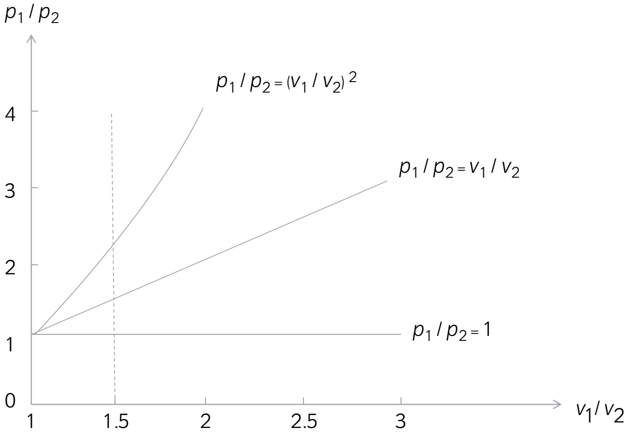

FIGURE 1 Different ranges of p_1/p_2 for given v_1/v_2

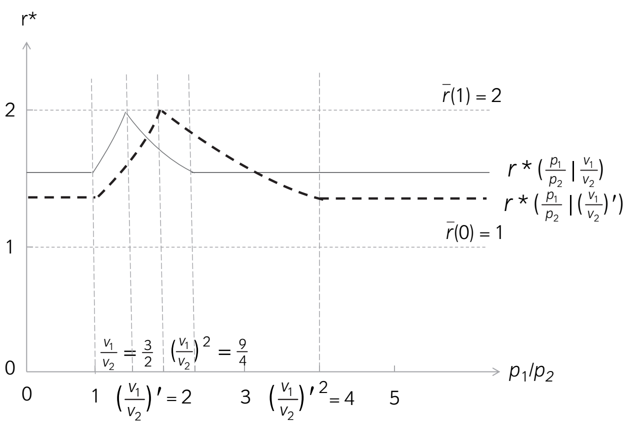

FIGURE 2 Optimal precision r^* as a function of p_1/p_2 for given v_1/v_2

Corollary 3 *(i) The bundle (F, r_F^*) is optimal if either $p_1/p_2 < 1$ or $p_1/p_2 > (v_1/v_2)^2$. (ii) The bundle (N, r_N^*) is optimal if $p_1/p_2 \in (1, (v_1/v_2)^2)$. (iii) Both (F, r_F^*) and (N, r_N^*) are optimal if and only if either $p_1/p_2 = 1$ or $p_1/p_2 = (v_1/v_2)^2$.*

We use two figures to illustrate the observations highlighted by Corollary 3.

In figure 1, we illustrate how the designer's optimal bundle would vary when p_1/p_2 varies for given v_1/v_2. Figure 2 demonstrates the optimal level of precision as a function of p_1/p_2 for given v_1/v_2. Without loss of generality, we fix v_1/v_2 to 3/2 in figure 1, which is represented by the vertical dotted line.

Full disclosure is optimal for all $p_1/p_2 < 1$, and the associated optimal bundle is a constant. At $p_1/p_2 = 1$, full disclosure and no disclosure are both optimal, and the optimal bundle also involves the same level of precision. As p_1/p_2 increases just above 1, no disclosure becomes uniquely optimal, and the corresponding optimal precision increases. This continues until p_1/p_2 reaches the line representing $p_1/p_2 = v_1/v_2$, where any r greater than or equal to 2 is optimal. As p_1/p_2 continues to increase, no disclosure remains optimal but the optimal precision decreases. This continues until p_1/p_2 reaches the curve representing $p_1/p_2 = (v_1/v_2)^2$. At this point, disclosure policy, again, becomes irrelevant, as in the case of $p_1/p_2 = 1$, and the optimal precision is the same under either disclosure policy. As p_1/p_2 increases further, full disclosure is uniquely optimal, and the optimal precision stays constant (and equal to the constant when $p_1/p_2 < 1$).

