R&D Contest Design with Resource Allocation and Entry Fees*

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Abstract

This paper explores the design of an R&D contest by a sponsor who can charge entry fees and allocate a fixed amount of productive resources across firms—e.g., access to computing infrastructure or laboratory equipment. The revenues collected through entry fees can fund the prize awarded to the winner. The posted prize, entry fees, and productive resources promised to potential entrants jointly determine firms' decisions to enter the competition and their effort supply. We characterize the respective optimal contests for two objectives: (i) maximizing total effort in the contest and (ii) maximizing the expected quality of the winning product. We show that the optimal contest induces the entry of only the two most efficient firms when the sponsor can jointly set entry fees and allocate productive resources. The resource allocation plan in the optimum may favor the initially more competent firm and thus promote a "national champion" instead of leveling the playing field, and the optimum depends on the nature of the R&D task and effort cost profiles of the firms. Our analysis sheds light on the role played by these instruments in shaping optimal research contests.

Keywords: Research Contest; Contest Design; Resource Allocation; Entry Fee.

JEL Classification Codes: C72, D72.

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1 Introduction

Inducement prize contests—which offer prizes to elicit efforts to achieve defined goals—are increasingly being recognized in the modern economic landscape as a cost-effective and efficient mechanism to procure technological solutions for specific needs, promote innovative research of scientific significance, or encourage entrepreneurial efforts toward socially valuable goals (see, e.g., Terwiesch and Ulrich, 2009). For example, Samsung strategically leverages its regular innovation challenges to acquire external expertise and foster new developments within its existing product lines. Toyota's Mobility Unlimited Challenge promotes the development of assistive devices for people with lower-limb paralysis. The NFL and Duke University set aside a prize purse to call for the innovative design of helmets that minimize injuries. The XPRIZE Foundation has sponsored numerous high-profile public innovation challenges "to encourage technological development to benefit humanity," and the U.S. government created an online platform, Challenge.gov, to facilitate the use of contest protocols and match federal agencies' needs with public innovators. The Department of Defense (DoD) engages in private R&D for defense technologies using competitive procurement exercises and awards contracts to private firms that develop prototypes of superior quality.

Due to their popularity and success, R&D contests have invigorated scholarly efforts to identify efficient ways to administer such competitions based on a wide range of perspectives and disciplines, from economics and operations management to information systems (see, e.g., Taylor, 1995; Fullerton and McAfee, 1999; Che and Gale, 2003; Terwiesch and Xu, 2008; Bimpikis, Ehsani, and Mostagir, 2019; Letina and Schmutzler, 2019; Benkert and Letina, 2020).

This paper analyzes the optimal design of an R&D contest to address two natural and practically relevant questions. First, suppose that a pool of heterogeneous firms may enter the competition. How many firms should be included in the competition? Should a contest encourage open entry or limit participation, in the presence of mixed observation in practice (see, e.g., Terwiesch and Xu, 2008; Boudreau, Lacetera, and Lakhani, 2011; Ales, Cho, and Körpeoğlu, 2017, 2021)? Open entry expands the sources of contribution, but excessive competition could diminish an individual firm's incentive. Second, suppose that the contest sponsor has a fixed amount of resources—e.g., access to computing infrastructure or laboratory equipment—that could improve firms' research productivity. How should she optimally allocate these limited resources among contenders? Resource allocation not only bolsters recipient firms' productivity, but also varies their relative competitiveness in the contest and alters their effort incentives (see, e.g., Brown, 2011; Bockstedt, Druehl, and Mishra, 2022). Should the allocation favor the initially weaker firm in order to even the playing field—as traditionally advocated by the contest literature—or should it advantage the frontrunner to cultivate a "national champion"—a strategy many governments have adopted in their industrial policies (see, e.g., Falck, Gollier, and Woessmann, 2011)?

For these purposes, we construct a research tournament model à la Fullerton and McAfee (1999), in which the quality of a firm's product is a random variable and the one with the best submission wins the prize—e.g., a cash prize (for example, the $1 million grand prize offered by Netflix in its competition for a more predictive algorithm, known as the Netflix Prize) or a procurement contract (as in the prototype competitions sponsored by DoD). The sponsor sets and announces the contest rule to firms in the first stage, and firms commit to their entry and effort choice in the second stage. The contest rule consists of three elements: (i) a prize for the winner; (ii) a fee required for entry, with the revenue collected from participants to fund the posted prize; and (iii) an allocation profile of a limited amount of productive resources. The contest rule shapes the competition and ultimately determines firms' willingness to participate and their effort supply upon entry. An optimally set rule allows the sponsor to select and maximally incentivize the most desirable entrants.

To the best of our knowledge, this study is the first formal analysis that integrates entry fees and resource allocation in the design of contests. Numerous observations inspire this approach. Entry fees, for instance, are required by a number of XPRIZE challenges—e.g., the Google Lunar XPRIZE and XPRIZE Carbon Removal—and the prizes offered by data science competitions organized by Kaggle are funded by entry fees. These contests not only reward winners with prizes, but also often provide participants with various resources that bolster their productivity. Entrants in Mozilla's Open Innovation Challenge, for instance, receive mentorship and are provided with Mozilla's development tools, and the IBM Watson AI XPRIZE opens IBM Watson's APIs to participants. The DoD's Small Business Innovation Research Program not only rewards winners with procurement contracts, but also provides an "implicit subsidy" to selected private contractors to support their development efforts (Lichtenberg, 1990). The DARPA Robotics Challenge charges an entry fee, but also provides access to DARPA's robotics lab and software to facilitate participating teams' development.

Our analysis accommodates diverse preferences for contest design. Specifically, the sponsor sets the contest rule to maximize either (i) the total effort of the contest (effort-maximizing contest) or (ii) the expected quality of the winning product (quality-maximizing contest). The first objective is broadly assumed in the literature on contest design. Imagine a nonprofit organization—e.g., the XPRIZE Foundation—that aims to rally social effort toward or stimulate ideas about a fundamental challenge, such as rainforest conservation or decarbonization, in which case the first objective tends to be more relevant. In contrast, imagine a pharmaceutical company that seeks a cost-efficient method to synthesize an ingredient in its drugs, in which case the second objective would presumably apply (see, e.g., Taylor, 1995; Terwiesch and Ulrich, 2009; Stouras, Hutchison-Krupat, and Chao, 2022).

Summary of the Results We begin with a baseline model in which the sponsor imposes a uniform entry fee on all participating firms. We show that the optimal contest always involves only two active firms regardless of the sponsor's objective, with the two most competent firms entering the contest. That is, limited entry is optimal regardless of the prevailing design objective when the sponsor is able to charge an entry fee, top up her prize purse with the revenue, and allocate productive resources.

The two design objectives are not aligned and generate diverging implications with respect to the optimal resource allocation. When the sponsor aims to maximize the total effort of the contest, the resource allocation plan fully levels the playing field, such that the two firms win with equal probability (Proposition 1). That is, the initially weaker firm is prioritized for resource allocation, which closes the gap between firms in terms of their competence and creates an even race. The result thus reflects the conventional wisdom in the contest literature of leveling the playing field. In contrast, when the sponsor is concerned about the expected quality of the winning product, she may promote a "national champion": The initially more competent firm receives more resources, which enlarges the gap in competitiveness and results in a more lopsided competition (Proposition 2). We demonstrate that the optimum depends on the nature of the R&D task—i.e., the level of difficulty or uncertainty of the task—and the degree of heterogeneity of the two most competent firms.

