CONTEST DESIGN AND OPTIMAL ENDOGENOUS ENTRY

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This paper derives the effort-maximizing contest rule and the optimal endogenous entry in a context where potential participants bear fixed entry costs. The organizer is allowed to design the contest under a fixed budget with two strategic instruments: the value of the prize purse and a monetary transfer (entry subsidy/fee) to each participating contestant. The results show that the optimally designed contest attracts exactly two participating contestants in its unique subgame perfect equilibrium and extracts all the surplus from participating contestants. The direction (subsidy or fee) and amount of the monetary transfer depend on the magnitude of the entry cost. (*JEL* C7, D7)

I. INTRODUCTION

A contest is a situation in which economic agents expend costly and nonrefundable resources in order to win a limited number of prizes. Numerous academic surveys and anecdotal accounts have shown that a wide variety of competitive activities can be viewed as winner-take-all contests.[1] It has been widely recognized in the literature that contestants' incentive to exert effort and the resultant rent dissipation depend largely on the competitive environment as defined by the rules of the contest. Therefore, a forward-looking organizer must set the rules of a contest strategically such that the contest structure best serves his/her interests.

While a contest organizer may have diverse objectives, enormous academic resources have been devoted to the design of contests that maximize the effort that has to be exerted by contestants (Baye, Kovenock, and de Vries 1993; Gradstein and Konrad 1999; Rosen 1986) This paper follows in the same vein and investigates the design of the effort-maximizing contest. The model that is proposed in this paper pertains to the induction of maximal total effort from a fixed pool of potential contestants, with the contest organizer being financially bound by a fixed budget. However, this paper departs from the existing contest literature in two main aspects.

First, it is assumed in this paper that to participate in the contest, a contestant must bear a fixed entry cost in addition to the cost of autonomous (productive) efforts that determine the probability of them winning the prize. The traditional modeling approach assumes that all invested resources contribute to contestants' productive efforts and help increase their likelihood of success. In reality, however, contestants often bear additional costs merely to participate, which do not relate directly to winning. To provide an analogy of this point, while an air ticket paves the way for American tennis star Venus Williams to arrive at the courts of the Australian Open, it does not contribute to her winning the championship. Similarly, a research company may have established the necessary laboratory equipment and developed the project-specific

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1. Explanatory examples include research tournaments, political lobbying, sports races, and promotion tournaments in a firm's internal labor market.

ABBREVIATIONS

DoD: U.S. Department of Defense
R&D: Research and Development

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knowledge required to participate in an innovation tournament, but its success depends largely on its subsequent efforts and the value of its creative input.

The above examples indicate that a potential contestant would join a contest if and only if the expected payoffs from participation are higher than the entry costs. Thus, unlike the typical setting where there is generally a given pool of active contestants, in the context of this paper, the number of participating contestants is endogenously determined by the contest structure.[2,3]

Second, we allow the effort-maximizing contest organizer to design the competition with two strategic instruments: *the value of the (unique) winner's purse* and *a direct monetary transfer to each participating contestant*. The monetary transfer may be an entry subsidy aimed at mitigating contestants' entry costs.[4] By way of contrast, when the transfer moves into the opposite direction, it turns into an entry fee. This phenomenon is widely observed in many real-life tournament settings.[5] The contest organizer has to allocate his/her limited resources between the prize purse and the monetary transfer. This key feature makes our model differ from the typical setting in existing literature that assumes a given prize purse.

Conventional wisdom suggests that in an imperfectly discriminatory contest (a) a larger number of contestants lead to greater total effort and (b) a more generous winner's purse causes each contestant to exert more effort. These insights, however, lead to a paradoxical situation where no clear implications can be provided to the contest organizer with a limited budget. An entry subsidy encourages more participation on the one hand while absorbing funds that would otherwise be used to award the winner on the other. It is then called into question whether an entry subsidy is a desirable way to promote the effort outlay. Despite the fact that entry fees discourage participation, the revenue earned nevertheless enriches the winner's purse and promotes competition among participating contestants. Hence, the direction and amount of the optimal monetary transfer have yet to be identified, and the desirable number of participants in the contest remains foggy.

We construct a three-stage model to explore the properties of the optimally designed contest. Consider a fixed pool of identical potential contestants who may choose to compete for a unique prize. In the first stage of the contest, the organizer, who is subject to a fixed budget Γ_0, announces the value of the winner's purse V, as well as the amount of money to be transferred S, to each participating contestant. In the second stage, the potential contestants are informed about the rules of the contest as indicated by the contest organizer's strategy pair (V, S).[6] They then make their entry decisions and incur a fixed participation cost $C > 0$ if they enter the contest. In the third stage, all participants choose their effort outlays simultaneously, and a unique winner is found through a stochastic selection procedure.