In figure 2, the solid curve depicts the optimal precision for $v_1/v_2 = 3/2$ when p_1/p_2 increases. As discussed above, the optimal precision remains constant for $p_1/p_2 < 1$ and $p_1/p_2 > (v_1/v_2)^2$, while it increases first and then decreases when p_1/p_2 is between the two cutoffs, peaking at $p_1/p_2 = \frac{v_1}{v_2}$. In the figure, we further illustrate the optimal precision for a greater $v_1/v_2 (= 2)$ for comparison by the broken curve marked by "$r^*(p_1/p_2)$ for $v_1/v_2 = 2$." The same nonmonotonic pattern is observed. Because of the larger $v_1/v_2 (= 2)$, the curve representing the optimal precision is shifted down for $p_1/p_2 < 1$, shifts down and to the right for the part between the cutoffs 1 and $(v_1/v_2)^2$ and remains below the original curve thereafter. A closer inspection on these curves leads to an interesting observation: whenever it's optimal to hide information about entry, i.e., when p_1/p_2 is between the cutoffs 1 and $(v_1/v_2)^2$, it is also optimal to raise the level of precision.

4.2. Impact of entry probabilities on expected total effort

As shown above, contestants' entry probabilities discount the incentives provided by prizes under the no disclosure policy, which, paradoxically, varies the balance of strength between contestants and could intensify the competition. Our analysis has assumed fixed entry probabilities. One intriguing extension is to allow the designer to manipulate potential contestants' entry probabilities.[12] In this part, we further discuss the role played by contestants' entry probabilities in inducing effort, which further sheds light on the balance effect under no disclosure.

Consider hypothetically that the designer manages to decrease the probability of one contestant's entering the contest. With full disclosure, it is obvious that the total effort should increase accordingly. Two competing effects, however, loom large with no disclosure. On the one hand, a negative level effect arises because: (1) a contestant's lower entry probability further diminishes the incentive provided by the prize to her opponent as the absence of competition becomes more likely and (2) the contestant herself is less likely to contribute her own effort. On the other hand, a positive balance effect could arise, as long as the change in entry probability leads to a more even playing field, as shown previously. Our analysis leads to the following:

PROPOSITION 6 *A decrease in p_i, contestant i's entry probability, strictly decreases the expected total effort under no disclosure.*

12 We thank the editor and one anonymous referee for pointing out these interesting questions.

Proposition 6 shows that the negative level effect always outweighs the positive balance effect, if any. A unilateral decrease in one contestant's entry probability always leads the expected total effort to fall. This observation inspires a more interesting question. We now conduct the following thought experiment that manipulates both contestants' entry probabilities simultaneously while keeping p_1p_2 constant. In this case, any such change will not affect the expected total effort under full disclosure but does affect the expected total effort under no disclosure through the balance effect. We thus have the following proposition.

PROPOSITION 7 *For any $v_1/v_2 > 0$, an increase in p_1 and a decrease in p_2 such that p_1p_2 is kept unchanged will increase the expected total effort under no disclosure if and only if $p_1/p_2 < v_1/v_2$, i.e., $p_2v_1 > p_1v_2$.*

Proposition 7 further highlights the balance effect under no disclosure identified in this paper. When $p_2v_1 \geqslant p_1v_2$, increasing p_1 and decreasing p_2 while keeping p_1p_2 fixed make the competition more balanced without changing the sum of values. As a result, the positive balance effect would enhance the expected total effort under no disclosure.

Proposition 7, together with proposition 3, alludes to a possible approach to enhancing effort supply: manipulate contestants' entry probabilities and adopt a no disclosure policy to take advantage of the balance effect. Without loss of generality, assume $v_1 > v_2$. The designer should increase p_1 and decrease p_2 while keeping p_1p_2 fixed when $p_1/p_2 \in (1, v_1/v_2)$ and at the same time switch from full disclosure to no disclosure. When $p_1/p_2 \in (v_1/v_2, (v_1/v_2)^2)$, the designer should try the opposite changes in entry probabilities and at the same time switch from full disclosure to no disclosure.