The distribution of a firm's product quality depends not only on its effort, but also the resources it receives. More specifically, a firm's effort and the resources available are complementary to each other in producing a high-quality submission. As a result, the sponsor is compelled to spend more resources on the more competent firm when she is concerned about maximizing the winning product's quality: The more competent firm bears a lower marginal effort cost and presumably expends a higher effort, so an allocation plan that prioritizes the initial favorite ensures allocative efficiency. However, this further upsets the competitive balance of the contest and tends to soften the competition, as the conventional wisdom of the contest literature would predict.

The concern about allocative efficiency does not arise when the sponsor maximizes total effort, so the usual prediction of leveling the playing field is preserved. The quality-maximizing contest must reconcile the trade-off between firms' effort incentives and allocative efficiency, and thus a level playing field can be suboptimal. These findings highlight the costly nature of creating a level playing field as a catalyst for competition, which can compromise allocative efficiency. In addition, our findings emphasize the need to consider the specific nature of the research problem and the profiles of the contenders when designing the competition framework.

Our results shed light on the nature of these popularly adopted design instruments. Suppose, for instance, that the sponsor is unable to collect entry fees and can only allocate productive resources. The effort-maximizing contest should always involve at least three active firms whenever possible, as shown in the literature—e.g., Franke, Kanzow, Leininger, and Schwartz (2013) and Fu and Wu (2020). We also demonstrate that without entry fees, a national champion is more likely to emerge when the sponsor is concerned about the expected quality of the winning product. Put differently, the ability to collect entry fees and use the revenue to supplement the prize purse enables the sponsor to create more balanced competition through resource allocation. This subtlety inspires us to further explore the role played by entry fees. We extend our model to allow for discriminatory entry fees, such that the sponsor may condition entry fees on firms' identities (Propositions 3 and 4). The results and economic logic are presented and discussed in Section 4.

Our results yield ample implications for the practice of the administration of contest-like competitive events, which are discussed in Section 5.

Related Literature Our paper is the first study in the literature to examine contest design that incorporates both entry fees and resource allocation. This novel approach enables us to explore two critical and inherent questions: (i) Should a contest encourage broader participation or restrict entry? (ii) When a sponsor has the ability to distribute productive resources, should the emphasis be on bolstering the frontrunner to capitalize on its efficiency, or should support be given to the underdog to ensure a more equitable competition?

A classical question in the literature on contest design concerns comparing the optimal size of the competition—such as open versus restricted entry—with the optimal selection of participating contenders. Terwiesch and Xu (2008) demonstrate that the sponsor's design objective—maximizing the average quality of solutions or best solution—plays a critical role for the choice between open entry and restricted access. Körpeoğlu and Cho (2018) suggest an additional positive incentive effect of open entry. Ales, Cho, and Körpeoğlu (2021) demonstrate how the choice between open and restricted entry depends on the properties of innovation production technology—i.e., the weight and distribution of random terms—and the number of potential contributors.[1] A burgeoning strand of the literature examines this question in empirical contexts. Chen, Pavlou, Wu, and Yang (2021), for instance, focus on how contestants' entry is affected by the posted prize and the duration of the contest. Boudreau, Lacetera, and Lakhani (2011) investigate how the optimal size of a contest depends on the nature and uncertainty of the research problem.

Our paper joins the strand of literature that demonstrates the merit of limiting participation, such as Baye, Kovenock, and De Vries (1993); Fullerton and McAfee (1999); and Che and Gale (2003). In particular, Fullerton and McAfee (1999) suggest that a contest organizer strategically sets entry to filter entrants and show that a contest of two active contenders can be optimal when the cost profile of eligible firms meets certain conditions. In contrast, we establish the optimality of minimum entry without restrictions on firms' cost structures, which is achieved by the joint use of entry fees and resource allocation.[2,3] The setting of joint design differentiates our study from prior contributions, since none of them allow for the allocation of productive resources.

Our paper is naturally linked to the growing literature that treats contestants' participation as an endogenous choice (e.g., Ales, Cho, and Körpeoğlu, 2017; Mihm and Schlapp, 2019). A handful

[1] Stouras, Hutchison-Krupat, and Chao (2022) examine the same problem. In contrast to the majority of these studies, they assume that potential contenders' types are privately known and consider the optimal reward structure that attracts the most desirable participants.

[2] In the model of Fullerton and McAfee (1999), quality maximization and effort maximization perfectly coincide, since the total effort determines the distribution of the quality of the winning product. In contrast, our setting—by allowing for resource allocation—causes the two objectives to diverge.

[3] Terwiesch and Xu (2008), in contrast, contend that broader participation allows the contest organizer to secure more diverse solutions for the problem she aims to tackle. Boudreau, Lacetera, and Lakhani (2011) demonstrate empirically that the proper number of participants depends on the nature of the underlying research problem, as well as the uncertainty it entails.

of papers assume that participation requires a fixed and exogenous entry cost (e.g., Fu, Jiao, and Lu, 2015; Boosey, Brookins, and Ryvkin, 2020; Stouras, Hutchison-Krupat, and Chao, 2022). We instead assume an endogenously set entry fee and that the revenues from fees are added to the prize purse, which puts our paper in the company of Fullerton and McAfee (1999), Taylor (1995), Moldovanu, Sela, and Shi (2012), and Hammond, Liu, Lu, and Riyanto (2019).[4,5] None of these studies involve the allocation of productive resources among firms.

The literature has long recognized the important incentive effects of the evenness of a contest. Brown (2011) and Bockstedt, Druehl, and Mishra (2022) empirically investigate how the presence of "star" contenders affects the incentives and performance of the peers. In our setting, the resource allocation profile endogenously determines firms' relative competitiveness, which affects their incentives and payoffs in the contest and, in turn, is a factor in their entry decisions. Our paper is closely related to the immense literature on contests with identity-dependent preferential treatment, such as Franke, Kanzow, Leininger, and Schwartz (2013, 2014); Drugov and Ryvkin (2017); and Fu and Wu (2020). These studies typically view the identity-dependent biases imposed on contestants' effort entry as a nominal scoring rule. In contrast, the resources allocated to a firm not only increase its relative competitiveness but also its actual output. A handful of studies—e.g., Fu, Lu, and Lu (2012); Deng, Fu, and Wu (2021); and Gao, Fan, Huang, and Chen (2022)—study similar productive resource allocation problems, with productive resources playing a role in determining contenders' relative competitiveness. However, none of these studies involve entry fees.

The resource allocation profile set by the designer determines the mapping of firms' efforts to the probabilities of their winning the contest. This links our paper to studies that endogenize the winning probability specification of a contest (i.e., contest success function) through optimal contest design. Letina, Liu, and Netzer (2023), for instance, let a designer decide how to allocate prizes based on the noisy signals of contestants' efforts and find that the optimum boils down to a nested Tullock contest.