The main findings of this analysis are summarized as follows:

1. "*It takes (exactly) two to tango*": the optimally designed contest induces exactly two contestants to participate regardless of the direction or the amount of the monetary transfer in equilibrium.
2. "*Full 'net rent' dissipation*": the optimally designed contest, regardless of the direction or the amount of monetary transfer, fully dissipates the "net rent" available in the contest.

This paper is connected to a few strands of economic literature on contests and tournaments. First, it is inspired by and linked closely to the seminal works of Baye, Kovenock, and de Vries (1993), Fullerton and McAfee (1999), and Che and Gale (2003). These papers establish the optimality of two participants in a wide

2. An imperfectly discriminatory contest with concave contest technology does not involve endogenous entry if no fixed cost is incurred upon entry. The interior equilibrium would guarantee that all participating contestants would receive positive expected payoffs.

3. Exceptions are explored in the seminal works of Baye, Kovenock, and de Vries (1993) and Fullerton and McAfee (1999).

4. For instance, the U.S. Department of Defense (DoD) substantially subsidizes military research and development (R&D) activities conducted by contractors competing for procurement contracts. The DoD's subsidies for independent military R&D projects have been empirically documented by Lichtenberg (1988).

5. One such example is the National Scholastic Surfing Association National Tournament, where an entry fee applies to participating teams and individuals.

6. A participating contestant receives an entry subsidy if $S > 0$ but pays an entry fee if $S < 0$. The contest (V, S) needs to be *feasible* in the sense that the prize V cannot be greater than the total resources available to the contest organizer (including the revenue collected from the entry fees).

variety of settings when the contest organizer attempts to narrow down the set of participants. Our analysis departs from these papers in two main aspects. First, unlike Baye, Kovenock, and de Vries (1993) and Fullerton and McAfee (1999) who assume a given prize, we allow the contest organizer to flexibly allocate his/her resources between the prize purse and a strategic monetary transfer (entry fee/subsidy). Second, instead of directly controlling the number of participants (such as Che and Gale 2003), we allow the contest organizer to induce desired voluntary entry by offering the optimal bundle of prize purse and the monetary transfer. Thus, our paper is also related to the literature on optimal prize allocation. Both Moldovanu and Sela (2001) and Siegel (2007) consider how the contest organizer maximizes overall effort by splitting a fixed budget into a number of prizes. In addition, our paper is linked to Taylor (1995), who shows that an entry fee could benefit the contest organizer.

This section of the paper has introduced the topic of contest design. Section II sets up the model, while in Section III, the formal analysis is presented and the results are briefly discussed. Concluding remarks are presented in Section IV.

II. PRELIMINARIES

This section considers the design of a winner-take-all contest within a three-stage framework with endogenous entry.

The contest organizer begins with a fixed budget of Γ_0 with which to fund a contest. A fixed pool of M ($\geq$3) identical risk-neutral potential contestants demonstrate interest in the contest. In the first stage, the organizer announces the rules and commits to a prize purse V ($\geq$0) and a direct monetary transfer $S \in \Re$ to each participating contestant. For the ease of notation, a contest will be denoted by (V, S), which also represents the contest organizer's strategy.[7,8] In the second stage, contestants decide whether or not to participate. It is assumed that they enter the contest sequentially and that they are fully aware of the number of current participants.[9] Each contestant incurs a fixed participation cost of $C > 0$ upon entry but is either rewarded with an entry subsidy S when $S > 0$ or is charged an entry fee $|S|$ when $S < 0$. In the third stage, all contestants simultaneously submit their effort entries.

In the event that there are no participants, the organizer simply keeps the prize. The set of contestants is denoted by Ω_N when N ($\geq$1) of the M ($\geq$3) potential contestants participate in the contest (V, S). In the event that there is only one contestant, this contestant automatically receives the prize V regardless of the amount of effort exerted.

When there are at least two participants in a contest, the probability that a contestant $i \in \Omega_N$ wins the unique prize is

$$p_i(e_i, \boldsymbol{e}_{-i}; \Omega_N) = \frac{f(e_i)}{f(e_i) + \sum_{j \in \Omega_N, j \neq i} f(e_j)}, \quad (1)$$

[10]

where e_i is i's effort and $\boldsymbol{e}_{-i}$ denotes the effort vector of the other participating contestants.[11] The impact function $f(\cdot)$ represents the technology of the contestants. To guarantee the existence of a unique symmetric pure-strategy equilibrium, $f(\cdot)$ is assumed to be strictly increasing and weakly concave, with $f(0) = 0$ and $f'(0) > 0$. We define $H(\cdot) \equiv \frac{f(\cdot)}{f'(\cdot)}$. The inverse function of $H(\cdot)$ is denoted by $H^{-1}(\cdot)$. Due to the concavity of $f(\cdot)$, $H^{-1}(\cdot)$ must be strictly increasing. In addition, $\frac{dH^{-1}(x)}{dx} \in (0, 1)$. If all the participating contestants exert zero effort, it is assumed that the prize will be given away at random.