5. Concluding remarks

This paper complements the contest literature on stochastic entry by investigating a setting with asymmetric contestants. Contestants are allowed to be asymmetric in two dimensions, i.e., their valuations of the prize and their entry probabilities. The optimal disclosure policy depends on both contestants' valuations and their entry probabilities but is independent of the size of precision r.

To the best of our knowledge, the present study is the first effort in the literature to allow for asymmetries in contests with stochastic entry and to identify the roles played by these asymmetries. Lim and Matros (2009) and Fu et al. (2011) establish the disclosure-independence principle in Tullock contests with symmetric contestants and symmetric entry probabilities. In a more general setting in which no restriction is imposed on the size of r, we demonstrate that the equivalence result continues to hold even if contestants have different valuations of the prize as long as they enter with the same probability. However, the equivalence does not hold, in general, when they enter with different probabilities.

Our study leaves room for further extensions. The analysis of optimal disclosure policy in contests has so far been limited to complete-information settings. One natural extension is to allow contestants' valuations or entry costs to be privately known. McAfee and McMillan (1987) establish that in a private-value auction with risk-neutral contestants and symmetric entry, a seller's revenue does not depend on whether the actual number of buyers is revealed. A similar result is expected to hold in private-value all-pay auctions. It remains unknown, however, whether the result would hold if the entry pattern is asymmetric.

Another possible extension is to allow contestants to enter the contest endogenously. The prevailing disclosure policy affects not only contestants' incentives to exert effort but also their entry patterns. The optimal policy deserves serious research effort. The technical complexity, due to the discontinuous nature of imperfectly discriminatory contests, prevents us from carrying out the analysis in the current study. In addition, multiple equilibria could emerge under either policy and cause additional complications since revenue ranking depends crucially on equilibrium selection. One may resort to laboratory experiments to explore which equilibrium arises in real life. We will attempt this in the future. Finally, we focus on two-contestant case. Generalizations in multi-player contests are technically challenging. The existing technique in the contest literature loses its bite in characterizing equilibrium under no disclosure.

Appendix A: Proof of proposition 1

The effort function is a direct application of Alcade and Dahm (2010) and Wang (2010). The total effort of the contest, in case (i), i.e., $r \leqslant \bar{r}(v_2/v_1)$, is given by:

$$\begin{aligned} R_{F1} &= p_1 p_2 [B(v_1, v_2; r) + \tilde{B}(v_1, v_2; r)] \\ &= p_1 p_2 \left[\frac{r {v_1}^{r+1} {v_2}^{r}}{({v_1}^r + {v_2}^r)^2} + \frac{r {v_1}^{r} {v_2}^{r+1}}{({v_1}^r + {v_2}^r)^2} \right] \\ &= \frac{p_1 p_2 r {v_1}^r {v_2}^r (v_1 + v_2)}{({v_1}^r + {v_2}^r)^2}. \end{aligned} \tag{A1}$$

In case (ii), i.e., $\bar{r}(v_2/v_1) < r \leqslant 2$, since contestant 2 is adopting a mixed strategy, expectation should be taken on his strategy:

$$\begin{aligned} R_{F2} &= p_1 p_2 [B(v_1, v_2; r) + \tilde{B}(v_1, v_2; r)] \\ &= p_1 p_2 \left[\left(\frac{1}{r-1} \right)^{\frac{1}{r}} \left(1 - \frac{1}{r} \right) v_2 + \left(1 - \frac{1}{r} \right) v_2 \frac{v_2}{v_1} \left(\frac{1}{r-1} \right)^{\frac{1}{r}} \right] \\ &= \frac{p_1 p_2 v_2 (v_2 + v_1)(r-1)^{1-\frac{1}{r}}}{r v_1}. \end{aligned} \tag{A2}$$