In our setting, the entry fee is a source of revenue to fund the prize purse, which also links our study to the extensive literature on the optimal prize structure in contests, such as Moldovanu and Sela (2001); Kalra and Shi (2001); Terwiesch and Xu (2008); Ales, Cho, and Körpeoğlu (2017); and Stouras, Hutchison-Krupat, and Chao (2022). These studies typically focus on the choice between a winner-take-all contest and multiple prizes. In contrast, we focus on a single prize and search for the entry fees that induce the optimal entry and generate the associated optimal size of the prize.

[4] Azmat and Möller (2009, 2018) allow each contestant to choose which contest to enter when multiple contests are available.

[5] Instead of the explicit decision of entry, Lemus and Marshall (2021) empirically examine a dynamic contest setting in which contestants decide whether to continue their participation.

2 The Model

A sponsor organizes an R&D contest to acquire an innovative product. She posts a prize of a value $V > 0$—e.g., a procurement contract—for the winner. A pool of $n \geq 2$ firms are interested in the competition. A firm bears an exogenous fixed entry cost $\gamma > 0$—e.g., the costs of preparation for the project and forgone revenues from alternative engagement—as well as paying an entry fee $\phi \geq 0$ to the sponsor. Each firm $i \in \mathcal{N} := \{1, \ldots, n\}$, on entry, commits to its effort $x_i > 0$ to develop the product sought by the sponsor. In the case in which a firm i chooses to opt out of the research contest, we set $x_i = 0$. The effort incurs a constant marginal effort cost $c_i > 0$. Assume without loss of generality that firms are ordered such that $c_1 \leq \cdots \leq c_n$, with a lower marginal cost to imply a greater level of innate ability.

The sponsor is endowed with an initial (monetary) prize purse $b > 0$ and a fixed amount of (non-monetary) productive resources—e.g., the mentorship Mozilla provides to development teams, use of DAPRA's robotics lab, or access to IMB Watson's application programming interfaces (APIs)—which improve the productivity of a recipient, and we normalize to unity. The sponsor sets the entry fee and splits and allocates her endowed productive resources among participating firms.

R&D Contest The winner is selected through a standard "best of simultaneous submissions" R&D contest à la the research tournament model of Fullerton and McAfee (1999). The sponsor awards the prize to the firm that presents a product of the highest quality. The quality q_i of a firm i's product is randomly drawn from a distribution with cumulative distribution function (CDF) $[F(q_i)]^{\alpha_i x_i^r}$, with $r \in (0, 1]$, where $\alpha_i \geq 0$ is the amount of productive resource firm i receives from the sponsor and $F(\cdot)$ is a continuous CDF on a support $\left[\underline{q}, \overline{q}\right]$. The term $\alpha_i x_i^r$ can intuitively be interpreted as the number of effective trials or draws of ideas, with the quality of each trial or draw following the distribution $F(\cdot)$. The firm simply presents the output of the most successful trial or draw—with a quality q_i—as its submission to the contest.

A larger $\alpha_i x_i^r$ implies that a higher q_i is more likely to be realized and firm i is more likely to leapfrog its opponents. The resource α_i can presumably be viewed as a capital input that improves the firm's efficiency—e.g., access to equipment, laboratory facilities, or computing infrastructure. The effort x_i can conveniently be interpreted as a labor input sunk by the firm—e.g., the time, energy, and intellectual resources dedicated to the project. We assume the term $\alpha_i x_i^r$ to be concave—i.e., $r \leq 1$—which describes a development process with diminishing marginal returns. To put this intuitively, doubling input cannot more than double the likelihood of a scientific discovery.

If no firm enters, the contest is cancelled. If only one firm enters, the entrant automatically wins the prize. Otherwise, by Fullerton and McAfee (1999) and Baye and Hoppe (2003), fixing an effort profile $\boldsymbol{x} := (x_1, \ldots, x_n)$, with $\sum_{j=1}^{n} \alpha_j \cdot x_j^r > 0$, each firm $i \in \mathcal{N}$ wins with a probability[6,7]

$$p_i(\boldsymbol{x}) := \Pr\left(q_i > \max_{j \neq i} q_j\right) = \frac{\alpha_i \cdot x_i^r}{\sum_{j=1}^{n} \alpha_j \cdot x_j^r}. \tag{1}$$

Contest Design Prior to the contest, the sponsor sets and publicly announces the contest rule—which is described by a triple $(V, \phi, \boldsymbol{\alpha})$—anticipating firms' responses in terms of entry and effort decisions. The contest rule consists of three elements: (i) the posted prize value $V > 0$; (ii) the entry fee $\phi \geq 0$; and (iii) a resource allocation profile $\boldsymbol{\alpha} := (\alpha_1, \ldots, \alpha_n) \geq (0, \ldots, 0)$. We assume that the entry fee is *uniform* for all firms and will relax this assumption in Section 4.

The sponsor is subject to two prevailing budget constraints. First, the productive resources she can provide to the firms are limited—i.e., $\sum_{i \in \mathcal{N}} \alpha_i = 1$. Second, the value of the posted prize, V, is bounded by her initial endowment b and the proceeds collected through entry fees. Denote by $k(V, \phi, \boldsymbol{\alpha})$ the number of entrants in the equilibrium for a given contest rule $(V, \phi, \boldsymbol{\alpha})$. The budget constraint for the prize purse is thus $V \leq b + k(V, \phi, \boldsymbol{\alpha})\phi$.

The sponsor can have two design objectives. She may intend to promote technological efforts for socially valuable missions—e.g., an XPRIZE challenge to discover clean and renewable energy in response to climate change. The sponsor, under such a circumstance, aims to maximize firms' total effort, which is given by

$$Z^* := \sum\nolimits_{i \in \mathcal{N}} x_i. \tag{2}$$

Alternatively, the sponsor can be concerned about the quality of the winning product—e.g., when the DoD procures military equipment from private contractors or Netflix searches for an algorithm for more precise predictions. Denote by $q_{max} = \max\{q_1, \ldots, q_n\}$ the quality of the winning product. For a given effort profile $\boldsymbol{x} \equiv (x_1, \ldots, x_n)$, q_{max} is the first-order statistic of the quality of firms' submissions q_i, which follows a distribution with CDF $\left[F(q_{max})\right]^{\sum_{i=1}^{n} \alpha_i \cdot x_i^r}$. The sponsor thus sets the contest rule $(V, \phi, \boldsymbol{\alpha})$ to maximize

$$Z^\star := \sum\nolimits_{i \in \mathcal{N}} \alpha_i \cdot x_i^r. \tag{3}$$

Timeline and Payoff The game proceeds in two stages. In the first, the sponsor announces the contest rule $(V, \phi, \boldsymbol{\alpha})$. The contest takes place in the second stage. Firms observe $(V, \phi, \boldsymbol{\alpha})$ and simultaneously make their entry and effort decisions.

For a given effort profile $\boldsymbol{x} \equiv (x_1, \ldots, x_n)$, a firm i's expected payoff in a contest $(V, \phi, \boldsymbol{\alpha})$ is

[6] Another micro-foundation to obtain this success function is the following: Firm $i \in \mathcal{N}$ has a production technology in the form of $f_i(x_i) = \alpha_i \cdot x_i^r$, where α_i is the resource allocated to firm i. The sponsor receives a noisy signal s_i of firm i's performance or output, with $\log s_i = \log f_i(x_i) + \epsilon_i$, where ϵ_i follows a type I extreme-value distribution (i.e., Gumbel distribution). The prize is awarded to the firm with the highest signal.