Assume that the cost of effort equals the effort itself.[12] A potential contestant expects to receive a payoff of

7. The contest rule (V, S) can also be as follows. The organizer commits to a transfer S to every entrant, and the prize will be $V = \Gamma_0 + NS$, where N is the actual number of entrants. The entry and effort equilibrium remain the same as contest ($\Gamma_0 + NS$, S). According to Lemma 5, this setup leads to the same optimal contest. We thank an anonymous referee for pointing this out.

8. We assume that the contest organizer's resource has no alternative use other than inducing higher effort from contestants.

9. Sequential entry and complete information ensure that potential contestants play pure strategies (0 or 1 probability of entry) in the entry stage of the game.

10. This model, together with a ratio form success function, can be applied to a wide variety of contest settings. For instance, Baye and Hoppe (2003) establish strategic equivalence between Tullock rent-seeking contests and research tournaments, as well as patent races.

11. We assume that a nonparticipant will not be awarded the prize.

12. This is always the case when effort is measured by the expenditure of the contestants.

$$\pi_i(e_i, \boldsymbol{e}_{-i}; \Omega_N, V, S) = p_i(e_i, \boldsymbol{e}_{-i}; \Omega_N)V - e_i + S - C, \tag{2}$$

if he participates and exerts effort e_i, provided that the efforts of the other participating contestants are $\boldsymbol{e}_{-i}$. Every participating contestant will choose the level of effort to maximize his expected payoff.

Since all N ($\geq$1) participating contestants are identical, every individual contestant in symmetric equilibrium has the equilibrium probability $\frac{1}{N}$ of receiving the prize and receives an equilibrium payoff of $\pi(N, V, S) = \frac{1}{N}V - e(N, V, S) + S - C$, where $e(N, V, S)$ denotes the equilibrium effort as a function of N, V, and S. The results below indicate the participants' equilibrium individual effort and equilibrium payoff, which can be established through standard techniques.[13]

LEMMA 1. *In the unique symmetric Nash equilibrium of a contest* (V, S) *with N participating contestants, where* $N \geq 1$, *each contestant exerts an effort of*

$$e(N, V, S) = \begin{cases} 0 \text{ if } N = 1, \\ H^{-1}\left(\frac{V}{N}\left(1 - \frac{1}{N}\right)\right) \text{ if } N \geq 2 \end{cases}, \tag{3}$$

and each contestant receives an expected payoff of

$$\pi(N, V, S) = \begin{cases} V + S - C \text{ if } N = 1, \\ \frac{1}{N}V - H^{-1}\left(\frac{V}{N}\left(1 - \frac{1}{N}\right)\right) + S - C \text{ if } N \geq 2 \end{cases}. \tag{4}$$

13. Note that concavity of impact function $f(\cdot)$ ensures the existence of a unique symmetric interior equilibrium effort. Since $H^{-1}(0) = 0$ and $\frac{dh^{-1}(x)}{dx} \in (0, 1)$, we have $H^{-1}(x) \leq x$ when $x \geq 0$. This leads to that $\frac{1}{N}V - H^{-1}(\frac{V}{N}(1 - \frac{1}{N})) > \frac{1}{N}V - (\frac{V}{N}(1 - \frac{1}{N})) = \frac{V}{N^2} > 0$. This means that given that N contestants have participated, it is optimal for them to make the effort $H^{-1}(\frac{V}{N}(1 - \frac{1}{N}))$.

Based on Lemma 1, the equilibrium number of entrants in contest (V, S) is characterized in the following lemma.