In case (iii), i.e., $r > 2$, both players are adopting mixed strategy, expectation should be taken on their strategies. Although we do not have a closed form solution for the strategy μ^*, we know it will fully dissipate the rent in the symmetric game when both have valuation v_2. In such a symmetric game, players win the prize v_2 with the same probability. As a result, the expected effort from μ^* must be the same as the expected gain of winning, which is equal to $v_2/2$. Note that contestant 1 adopts μ^* for sure and contestant 2 adopts μ^* with probability v_2/v_1. Therefore, we can calculate the expected total effort as:

$$\begin{aligned} R_{F3} &= p_1 p_2[B(v_1, v_2; r) + \tilde{B}(v_1, v_2; r)] \\ &= p_1 p_2 \left[\frac{v_2}{2} + \frac{v_2}{v_1} \frac{v_2}{2} \right] \\ &= \frac{p_1 p_2 v_2 (v_2 + v_1)}{2 v_1}. \end{aligned} \tag{A3}$$

■

Appendix B: Proof of lemma 3 and proposition 2

The effort function is a direct application of Alcade and Dahm (2010) and Wang (2010). Note that in this case a contestant contributes effort as long as he enters, regardless of whether the other contestant. First consider the situation of $p_2 v_1 \geqslant p_1 v_2$.

In case (i):

$$\begin{aligned} R_{N1} &= p_1 B(p_2 v_1, p_1 v_2; r) + p_2 \tilde{B}(p_2 v_1, p_2 v_2; r) \\ &= p_1 \frac{r (p_2 v_1)^{r+1} (p_1 v_2)^r}{((p_2 v_1)^r + (p_1 v_2)^r)^2} + p_2 \frac{r (p_2 v_1)^r (p_1 v_2)^{r+1}}{((p_2 v_1)^r + (p_1 v_2)^r)^2} \\ &= \frac{r p_2^{r+1} v_1^r p_1^{r+1} v_2^r (v_1 + v_2)}{(p_2^r v_1^r + p_1^r v_2^r)^2}. \end{aligned} \tag{A4}$$

In case (ii), since contestant 2 is adopting a mixed strategy, expectation should be taken on his strategy:

$$\begin{aligned} R_{N2} &= p_1 B(p_2 v_1, p_2 v_2; r) + p_2 \tilde{B}(p_2 v_1, p_1 v_2; r) \\ &= p_1 \left(\frac{1}{r-1} \right)^{\frac{1}{r}} \left(1 - \frac{1}{r} \right) p_1 v_2 + p_2 \left[\left(1 - \frac{1}{r} \right) p_1 v_2 \frac{p_1 v_2}{p_2 v_1} \left(\frac{1}{r-1} \right)^{\frac{1}{r}} \right] \\ &= \frac{p_1^2 v_2 (v_1 + v_2)(r-1)^{1-\frac{1}{r}}}{r v_1}. \end{aligned} \tag{A5}$$

In case (iii), both players are adopting mixed strategy, expectation should be taken on their strategies. Although we do not have a closed form solution for the strategy μ^*, we know it will fully dissipate the rent in the symmetric game when both have valuation $p_1 v_2$. In such a symmetric game, players wins the prize $p_1 v_2$ with the same probability. As a result, the expected effort from μ^* must be the same as the expected gain of winning, which is equal to $p_1 v_2/2$. Note that contestant 1 adopts μ^* for sure and contestant 2 adopts μ^* with probability p_1v_2/p_2v_1. The expected effort from μ^* must be $p_1v_2/2$. Therefore, we can calculate the expected total effort as:

$$\begin{aligned} R_{N3} &= p_1 B(p_2v_1, p_1v_2; r) + p_2 \tilde{B}(p_2v_1, p_1v_2; r) \\ &= p_1 \frac{p_1v_2}{2} + p_2 \frac{p_1v_2}{p_2v_1}\frac{p_1v_2}{2} \\ &= \frac{p_1^2 v_2(v_1+v_2)}{2v_1}. \end{aligned} \tag{A6}$$