[7] In the case of $\sum_{j=1}^{n} \alpha_j \cdot x_j^r = 0$, we let $p_i(\boldsymbol{x}) = 1/|\{j \in \mathcal{N} | x_j > 0\}|$ if $x_i > 0$.

given by

$$\pi_i\left(\boldsymbol{x}; V, \phi, \boldsymbol{\alpha}\right) = \begin{cases} p_i(\boldsymbol{x}) \cdot V - c_i x_i - \phi - \gamma, & \text{if } x_i > 0, \\ 0, & \text{if } x_i = 0, \end{cases}$$

where $p_i(\boldsymbol{x})$ is defined in (1).

3 Analysis

This section characterizes the optimal contest. Before we proceed, two remarks are in order. First, the second-stage contest game, in general, does not yield a closed-form equilibrium solution. This nullifies the traditional implicit programming approach to optimal contest design, which requires a closed-form equilibrium solution for every possible contest rule (see, e.g., Franke, Kanzow, Leininger, and Schwartz, 2013). We adopt the technique developed by Fu and Wu (2020) to circumvent this challenge. Second, we assume that the sponsor has adequate budget to attract the participation of the set of firms she desires for all possible scenarios we will consider in the subsequent analysis. The following condition is imposed throughout the paper.

Assumption 1 ***(Sufficient Budget for Sponsor)*** $b > \max\{\underline{b}^*, \underline{b}^\star\}$*, where* $\underline{b}^*$ *and* $\underline{b}^\star$ *are to be defined later in* (4) *and* (6)*, respectively.*

3.1 Effort-maximizing Contests

We now let the sponsor choose the contest rule $(V, \phi, \boldsymbol{\alpha})$ to maximize the total effort of the contest, $Z^* \equiv \sum_{i\in\mathcal{N}} x_i$. When designing the contest, the sponsor needs to ensure *budget balance*, which requires that the prize be sufficiently funded by the sponsor's initial prize purse and the revenues of entry fees collected from participants. In addition, she is subject to a *participation constraint*, which ensures that the firms she targets would enter the competition.

Denote by $\boldsymbol{p}^* := (p_1^*, \ldots, p_n^*)$ the effort-maximizing equilibrium winning probabilities. Further define

$$\underline{b}^* := \frac{4\gamma}{2-r} > 0. \tag{4}$$

The following result can be obtained.

Proposition 1 ***(Effort-maximizing Research Contest with Uniform Entry Fees)*** *Suppose that the sponsor aims to maximize the total effort of the R&D contest and Assumption 1 is satisfied. The optimal contest* $(V^*, \phi^*, \boldsymbol{\alpha}^*)$ *is given by*

$$V^* = \frac{2b - 4\gamma}{r}, \phi^* = \frac{(2-r)b}{2r} - \frac{2\gamma}{r},$$

and

$$\boldsymbol{\alpha}^* = \left(\frac{c_1^r}{c_1^r + c_2^r}, \frac{c_2^r}{c_1^r + c_2^r}, 0, \ldots, 0\right).$$

The optimal contest induces a profile of equilibrium winning probabilities $\boldsymbol{p}^* \equiv (p_1^*, \ldots, p_n^*) = (1/2, 1/2, 0, \ldots, 0)$*: The two most competent firms enter the contest and they win with an equal probability.*

By Proposition 1, it takes (exactly) two to tango: The optimal R&D contest involves the two most efficient firms and fully levels the playing field, such that firms 1 and 2 win with equal probability. To achieve this, the sponsor arms the less efficient firm—i.e., firm 2—with a larger amount of resources, with $\alpha_2^* = c_2^r/(c_1^r + c_2^r) \geq c_1^r/(c_1^r + c_2^r) = \alpha_1^*$.

A smaller contest weakens competition in the contest and limits effort contribution, which tends to leave more surplus to participating firms. As a result, at least three firms will be kept active in the optimum if the designer optimizes the contest with only the choice of $\boldsymbol{\alpha}$, as Franke, Kanzow, Leininger, and Schwartz (2013) show. This contrasts with our result of involving only the two most competent firms. The entry fee plays a critical role in our setting and remedies this adverse effect: The sponsor collects revenue through entry fees to finance a larger prize purse, which bolsters the incentive provided to participating firms and motivates their investment. Without entry fees, the optimal contest has to involve broader participation—i.e., by requiring at least three active firms whenever feasible.

Resource allocation and the entry fee play complementary roles. By Proposition 1, the sponsor fully levels the playing field by spreading more resources to the ex ante weaker firm—i.e., firm 2—such that they win with an equal probability. The optimal contest fully extracts firms' surplus, which is achieved by the proper combination of resource allocation and entry fee.

3.2 Quality-maximizing Contests

Now suppose that the sponsor is concerned about the quality of the winning product, $Z^\star \equiv \sum_{i\in\mathcal{N}} \alpha_i \cdot x_i^r$. The following preliminary result paves the way for our formal characterization of the optimum. Let $(p_1^\star, p_2^\star)$ solve

$$\min_{p_1+p_2=1,\, p_1\geq p_2>0} \left(\frac{c_1^r p_1^{1-r}}{p_2^r} + \frac{c_2^r p_2^{1-r}}{p_1^r} \right) \times \left[1 - 2p_2\left(1 - rp_1\right)\right]^r. \tag{5}$$

Further define

$$\underline{b}^\star := \frac{\gamma}{p_2^\star\left(1 - rp_1^\star\right)}. \tag{6}$$

The following result ensues.

Proposition 2 ***(Quality-maximizing R&D Contest with Uniform Entry Fees)*** *Suppose that the sponsor aims to maximize the expected quality of the winning product of the R&D contest and Assumption 1 is satisfied. The optimal contest induces two entrants—with the two ex ante most competent firms remaining active in the competition—and a profile of equilibrium winning* *probabilities* $\boldsymbol{p}^\star = (p_1^\star, p_2^\star, 0, \ldots, 0)$, *with* $(p_1^\star, p_2^\star)$ *defined above and* $p_1^\star \geq \frac{1}{2} \geq p_2^\star$. *The optimal contest rule* $(V^\star, \phi^\star, \boldsymbol{\alpha}^\star)$ *is given by*

$$V^\star = \frac{b - 2\gamma}{1 - 2p_2^\star(1 - rp_1^\star)}, \ \phi^\star = \frac{bp_2^\star\left(1 - rp_1^\star\right) - \gamma}{1 - 2p_2^\star\left(1 - rp_1^\star\right)},$$

and

$$\boldsymbol{\alpha}^\star = \left(\frac{\frac{c_1^r (p_1^\star)^{1-r}}{(p_2^\star)^r}}{\frac{c_1^r (p_1^\star)^{1-r}}{(p_2^\star)^r} + \frac{c_2^r (p_2^\star)^{1-r}}{(p_1^\star)^r}}, \frac{\frac{c_2^r (p_2^\star)^{1-r}}{(p_1^\star)^r}}{\frac{c_1^r (p_1^\star)^{1-r}}{(p_2^\star)^r} + \frac{c_2^r (p_2^\star)^{1-r}}{(p_1^\star)^r}}, 0, \ldots, 0 \right).$$

Similar to Proposition 1, Proposition 2 also predicts that the optimal contest involves only the two most competent firms. Despite the similarity, quality maximization stands in sharp contrast to effort maximization in terms of the underlying trade-offs. Note that the productive resources $\boldsymbol{\alpha} \equiv (\alpha_1, \ldots, \alpha_n)$ do not directly affect the sponsor's payoff when she maximizes total effort. In contrast, $\boldsymbol{\alpha}$ directly enters the objective function (3) and generates intrinsic value to the sponsor in a quality-maximizing contest. Because of the *complementarity* between the resource α_i and a firm's input x_i, the sponsor must avoid spreading costly and scarce resources across less productive firms. This concern compels her to limit the competition and induce the entry of only the two most efficient firms.