LEMMA 2. *A contest* (V, S) *attracts a unique number of* $N(V,S) = \arg \max_{\{\pi(N,V,S) \geq 0, 1 \leq N \leq M\}} \{N\}$ *contestants to participate if* $\pi(1, V, S) \geq 0$, *since* $\pi(N, V, S)$ *strictly decreases with* N ($\geq$1). *If* $\pi(1, V, S) < 0$, *the contest attracts a unique number of* $N(V, S) = 0$ *contestants.*

Proof. Clearly, $\pi(1, V, S) > \pi(2, V, S)$. To show that $\pi(N, V, S)$ strictly decreases with N for any $N \geq 2$, it is sufficient to show that function $g(x) = Vx - H^{-1}(x(1 - x)V)$ is increasing over the interval (0, 1/2]. Note that $\frac{dg(x)}{dx} = V - \frac{dH^{-1}(y)}{dy}|_{y = H^{-1}(x(1-x)V)} (1 - 2x) V \geq 0$ as $\frac{dH^{-1}(y)}{dy} \in (0, 1)$. Since N contestants enter the contest (V, S) if and only if $\pi(N, V, S) \geq 0$, $N(V, S)$ is the unique equilibrium number of entrants in contest (V, S) if $\pi(1, V, S) \geq 0$. It is then obvious that no one participates in the contest if $\pi(1, V, S) < 0$. Q.E.D. ■

The contest organizer has a total budget of Γ_0 available from his own pocket. He has the freedom either to split the budget between the prize purse and the payment of entry subsidies up to the budget limit or to fund the prize purse using the revenue from the entry fees. Such flexibility in resource allocation represents one of the main features of the analysis in this paper.

DEFINITION 1. A contest design (V, S) is *feasible* if and only if

$$0 \leq V \leq \Gamma_0 - N(V, S)S. \tag{5}$$

The feasibility condition (5) states that the prize purse cannot exceed the total resources available to the contest organizer. The total effort induced by contest (V, S) is $E \equiv N(V, S) \cdot e(N(V, S), V, S)$, where $N(V, S)$ is the equilibrium number of participants who enter the contest (V, S). In this paper, we assume that the contest organizer searches for the optimal feasible contest (V^*, S^*) that maximizes the total effort E exerted by the endogenously determined number of participating contestants.

III. ANALYSIS

The following is assumed to make the analysis more interesting.

ASSUMPTION 1. $C \leq \frac{\Gamma_0}{2}$.

The prize is assumed to be automatically awarded if there is only one contestant. This is why a contestant would exert zero effort if he turns out to be the unique participant. Therefore, a contest rule cannot create an optimal situation if less than two contestants participate. Assumption 1 guarantees that the contest organizer can induce the entry of at least two participants by providing an entry subsidy, as shown by the following lemma.

LEMMA 3. *A* feasible *contest that induces at least two contestants to participate exists if and only if* Assumption 1 *holds.*

Proof. Sufficiency: Let S_0 denote the solution of

$$(6)\quad \pi(2, \Gamma_0 - 2S, S) = \frac{\Gamma_0}{2} - H^{-1}\left(\frac{\Gamma_0 - 2S}{4}\right) - C = 0.$$

First, note that $\frac{\Gamma_0}{2} - H^{-1}\left(\frac{\Gamma_0 - 2S}{4}\right) - C$ increases with S. Second, when $S = \frac{\Gamma_0}{2}$, $\pi(2, \Gamma_0 - 2S, S) = \frac{\Gamma_0}{2} - C \geq 0$ based on Assumption 1. Third, when $S = \frac{\Gamma_0}{2} - 2H\left(\frac{\Gamma_0}{2}\right)$, $\pi(2, \Gamma_0 - 2S, S) = -C < 0$. Thus, there exists a unique solution $S_0 \in \left(\frac{\Gamma_0}{2} - 2H\left(\frac{\Gamma_0}{2}\right), \frac{\Gamma_0}{2}\right]$ for Equation (6).

Set $S = S_0$ and $V = \Gamma_0 - 2S_0 \geq 0$. Since $\pi(2, \Gamma_0 - 2S_0, S_0) = 0$, we have $N(\Gamma_0 - 2S_0, S_0) = 2$ by Lemma 2, which indicates that two contestants will participate in the contest $(\Gamma_0 - 2S_0, S_0)$. In addition, contest $(\Gamma_0 - 2S_0, S_0)$ is feasible according to Definition 1. It has thus been shown that Assumption 1 represents a sufficient condition for the existence of a feasible contest that induces at least two contestants to participate.