The situation of $p_2v_1 \leqslant p_1v_2$ can be calculated similarly. ■

Appendix C: Proof of proposition 4

Let $z=\alpha^r \in (0,1)$, where $r>0$ and $\alpha = v_2/v_1 \in (0,1)$. Note that $1+\alpha^r + rln\alpha - r\alpha^r ln\alpha$ can be written as $f(z)=1+z+lnz-zlnz$. In addition, also note that $f'(z)=1/z-\ln z>0$, thus $f(z)$ increases with z. Note $f(1)=2$ and $\lim_{z\to 0^+} f(z) = -\infty$. There exists a unique z_0 such that $f(z_0)=0$. Fix $\alpha = v_2/v_1 \in (0,1)$. If $z=\alpha^r \leqslant z_0$, $dR_{F1}/dr \leqslant 0$; if $z=\alpha^r \geqslant z_0$, $dR_{F1}/dr \geqslant 0$. This means that when $r\to 0^+$, i.e., $z\to 1^-$, $dR_{F1}/dr \geqslant 0, \forall \alpha<1$.

We now identify a threshold $\hat{\alpha}$ such that $\alpha^r \geqslant z_0, \forall r \in (0, \bar{r}(\alpha)], \forall \alpha \in [\hat{\alpha}, 1]$. Since α^r increases with α and decreases with r, $\hat{\alpha}$ must satisfy $\hat{\alpha}^{\bar{r}(\hat{\alpha})} = z_0$. Note that $\bar{r}(\hat{\alpha}) = 1+\hat{\alpha}^{\bar{r}(\hat{\alpha})}$; we thus have $\bar{r}(\hat{\alpha}) = 1+z_0$. Thus, $\hat{\alpha} = z_0^{1/\bar{r}(\hat{\alpha})}$.

For $\alpha = v_2/v_1 > \hat{\alpha}$, we then must have $dR_{F1}/dr \geqslant 0$ for $r \in (0, \bar{r}(\alpha)]$.

For $\alpha = v_2/v_1 < \hat{\alpha}$, then denote the solution to $\alpha^r = z_0$ as $\hat{r}(\alpha)$. We have $\hat{r}(\alpha)$ increases with α and $\hat{r}(\hat{\alpha}) = \bar{r}(\hat{\alpha})$. Note that $\hat{r}(\alpha) < \bar{r}(\alpha) < 2$ for $\alpha = v_2/v_1 < \hat{\alpha}$. This is clear for the following arguments. Suppose $\hat{r}(\alpha) > \bar{r}(\alpha)$. Then we have $\alpha^{\bar{r}(\alpha)} > \alpha^{\hat{r}(\alpha)} = z_0$, which leads to $\bar{r}(\alpha) = 1+\alpha^{\bar{r}(\alpha)} > 1+z_0 = \bar{r}(\hat{\alpha})$. However, this conflicts with the fact that $\bar{r}(\cdot)$ is increasing. Thus, R_{F1} first increases in r up to $\hat{r}(v_2/v_1)$ and then decreases until $\bar{r}(\frac{v_2}{v_1})$ when $\frac{v_2}{v_1} < \hat{\alpha}$.

Since $dR_{F2}/dr < 0$ and $dR_{F3}/dr = 0$ and the profit function is continuous in r, the proposition thus follows. ■

Appendix D: Proof of proposition 6

For the case of full disclosure, it is obvious that the total effort is increasing in contestants' entry probabilities according to proposition 1. For the no disclosure case, we will also show that the total effort is increasing in contestants' entry probabilities. Without loss of generality, let us focus on the effect of p_1. According to proposition 2, since other parameter ranges are obvious and the total effort is a continuous function of p_1, we need only to verify that $k(p_1) = rp_2^{r+1} v_1^r p_1^{r+1}(v_1+v_2)/(p_2^r v_1^r + p_1^r v_2^r)^2$ is increasing in p_1 under two cases:

Case 1: $p_2v_1 \geqslant p_1v_2$ and $r \leqslant \bar{r}(p_1v_2/p_2v_1)$;
Case 2: $p_2v_1 \leqslant p_1v_2$ and $r \leqslant \bar{r}(p_2v_1/p_1v_2)$.