When allocating resources between heterogeneous firms, the sponsor must strike a balance between two competing effects. Prioritizing the weaker firm fuels competition, which we call the *competition effect*. However, this undermines *allocative efficiency*, which requires that the resources be concentrated on the more competent firm, since resources and effort are complementary. Tension between the two concerns tilts the optimum away from its counterpart of effort maximization and may even overturn the conventional wisdom by further upsetting the balance of the competition. A closer look at $(\alpha_1^\star, \alpha_2^\star)$ yields the following.

Corollary 1 ***(National Champion vs. Handicapping)*** *Suppose that* $c_1 < c_2$. *The following statements hold:*

(i) If $r \geq 1/2$, *then* $\alpha_1^\star < \alpha_2^\star$.

(ii) If $r < 1/2$, *then there exists a threshold* ℓ*—which depends on* r*—such that* $\alpha_1^\star \gtreqless \alpha_2^\star$ *if and only if* $c_2/c_1 \gtreqless \ell$.

The sponsor may create a national champion by prioritizing the more competent firm in resource allocation, which further upsets the competitive balance of the contest. Figure 1 depicts the comparison between $\alpha_1^\star$ and $\alpha_2^\star$ in the optimum. The horizontal axis measures the degree of heterogeneity between the two most efficient firms, $\log(c_2/c_1)$, and the vertical axis traces the value of the exponential term r.

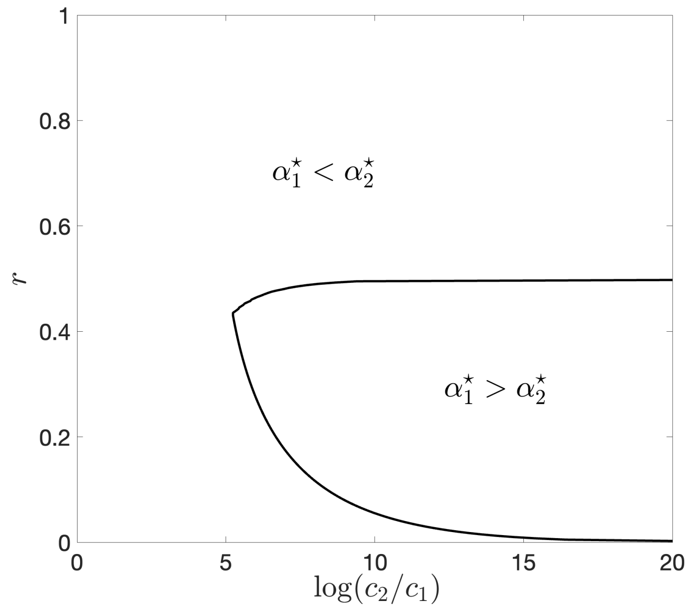

Figure 1: Quality-maximizing Resource Allocation Scheme.

Recall that $\alpha_i x_i^r$ is interpreted as the number of trials. The term r thus measures how effectively effort x_i can be converted into output and provides an intuitive account of the R&D task's technological nature. A more challenging R&D task or a more strenuous R&D process can intuitively be described as a smaller r, since a given input is less likely to deliver high-quality trials. For instance, a research project that aims for major scientific discovery—e.g., a universal flu vaccine—can be described by a small r; in contrast, an effort to incrementally improve an engineering process involves less uncertainty and presumably implies a larger r. By Corollary 1 and Figure 1, $\alpha_1^\star > \alpha_2^\star$ when $r < 1/2$ and $c_2/c_1 > \ell$. That is, the optimal contest favors the more competent firm if and only if (i) the R&D process is sufficiently difficult and (ii) firms are substantially heterogeneous.

To understand the result, first note that the competition effect wanes when the difficulty of the task increases—i.e., with a smaller r: The additional incentive provided by a level playing field is diminished by the lower marginal return to effort, so a more even race incentivizes firms less effectively. As a result, $\alpha_1^\star < \alpha_2^\star$ may not hold when r falls below 1/2. Second, an increase in the degree of heterogeneity between firms—i.e., a larger c_2/c_1—magnifies the loss of allocative efficiency when assigning resources to the weaker firm, which further diminishes the appeal of a level playing field. A national champion—i.e., $\alpha_1^\star > \alpha_2^\star$—thus emerges when the R&D process is sufficiently difficult and the degree of heterogeneity between firms is significant, i.e., $c_2/c_1 > \ell$.

Again, the entry fee plays a critical role as a design instrument. For instance, Deng, Fu, and Wu (2021) consider a resource allocation problem in an R&D contest, but their setting does not involve the use of entry fees; they show that a national champion arises in the optimum whenever r falls below 1/2. Entry fees render a national champion less likely: By Corollary 1, a national champion requires not only $r < 1/2$ but also $c_2/c_1 > \ell$. The revenue from entry fees enlarges the prize purse, which amplifies effort incentives, and thereby magnifies the competition effect of a more level playing field. Meanwhile, concern about allocative efficiency can be ameliorated because less efficient firm 2 contributes more effort when a larger prize is in place. We then observe that the entry fee catalyzes even races in the optimum.

4 Discriminatory Entry Fees

We now relax the assumption of uniform entry fees and allow them to depend on firms' identities. Denote by $\phi_i \geq 0$ the entry fee imposed for a firm $i \in \mathcal{N}$ and let $\boldsymbol{\phi} := (\phi_1, \ldots, \phi_n)$. We first consider the optimal contest that maximizes total effort, then proceed to the case of quality maximization.

4.1 Effort-maximizing Contests

Similar to the analysis in Section 3.1, the sponsor chooses $(V, \boldsymbol{\phi}, \boldsymbol{\alpha})$ to maximize $Z^* \equiv \sum_{i \in \mathcal{N}} x_i$, subject to firms' participation constraint and her own budget constraint.