Necessity: We prove it by contradiction. Suppose that there exists a feasible contest (V, S) that induces N (≥ 2) contestants when $C > \frac{\Gamma_0}{2}$. By Lemma 1, a contestant would receive an expected payoff

$$\pi(N, V, S) = \frac{V}{N} - H^{-1}\left(\frac{V}{N}\left(1 - \frac{1}{N}\right)\right) + S - C = \frac{V + SN}{N} - C - H^{-1}\left(\frac{V}{N}\left(1 - \frac{1}{N}\right)\right),$$

which implies $\pi(N, V, S) \leq \frac{\Gamma_0}{N} - C - H^{-1} \times \left(\frac{V}{N}\left(1 - \frac{1}{N}\right)\right)$ by the fact $V + SN \leq \Gamma_0$. Thus, no participant could ex ante break even if $\frac{\Gamma_0}{2} - C < 0$. Thus, a feasible contest with at least two contestants could occur only if Assumption 1 holds. Q.E.D. ■

Clearly, when C is small, more than two potential contestants can be induced to participate. The contest organizer therefore has more freedom in terms of the (desirable) number of participants he can attract. Lemmas 4 and 5 characterize two intuitive necessary conditions for the optimal feasible contest. The formal proofs are laid out in the Appendix.

LEMMA 4. *In the optimal feasible contest* (V^*, S^*)*, every participating contestant breaks even, that is,* $\pi(N(V^*, S^*), V^*, S^*) = 0$.

LEMMA 5. *In the optimal feasible contest* (V^*, S^*)*, the contest organizer must put all the resources available in the prize purse, that is,* $V^* = \Gamma_0 - N(V^*, S^*)S^*$.

From Lemma 4, it follows that the optimal contest causes all participating contestants to break even. By Lemma 5, it is optimal for the contest organizer, who anticipates contestants' entry activity in response to the announced contest rule, to exhaust all the resources available to the organizer. As a result, the effort-maximizing contest (V^*, S^*) must satisfy $V^* = \Gamma_0 - N(V^*, S^*)S^*$ and $\pi(N(V^*, S^*), V^*, S^*) = 0$.

A. Main Results

Next, the optimal number of entrants $N(V^*, S^*)$ needs to be revealed. In the optimal contest (V^*, S^*), each contestant receives an equilibrium payoff of

$$\pi(N(V^*,S^*),V^*,S^*) = \frac{\Gamma_0 - N(V^*,S^*)S^*}{N(V^*,S^*)} - e(N(V^*,S^*),V^*,S) + S^* - C.$$

Thus,

$$\begin{aligned} N(V^*,S^*)\pi(N(V^*,S^*),V^*,S^*) &= (\Gamma_0 - N(V^*,S^*)S^*) \\ &\quad - N(V^*,S^*)e(N(V^*,S^*),V^*,S^*) \\ &\quad + N(V^*,S^*)S^* - N(V^*,S^*)C, \end{aligned}$$

which leads to

$$\begin{aligned} E &= N(V^*,S^*)e(N(V^*,S^*),V^*,S^*) \\ &= \Gamma_0 - N(V^*,S^*)\pi(N(V^*,S^*),V^*,S^*) \\ &\quad - N(V^*,S^*)C. \end{aligned}$$

Combining it with Lemma 4, we can establish the following important fact:

$$E = \Gamma_0 - N(V^*,S^*)C. \tag{7}$$

Note the importance of Equation (7). It states that in the optimally designed contest, the equilibrium total effort is given by the difference between the total budget of the contest organizer and the total entry costs incurred by participating contestants. In addition, the right-hand side of Equation (7) strictly decreases with $N(V^*, S^*)$, the equilibrium number of participating contestants, for any $N(V^*, S^*) \geq 2$. Hence, it can be deduced that the equilibrium efforts are bound from above by $\bar{E} = \Gamma_0 - 2C$. The following result is now ready to be established.

THEOREM 1. *The unique optimal contest induces exactly two potential contestants to participate and induces the total effort of* $\bar{E} = \Gamma_0 - 2C$.

Proof. Equation (7) shows that only a contest that attracts two contestants to participate may induce the total effort of $\bar{E}$.

Thus, it is only necessary to show that a feasible contest (V^*, S^*) exists that induces exactly two participants and satisfies the conditions given by Lemmas 4 and 5. To this end, it is necessary only to show that there exists an S^* that satisfies the following condition:

$$\begin{aligned} \pi(2,\Gamma_0 - 2S^*,S^*) &= \frac{\Gamma_0}{2} - H^{-1}\left(\frac{\Gamma_0 - 2S^*}{4}\right) - C \\ &= \left(\frac{\Gamma_0}{2} - C\right) - H^{-1}\left(\frac{\Gamma_0 - 2S^*}{4}\right) = 0. \end{aligned} \tag{8}$$

The existence and uniqueness of such an S^* have been established in the proof of Lemma 3. Q.E.D. ■

Theorem 1 shows that a unique optimal contest exists that maximizes the amount of total effort exerted in the contest. The optimal contest attracts exactly two contestants and induces the maximal total effort, which fully dissipates the net rent available in the contest, that is, $\Gamma_0 - 2C$. The following theorem further characterizes the properties of (V^*, S^*).