$$\begin{aligned}k'(p_1)=&\frac{r(r+1)p_2^{r+1}v_1^r p_1^r(v_1+v_2)(p_2^r v_1^r+p_1^r v_2^r)-2rp_2^{r+1}v_1^r p_1^{r+1}(v_1+v_2)rp_1^{r-1}v_2^r}{(p_2^r v_1^r+p_1^r v_2^r)^3}\\=&rp_2^{r+1}v_1^r p_1^r(v_1+v_2)\frac{(r+1)(p_2^r v_1^r+p_1^r v_2^r)-2p_1 rp_1^{r-1}v_2^r}{(p_2^r v_1^r+p_1^r v_2^r)^3}\\=&rp_2^{r+1}v_1^r p_1^r(v_1+v_2)p_2^r v_1^r\frac{(r+1)+(1-r)\left(\frac{p_1v_2}{p_2v_1}\right)^r}{(p_2^r v_1^r+p_1^r v_2^r)^3}.\end{aligned}$$

Therefore, we need only to prove that $m(p_1)=(r+1)+(1-r)(p_1v_2/p_2v_1)^r$ is always positive.

For case 1, if $0\leqslant r\leqslant 1$, the $m(p_1)$ is obviously positive. If $1\leqslant r\leqslant \bar{r}(p_1v_2/p_2v_1)$:

$$m(p_1)\geqslant(r+1)+(1-r)=2\geqslant 0.$$

For case 2, remember that $\bar{r}(p_2v_1/p_1v_2)$ is determined by $\bar{r}(p_2v_1/p_1v_2)=1+(p_2v_1/p_1v_2)^{\bar{r}(p_2v_1/p_1v_2)}$. If $0\leqslant r\leqslant 1$, then obviously $m(p_1)\geqslant 0$. If $1\leqslant r\leqslant\bar{r}(p_2v_1/p_1v_2)$, then:

$$\begin{aligned}m(p_1)\geqslant&(r+1)+(1-r)\left(\frac{p_1v_2}{p_2v_1}\right)^{\bar{r}\left(\frac{p_2v_1}{p_1v_2}\right)}\\=&(r+1)+\frac{1-r}{\bar{r}\left(\frac{p_2v_1}{p_1v_2}\right)-1}\\\geqslant&\left(\bar{r}\left(\frac{p_2v_1}{p_1v_2}\right)+1\right)+\frac{1-\bar{r}\left(\frac{p_2v_1}{p_1v_2}\right)}{\bar{r}\left(\frac{p_2v_1}{p_1v_2}\right)-1}\\=&\bar{r}\left(\frac{p_2v_1}{p_1v_2}\right)\geqslant 0.\end{aligned}\tag{A7}$$

The second inequality follows from the fact that (A7) is decreasing in r. ■

D1. Proof of proposition 7

Let $t=p_2v_1/(p_2v_1+p_1v_2)$. If $p_1/p_2<v_1/v_2$, i.e., $p_2v_1>p_1v_2$, we have $t>1/2$. By lemma 4 and property 1(i), $R_N=p_1p_2\ (v_1+v_2)[B(t,1-t;r)+\tilde{B}(t,1-t;r)]$. $B(t,1-t;r)+\tilde{B}(t,1-t;r)$ decreases in t by property 1(iii). We thus have an increase in p_1 and a decrease in p_2 such that p_1p_2 is kept unchanged (i.e., a decrease in t) would increase R_N.

If $p_1/p_2\geqslant v_1/v_2$, i.e., $p_2v_1\leqslant p_1v_2$, we have $t\leqslant 1/2$. By lemma 4 and property 1(i), $R_N=p_1p_2(v_1+v_2)(B(1-t,t;r)+\tilde{B}(1-t,t;r))$. In this case, $B(1-t,t;r)+\tilde{B}(1-t,t;r)$ is increasing in $t(\leqslant 1/2)$ by property 1(iii). Thus, an increase in p_1 and a decrease in p_2 such that p_1p_2 is kept unchanged (i.e., a decrease in t) would decrease R_N. ■

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