Proposition 3 ***(Effort-maximizing Contest with Discriminatory Entry Fees)*** *Suppose that the sponsor aims to maximize the total effort of the R&D contest and is allowed to impose discriminatory entry fees. Moreover, Assumption 1 is satisfied. The optimal contest involves two active firms and induces a profile of equilibrium winning probabilities* $\widehat{\boldsymbol{p}}^* = (\widehat{p}_1^*, \widehat{p}_2^*, 0, \ldots, 0)$*, where* $(\widehat{p}_1^*, \widehat{p}_2^*) > (0, 0)$ *satisfies*

$$\widehat{p}_1^* + \widehat{p}_2^* = 1, \text{ and } b \times \min_{i \in \{1,2\}} \left\{1 - r\widehat{p}_i^*\right\} - 2\gamma \geq 0. \tag{7}$$

The corresponding contest—which we denote by $\left(\widehat{V}^*, \widehat{\boldsymbol{\phi}}^*, \widehat{\boldsymbol{\alpha}}^*\right)$*—involves*

$$\widehat{V}^* = \frac{b - 2\gamma}{2r\widehat{p}_1^*\widehat{p}_2^*}, \widehat{\boldsymbol{\phi}}^* = \left(\frac{b\left(1 - r\widehat{p}_2^*\right) - 2\gamma}{2r\widehat{p}_2^*}, \frac{b\left(1 - r\widehat{p}_1^*\right) - 2\gamma}{2r\widehat{p}_1^*}, 0, \ldots, 0\right),$$

and

$$\widehat{\boldsymbol{\alpha}}^* = \left(\frac{\frac{c_1^r(\widehat{p}_1^*)^{1-r}}{(\widehat{p}_2^*)^r}}{\frac{c_1^r(\widehat{p}_1^*)^{1-r}}{(\widehat{p}_2^*)^r} + \frac{c_2^r(\widehat{p}_2^*)^{1-r}}{(\widehat{p}_1^*)^r}}, \frac{\frac{c_2^r(\widehat{p}_2^*)^{1-r}}{(\widehat{p}_1^*)^r}}{\frac{c_1^r(\widehat{p}_1^*)^{1-r}}{(\widehat{p}_2^*)^r} + \frac{c_2^r(\widehat{p}_2^*)^{1-r}}{(\widehat{p}_1^*)^r}}, 0, \ldots, 0\right).$$

By Proposition 3, the optimal R&D contest again involves two active firms when the sponsor can charge discriminatory entry fees. Two remarks are in order. First, the optimal contest is not unique, which stands in stark contrast to our previous findings. Multiple contests exist that generate maximum total effort while inducing different winning probability profiles in the equilibrium. The effort-maximizing contest in Section 3.1—which charges a uniform entry fee and induces an even contest with $(p_1, p_2) = (1/2, 1/2)$—remains one of the optima.[8]

This observation leads to the second remark: The sponsor does not (strictly) benefit from the opportunity to charge discriminatory entry fees. Relaxing the constraint of uniform entry fees allows for multiple optima, but none of them strictly outperforms the original optimum in Proposition 1. The sponsor can charge a uniform entry fee and set $\widehat{\boldsymbol{\alpha}}^*$ to level the playing field, as she does in Section 3.1. She can also set $\widehat{\boldsymbol{\alpha}}^*$ to induce uneven winning odds and impose a customized entry fee equal to the surplus each active firm expects to earn in the contest. Regardless, all of these candidate contests fully extract firms' surplus.

4.2 Quality-maximizing Contest

Now suppose that the sponsor is concerned about the quality of the winning product, i.e., maximizing $Z^\star \equiv \sum_{i\in\mathcal{N}} \alpha_i \cdot x_i^r$. The following result ensues.

Proposition 4 ***(Quality-maximizing R&D Contest with Discriminatory Entry Fees)*** *Suppose that the sponsor aims to maximize the expected quality of the winning product of the R&D contest and can charge discriminatory entry fees. Moreover, Assumption 1 is satisfied and* $b < \frac{2\gamma}{1-r}$.[9] *Then the optimal contest involves two active firms.*

(i) If $c_1 < c_2$*, then the optimal contest induces an equilibrium winning probability profile* $\widehat{\boldsymbol{p}}^\star = (\widehat{p}_1^\star, \widehat{p}_2^\star, 0, \ldots, 0)$*, with*

$$\widehat{p}_1^\star = 1 - \widehat{p}_2^\star = \frac{1}{r} - \frac{2\gamma}{rb}. \tag{8}$$

The corresponding contest rule—which we denote by $(\widehat{V}^\star, \widehat{\boldsymbol{\phi}}^\star, \widehat{\boldsymbol{\alpha}}^\star)$*—is*

$$\widehat{V}^\star = \frac{b-2\gamma}{2r\widehat{p}_1^\star\widehat{p}_2^\star},\ \widehat{\boldsymbol{\phi}}^\star = \left(\frac{b(1-r\widehat{p}_2^\star)-2\gamma}{2r\widehat{p}_2^\star}, 0, 0, \ldots, 0\right),$$

and

$$\widehat{\boldsymbol{\alpha}}^\star = \left(\frac{\frac{c_1^r(\widehat{p}_1^\star)^{1-r}}{(\widehat{p}_2^\star)^r}}{\frac{c_1^r(\widehat{p}_1^\star)^{1-r}}{(\widehat{p}_2^\star)^r} + \frac{c_2^r(\widehat{p}_2^\star)^{1-r}}{(\widehat{p}_1^\star)^r}}, \frac{\frac{c_2^r(\widehat{p}_2^\star)^{1-r}}{(\widehat{p}_1^\star)^r}}{\frac{c_1^r(\widehat{p}_1^\star)^{1-r}}{(\widehat{p}_2^\star)^r} + \frac{c_2^r(\widehat{p}_2^\star)^{1-r}}{(\widehat{p}_1^\star)^r}}, 0, \ldots, 0\right).$$

(ii) If $c_1 = c_2$*, there exist multiple contest rules that generate the maximum expected quality of the winning product, and are the same as those provided in Proposition 3.*

Analogous to Proposition 2, Proposition 4 states that the optimal R&D contest involves exactly two entrants. Further, it can be verified that the more competent firm stands a better chance to

[8] To see this, note that if a tuple $(p_1, p_2) = (p_1^\natural, p_2^\natural)$ satisfies constraint (7), then $(p_1, p_2) = (p_2^\natural, p_1^\natural)$ also satisfies the constraint.

[9] The assumption $b < \frac{2\gamma}{1-r}$ is imposed to guarantee that the equilibrium winning probability of the most efficient firm in (8) is smaller than one. Otherwise, a maximum does not exist when the two most efficient firms are heterogeneous; moreover, the supremum can be approached arbitrarily closely by giving the second most efficient firm an infinitesimal amount of winning probability and the most efficient firm complementary probability.

win the contest—i.e., $\widehat{p}_1^\star > 1/2 > \widehat{p}_2^\star$—whenever the two most competent firms are heterogeneous; i.e., $c_1 < c_2$. It is noteworthy, however, that the optimal contest collects the entry fee only from firm 1 in this case. Similar to Proposition 2, the sponsor may choose to either cultivate a national champion or favor the underdog in the optimum, depending on the discriminatory power r and the degree of firm heterogeneity c_2/c_1. We obtain the following result, which paves the way for more detailed discussion of the underlying logic.