THEOREM 2. *The optimally designed contest awards a unique equilibrium prize purse of* $V^* = 4H\left(\frac{\Gamma_0}{2} - C\right)(>0)$. *When* $C \leq \frac{\Gamma_0}{2} - H^{-1}\left(\frac{\Gamma_0}{4}\right)$, *the contest organizer charges an entry fee of* $S^* = \left[\frac{\Gamma_0}{2} - 2H\left(\frac{\Gamma_0}{2} - C\right)\right](\leq 0)$ *to each contestant. When* $\frac{\Gamma_0}{2} - H^{-1}\left(\frac{\Gamma_0}{4}\right) < C \leq \frac{\Gamma_0}{2}$, *the contest organizer awards an entry subsidy of* $S^* = \left[\frac{\Gamma_0}{2} - 2H\left(\frac{\Gamma_0}{2} - C\right)\right](\geq 0)$ *to each contestant.*

Proof. Equation (8) implies $S^* = \frac{\Gamma_0}{2} - 2H\left(\frac{\Gamma_0}{2} - C\right)$. This leads to $V^* = \Gamma_0 - 2S^* = 4H\left(\frac{\Gamma_0}{2} - C\right)$. Thus, $S^* \geq 0$ if and only if $\frac{\Gamma_0}{2} - 2H\left(\frac{\Gamma_0}{2} - C\right) \geq 0$. Q.E.D. ■

Theorems 1 and 2 characterize the unique subgame perfect equilibrium. It is worth pointing out that the critical value $\frac{\Gamma_0}{2} - H^{-1}\left(\frac{\Gamma_0}{4}\right)$ represents an individual contestant's equilibrium surplus $\pi(2, \Gamma_0, 0)$ in a feasible contest $(V, S) = (\Gamma_0, 0)$ with two participating contestants. At least two contestants are willing to participate in the contest (Γ_0, 0) when $C \leq \pi(2, \Gamma_0, 0)$. An entry fee can then be imposed to enhance the prize purse while maintaining sufficient participation (two contestants). On the other hand, when $C > \pi(2, \Gamma_0, 0)$, only one contestant is willing to participate in $(\Gamma_0, 0)$. Thus, an entry subsidy is required in this situation in order to maintain sufficient participation.

It has been assumed so far that the contest organizer attempts solely to maximize the amount of total effort exerted. However, a contest organizer may seek other objectives as well, such as maximizing the effort exerted by each individual (symmetric) contestant. For example, the organizer of a design competition would be more concerned about the quality of the potential supplier who secures the contract rather than the overall amount of effort exerted by the entire pool of competitors. It turns out that the optimal contest that has been derived above serves this objective as well. Equation (7) implies that the individual effort of a participating contestant is bound above by $\bar{e} = \frac{\Gamma_0}{2} - C$, which can be achieved if and only if the contest is organized as defined by Theorem 2.

B. Discussion

As revealed in Theorem 2, one key feature of the optimally designed contest in our analysis lies in the transfer between the contest organizer and participating contestants as a strategic instrument. This transfer plays a key role in extracting the surplus of the participating contestants. When a fixed entry cost exists for the contestants, the optimal transfer S^* leads to a unique optimally designed contest and further pins down the optimal number of participating contestants at "2."

The fixed entry cost C in our previous analysis is essential for determining the optimal contest structure. As Equation (7) implies, the main results discussed in the earlier sections of this paper stem from the existence of a positive entry cost, while the optimal contest rule depends largely on the size of the fixed cost. Theorem 2 states that an entry subsidy is desirable in order to invite participation and to maintain a sufficient level of competition if and only if the entry cost is prohibitively high. Thus, the result applies directly to the design competition for military procurement: R&D projects with a military purpose would arguably require substantial initial setup investment, which could play a large part in deterring the entry of independent contractors notwithstanding the generous potential rewards. Thus, a subsidy would be an effective way to maintain the optimal amount of competition.

An entry subsidy would not be justifiable (from the viewpoint of the contest organizer) if the level of the fixed entry cost falls below the threshold value $\frac{\Gamma_0}{2} - H^{-1}\left(\frac{\Gamma_0}{4}\right)$. The contest organizer would instead charge an optimal entry fee to restrict the level of participation to the unique optimum of two participants and attach the inflow of cash to the winner's purse. For example, although a civilian R&D project may involve a fixed setup cost, the investment may not be completely sunk because it could most likely be used for alternative purposes. Consequently, an entry fee that restricts entry may successfully enhance the quality of competition for the design of a civilian product through enhanced prize value.[14]

C. Extension with No Entry Cost

Although it has explicitly been assumed that $C > 0$, the analysis up to Equation (7) applies to the limiting case where $C = 0$, in which participation involves no sunk costs. As Equation (7) implies, when $C = 0$ the entry of additional participants does not reduce the maximum amount of effort that can be possibly induced in an optimally designed contest. Hence, the optimal contest structure would not be unique, and the optimal number of participating contestants would not necessarily be 2. The contest organizer can allow any number of contestants (but no less than 2) to participate and simply charge each of them an appropriate entry fee to extract all the expected surplus they would enjoy from the contest. Thus, the optimal monetary transfer (entry fee) S^* satisfies

$$(9)\quad \frac{1}{N}V^* - H^{-1}\left(\frac{V^*}{N}\left(1 - \frac{1}{N}\right)\right) + S^* = 0,$$
$$\forall N \in \{2, \ldots, M\}.$$

14. Nalebuff and Stiglitz (1983) allow losing contestants to receive negative prizes, and they show that negative prizes could help further extract contestants' surplus. The equilibrium entry fee plays a similar role. However, losers' tribute does not accrue to the prize purse in the setting of Nalebuff and Stiglitz (1983), while the revenue from entry fee in our model is fully dedicated to winner's purse in the unique subgame perfect equilibrium.

Since the contest organizer at the optimum directs all revenue toward the prize purse, Equation (9) is equivalent to

$$(10)\quad \frac{\Gamma_0}{N} = H^{-1}\left(\frac{(\Gamma_0 - NS^*)}{N}\left(1-\frac{1}{N}\right)\right),\quad N \in \{2,\ldots,M\}.$$

THEOREM 3. *When a pool of $M \geq 2$ potential contestants are up for a contest and each of them bears zero entry costs, the optimal contest can take a variety of forms ($V^*(N)$, $S^*(N)$), $\forall N \in \{2, \ldots, M\}$. In an optimal contest ($V^*(N)$, $S^*(N)$), the contest organizer charges an entry fee of*

$$(11)\quad S^*(N) = \frac{\Gamma_0}{N} - \frac{N}{N-1}H\left(\frac{\Gamma_0}{N}\right) < 0$$

and awards a prize of

$$(12)\quad V^*(N) = \frac{N^2}{N-1}H\left(\frac{\Gamma_0}{N}\right) > 0.$$

In the contest ($V^*(N)$, $S^*(N)$), exactly N contestants participate and each of them enjoys zero surplus. All these contests induce the same total amount of effort, Γ_0, which fully dissipates the total rent.

Theorem 3 defines a wide variety of optimal contest structures that differ in terms of their entry fees, prize purse, and the equilibrium level of participation. When contestants bear negligible entry costs, the contest organizer has complete flexibility to design the contest. Optimally designed contests may attract any feasible level of participation, and yet they all yield an equivalent outcome where the entire budget, Γ_0, is fully dissipated. Thus, our analysis does not lose its bite in those settings where a "more-than-two" participation rate could be considered optimal as well.

D. Nonlinear Cost Function

An additional line of complication would arise if the cost function of each contestant is strictly convex in his/her effort e. Denote by $\zeta(e)$ the cost a contestant has to bear when she/he exerts an effort e. In a contest that exhausts the budget and extracts all surplus from N participants, we must have

$$(13)\quad N\zeta(e_N) = \Gamma_0 - NC.^{15}$$

The overall effort is then given by

$$(14)\quad E_N = Ne_N = N\zeta^{-1}\left(\frac{\Gamma_0}{N} - C\right).$$

It remains obscure how the total effort varies when the number of participants increases. As $\zeta^{-1}(\cdot)$ is strictly concave in its argument, $N\zeta^{-1}\left(\frac{\Gamma_0}{N} - C\right)$ may not decrease when N increases. The equilibrium could depend on the functional form of $\zeta^{-1}(\cdot)$, the amount of the entry cost C and the size of budget Γ_0. An extension on convex cost function would be interesting, but exploring the resulted equilibria is beyond the scope of this paper.

Nevertheless, our analysis with linear cost function clearly has no loss of generality when the effort of contestants is measured by their expenditure, which is a common situation in contests. To maximize the total expenditure exerted by these contestants, it is optimal to restrict the entry to exactly two contestants.