Corollary 2 ***(Discriminatory Entry Fees Render a National Champion More Likely)*** *Suppose that $c_1 < c_2$; then $\alpha_1^\star < \widehat{\alpha}_1^\star$.*

By Corollary 2, the optimal contest with discriminatory entry fees awards a larger share of productive resources to the ex ante stronger firm—i.e., $\alpha_1^\star < \widehat{\alpha}_1^\star$—than its counterpart with uniform entry fees. Recall that the sponsor must factor in allocative efficiency, which tempts her to provide more resources to the more competent firm to tap its superior productivity. Discriminatory entry fees afford the sponsor more flexibility in this respect. Uniform entry fees favor a level playing field: The entry fee cannot exceed the surplus firm 2 is able to secure from the contest, so a more uneven race leaves rent to firm 1 and limits the eventual prize purse. However, awarding more resources to less productive firm 2 wastes productivity and jeopardizes the allocative efficiency of the contest. Discriminatory entry fees offer a solution. The sponsor can privilege the more competent firm 1 in resource allocation—which advantages the firm in the competition—while confiscating its rent by charging a larger entry fee ϕ_1. In the optimal contest, firm 2 ends up with zero surplus and pays zero entry fee, which leaves it indifferent between participating in or staying out of the contest. A lopsided contest can increase firm 1's surplus—which, however, is absorbed by a high entry fee; the revenue tops up the prize purse, which, in turn, motivates the two firms to invest in their effort. As a result, Corollary 1 states that firm 1 tends to receive more resources when the entry fee is not forced to be uniform.

Two remarks are in order before we close this section. First, in contrast to Proposition 3, a quality-maximizing sponsor *strictly* benefits from the flexibility to charge discriminatory entry fees when the top two candidates are heterogeneous. Second, a closer look at (7) and (8) reveals that the optimal contest established in Proposition 4 not only maximizes the expected quality of the winning product but also the total effort of the R&D contest.[10] The flexibility to impose identity-dependent entry fees allows the sponsor to fully extract surplus and maximizes the incentive the contest provides.

[10] Note that there does not always exist a contest that maximizes both total effort and the expected quality of the winning product under the constraint of a uniform entry fee. By Corollary 1, a quality-maximizing contest cultivates a national champion and gives the most efficient firm a higher equilibrium winning probability when $r < 1/2$ and $c_2/c_1 > \ell$, while according to Proposition 1, an effort-maximizing contest would completely level the playing field.

5 Discussions, Implications, and Conclusions

In this paper, we explore the design of an R&D contest by a sponsor who can (i) charge entry fees and (ii) allocate a fixed amount of productive resources across firms. To the best of our knowledge, this is the first paper in the literature to examine the joint contest design problem with the two popular and intuitive instruments. Our analysis sheds light on the role played by these instruments in providing incentives and shaping optimal contests.

Our results generate useful implications for the practical design of contest mechanisms. First, we show that restricting entry is optimal in a broad context. The optimal contest involves two active firms regardless of the sponsor's objective—maximizing either total effort or the expected quality of the winning product—which can be achieved by the combination of entry fees and strategic allocation of productive resources (Propositions 1 to 4).

Second, entry fees play subtle roles that could accrue to the benefit of a contest sponsor. The sponsor can use entry fees to select the optimal set of entrants and also elicit revenue to fund the prize purse, which strengthens the incentives provided to competing firms. Further, we show that the sponsor would be encouraged to favor the more competent firm when allocating her resources if she is able to charge an entry fee, as the discussion in Section 3.2 demonstrates.

Third, the conventional wisdom of leveling the playing field may not universally hold for quality maximization. The sponsor has to strike a balance between the competition effect—which requires a level playing field—and allocative efficiency, which requires that the ex ante more competent firm be prioritized in resource allocation. Creating a level playing field creates more competition, but compromises allocative efficiency. As a result, a sponsor must be alerted to the risk of wasting scarcely available resources on less productive candidates and carefully examine the technological nature of the research project and profile of the contenders.

Fourth, Proposition 2 provides a guideline for allocating productive resources to competing firms. A national champion is more likely to emerge in the optimum when the contest pursues a more difficult or riskier project. For instance, compare a regular engineering solution that streamlines the production process—e.g., a project intensification project to achieve a higher-yield, more reliable pharmaceutical process—and a fundamental scientific discovery—e.g., a potentially revolutionary technology for cancer treatment. The former calls on more balanced resource allocation, while the latter should concentrate limited resources more on the industry leader. Echoing Boudreau, Lacetera, and Lakhani (2011), our paper emphasizes that the design of contests must take into account the nature of the research problem. Similarly, compare a project that serves a mature sector and one for a nascent industry: The former favors a level playing field, while the latter might require a frontrunner.

Further, more caution is required when large gap exists between firms in terms of competence. An even race can lose its appeal in the face of substantial initial asymmetry: Balancing the contest requires allocating even more resources to less productive firms to close a wider competence gap.

Sponsors must thoroughly assess the project's nature to devise an effective resource allocation plan.

Last, discriminatory entry fees afford a contest sponsor more flexibility to boost the performance of the contest. The sponsor can further prioritize the ex ante more competent firm in resource allocation to tap its superior productivity, while expropriating its surplus through a higher entry fee. Discriminatory entry fees can easily be implemented by individualized invitations, cash subsidies to selected firms, or rebates.

References

Ales, L., S.-H. Cho, and E. Körpeoğlu (2017): "Optimal award scheme in innovation tournaments," *Operations Research*, 65, 693–702.

——— (2021): "Innovation tournaments with multiple contributors," *Production and Operations Management*, 30, 1772–1784.

Azmat, G. and M. Möller (2009): "Competition among contests," *RAND Journal of Economics*, 40, 743–768.

——— (2018): "The distribution of talent across contests," *Economic Journal*, 128, 471–509.

Baye, M. R. and H. C. Hoppe (2003): "The strategic equivalence of rent-seeking, innovation, and patent-race games," *Games and Economic Behavior*, 44, 217–226.

Baye, M. R., D. Kovenock, and C. G. De Vries (1993): "Rigging the lobbying process: An application of the all-pay auction," *American Economic Review*, 83, 289–294.

Benkert, J.-M. and I. Letina (2020): "Designing dynamic research contests," *American Economic Journal: Microeconomics*, 12, 270–289.

Bimpikis, K., S. Ehsani, and M. Mostagir (2019): "Designing dynamic contests," *Operations Research*, 67, 339–356.

Bockstedt, J., C. Druehl, and A. Mishra (2022): "Incentives and stars: Competition in innovation contests with participant and submission visibility," *Production and Operations Management*, 31, 1372–1393.

Boosey, L., P. Brookins, and D. Ryvkin (2020): "Information disclosure in contests with endogenous entry: An experiment," *Management Science*, 66, 5128–5150.

Boudreau, K. J., N. Lacetera, and K. R. Lakhani (2011): "Incentives and problem uncertainty in innovation contests: An empirical analysis," *Management Science*, 57, 843–863.

Brown, J. (2011): "Quitters never win: The (adverse) incentive effects of competing with superstars," *Journal of Political Economy*, 119, 982–1013.

Che, Y.-K. and I. Gale (2003): "Optimal design of research contests," *American Economic Review*, 93, 646–671.

Chen, P.-Y., P. Pavlou, S. Wu, and Y. Yang (2021): "Attracting high-quality contestants to contest in the context of crowdsourcing contest platform," *Production and Operations Management*, 30, 1751–1771.