IV. CONCLUDING REMARKS

This paper has investigated the design of an effort-maximizing contest where contestants bear a fixed entry cost and have the freedom to decide whether or not to participate. The findings indicate that an entry fee (subsidy) is essential for optimal contest design. Contest organizers subsidize entry when contestants bear substantial entry costs while charging an entry fee to fund the prize purse when the entry cost is sufficiently low. Interestingly, when effort cost function is linear, the optimally designed contest invites exactly two participants as long as the entry cost is positive. Thus, this paper provides a clear rationale for the contest structure involving only two contestants that is widely assumed in contest literature. This optimal participation is attributed to the presence of a fixed entry cost for the potential contestants. In the absence of a fixed entry cost, the contest organizer does not have to restrict the number of participating

15. The proof is similar to that of Equation (7).

contestants to exactly 2. There exist a variety of forms that an optimal contest can take, which differ in entry fees, prize purse, and the number of participants. However, they all fully dissipate the budget of the contest organizer through charging an entry fee and thus induce the same amount of total effort.

This framework leaves tremendous room for the extension of research. One possible avenue for further research is to allow for different types of contestants. In this paper, the contest organizer does not directly invite contestants but strategically induces desired voluntary entry by setting the (unique) optimal combination of prize purse and entry subsidy/fee. It should be noted that the implementation of the optimal entry through sequential moves is guaranteed only when contestants are homogeneous. When contestants are heterogeneous, voluntary and sequential entry alone may not guarantee the optimum. A more sophisticated selection mechanism (such as Fullerton and McAfee 1999) would be in demand. Indeed, this is a future research concern for the authors of this paper.

APPENDIX

Proof of Lemma 4

The lemma is proven by contradiction. Suppose the contrary that $\pi(N(V^*, S^*), V^*, S^*) > 0$. Two possible cases are considered.

Case I: $N(V^*, S^*) = M$.

In this case, there exists a transfer $S < S^*$ such that $\pi(M, V^*, S) > 0$ holds since $\pi(M, V^*, S)$ is continuous with respect to S. This leads to $\pi(M, V^* + M(S^* - S), S) > \pi(M, V^*, S) > 0$. Thus, $N(V^* + M(S^* - S), S) = M$. It is clear that $(V^* + M(S^* - S), S)$ is feasible since (V^*, S^*) is feasible. However, contest $(V^* + M(S^* - S), S)$ induces a larger amount of total effort since the prize is higher and the number of potential participants who enter the contest does not change.

Case II: $2 \leq N(V^*, S^*) < M$.

By Lemma 2, we must have $\pi(N(V^*, S^*), V^*, S^*) > 0$ and $\pi(N(V^*, S^*) + 1, V^*, S^*) < 0$. There must exist a $\varepsilon > 0$ which is small enough such that $\pi(N(V^*, S^*), V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon) > 0$ and $\pi(N(V^*, S^*) + 1, V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon) < 0$ because the function $\pi(N, V, S)$ is continuous with respect to all its arguments. We thus have $N(V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon) = N(V^*, S^*)$. It is clear that $(V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon)$ is feasible since (V^*, S^*) is feasible. However, $(V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon)$ induces a larger amount of total effort since the prize is higher and the number of potential participants who enter the contest does not change.

Based on the above arguments, $\pi(N(V^*, S^*), V^*, S^*) = 0$ for the optimal feasible contest (V^*, S^*). Q.E.D. ■

Proof of Lemma 5

The lemma is proven by contradiction. Suppose $V^* < \Gamma_0 - N(V^*, S^*)S^*$. We consider two possible cases.

Case I: $N(V^*, S^*) = M$.

The contest organizer has the option to allocate the balance of $(\Gamma_0 - N(V^*, S^*)S^*) - V^*$ to the prize without inducing the entry of additional participants while increasing the amount of total effort induced.

Case II: $2 \leq N(V^*, S^*) < M$.

In this case, we have $\pi(N(V^*, S^*), V^*, S^*) = 0$ by Lemma 4 and $\pi(N(V^*, S^*) + 1, V^*, S^*) < 0$ by the definition of $N(V^*, S^*)$. By the continuity of $\pi(N, V, S)$, there exists a small $\varepsilon > 0$ such that $V^* + \varepsilon \leq \Gamma_0 - N(V^*, S^*)S^*$, $\pi(N(V^*, S^*), V^* + \varepsilon, S^*) > 0$, and $\pi(N(V^*, S^*) + 1, V^* + \varepsilon, S^*) < 0$. Thus, the contest $(V^* + \varepsilon, S^*)$ is feasible and $N(V^* + \varepsilon, S^*) = N(V^*, S^*)$ holds. However, the contest $(V^* + \varepsilon, S^*)$ induces a larger amount of total effort since the prize is higher and the number of participants does not change. Based on the above arguments, $V^* = \Gamma_0 - N(V^*, S^*)S^*$ for the optimal feasible contest (V^*, S^*). Q.E.D. ■

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