Deng, S., Q. Fu, and Z. Wu (2021): "Optimally biased Tullock contests," *Journal of Mathematical Economics*, 92, 10–21.

Drugov, M. and D. Ryvkin (2017): "Biased contests for symmetric players," *Games and Economic Behavior*, 103, 116–144.

Falck, O., C. Gollier, and L. Woessmann (2011): *Industrial Policy for National Champions*, MIT Press.

Franke, J., C. Kanzow, W. Leininger, and A. Schwartz (2013): "Effort maximization in asymmetric contest games with heterogeneous contestants," *Economic Theory*, 52, 589–630.

——— (2014): "Lottery versus all-pay auction contests: A revenue dominance theorem," *Games and Economic Behavior*, 83, 116–126.

Fu, Q., Q. Jiao, and J. Lu (2015): "Contests with endogenous entry," *International Journal of Game Theory*, 44, 387–424.

Fu, Q., J. Lu, and Y. Lu (2012): "Incentivizing R&D: Prize or subsidies?" *International Journal of Industrial Organization*, 30, 67–79.

Fu, Q. and Z. Wu (2020): "On the optimal design of biased contests," *Theoretical Economics*, 15, 1435–1470.

Fullerton, R. L. and R. P. McAfee (1999): "Auctioning entry into tournaments," *Journal of Political Economy*, 107, 573–605.

Gao, P., X. Fan, Y. Huang, and Y.-J. Chen (2022): "Resource allocation among competing innovators," *Management Science*, 68, 6059–6074.

Hammond, R. G., B. Liu, J. Lu, and Y. E. Riyanto (2019): "Enhancing effort supply with prize-augmenting entry fees: Theory and experiments," *International Economic Review*, 60, 1063–1096.

Kalra, A. and M. Shi (2001): "Designing optimal sales contests: A theoretical perspective," *Marketing Science*, 20, 170–193.

Körpeoğlu, E. and S.-H. Cho (2018): "Incentives in contests with heterogeneous solvers," *Management Science*, 64, 2709–2715.

Lemus, J. and G. Marshall (2021): "Dynamic tournament design: Evidence from prediction contests," *Journal of Political Economy*, 129, 383–420.

Letina, I., S. Liu, and N. Netzer (2023): "Optimal contest design: Tuning the heat," *Journal of Economic Theory*, 105616.

Letina, I. and A. Schmutzler (2019): "Inducing variety: A theory of innovation contests," *International Economic Review*, 60, 1757–1780.

Lichtenberg, F. R. (1990): "US government subsidies to private military R&D investment: The defense department's independent R&D policy," *Defence and Peace Economics*, 1, 149–158.

Mihm, J. and J. Schlapp (2019): "Sourcing innovation: On feedback in contests," *Management Science*, 65, 559–576.

Moldovanu, B. and A. Sela (2001): "The optimal allocation of prizes in contests," *American Economic Review*, 91, 542–558.

Moldovanu, B., A. Sela, and X. Shi (2012): "Carrots and sticks: Prizes and punishments in contests," *Economic Inquiry*, 50, 453–462.

Stouras, K. I., J. Hutchison-Krupat, and R. O. Chao (2022): "The role of participation in innovation contests," *Management Science*, 68, 4135–4150.

Taylor, C. R. (1995): "Digging for golden carrots: An analysis of research tournaments," *American Economic Review*, 85, 872–890.

Terwiesch, C. and K. T. Ulrich (2009): *Innovation Tournaments: Creating and Selecting Exceptional Opportunities*, Harvard Business Press.

Terwiesch, C. and Y. Xu (2008): "Innovation contests, open innovation, and multiagent problem solving," *Management Science*, 54, 1529–1543.

Appendix: Proofs

Proof of Proposition 1

Proof. With simple algebraic transformation, the first-order condition $\partial\pi_i(\boldsymbol{x}; V, \phi, \boldsymbol{\alpha})/\partial x_i = 0$ for a firm $i \in \mathcal{N}$ that chooses to exert a strictly positive effort can be expressed as follows:

$$x_i = rp_i(\boldsymbol{x})\left[1 - p_i(\boldsymbol{x})\right] \times \frac{V}{c_i}. \tag{9}$$

Note that the above condition continues to hold for a firm that opts out of the contest, since an inactive firm simply stands zero chance of winning.

Denote by $\mathcal{N}_+(\boldsymbol{p})$ and $k(\boldsymbol{p})$, respectively, the set and number of firms with strictly positive equilibrium winning probabilities:

$$\mathcal{N}_+(\boldsymbol{p}) := \left\{i = 1, \ldots, n \,|\, p_i > 0\right\} \tag{10}$$

and

$$k(\boldsymbol{p}) := \left|\mathcal{N}_+(\boldsymbol{p})\right|. \tag{11}$$

Simple algebra would verify the following lemma, which establishes a correspondence between firms' equilibrium winning probabilities $\boldsymbol{p} \equiv (p_1, \ldots, p_n)$ and the resource allocation profile $\boldsymbol{\alpha} \equiv (\alpha_1, \ldots, \alpha_n)$.

Lemma 1 *Consider a second-stage contest and ignore for now firms' participation constraints; or equivalently, consider a second-stage research contest, with $\phi = \gamma = 0$. Any profile of the equilibrium winning probabilities $\boldsymbol{p} \equiv (p_1, \ldots, p_n) \in \Delta^{n-1}$, with $p_i \neq 1$ for all $i \in \mathcal{N}$, can be induced by the following resource allocation profile $\boldsymbol{\alpha}(\boldsymbol{p}) \equiv \big(\alpha_1(\boldsymbol{p}), \ldots, \alpha_n(\boldsymbol{p})\big)$:*

$$\alpha_i(\boldsymbol{p}) = \begin{cases} \frac{c_i^r p_i^{1-r}}{(1-p_i)^r} \times \frac{1}{\eta(\boldsymbol{p})}, & \text{if } p_i > 0, \\ 0, & \text{if } p_i = 0, \end{cases}$$

where $\eta(\boldsymbol{p}) := \sum_{j \in \mathcal{N}_+(\boldsymbol{p})} \frac{c_j^r p_j^{1-r}}{(1-p_j)^r}$.

Lemma 1 enables us to reformulate the optimization problem and treat the distribution of equilibrium winning probabilities $\boldsymbol{p}$ as the design variable. Instead of searching for the optimal $(V, \phi, \boldsymbol{\alpha})$, the sponsor literally chooses $(V, \phi, \boldsymbol{p})$ to maximize Z^* as specified in (2), i.e.,

$$Z^* \equiv \sum_{i \in \mathcal{N}} x_i = \sum_{i \in \mathcal{N}} \left[rp_i(1 - p_i)\frac{V}{c_i}\right],$$

subject to the following constraints:

$$\sum_{i\in\mathcal{N}} p_i = 1, \text{ and } p_i \geq 0, \text{ for all } i \in \mathcal{N}, \tag{12}$$

$$\min_{i\in\mathcal{N}_+(\boldsymbol{p})} \Big\{ p_i \left[1 - r(1-p_i)\right] \times V \Big\} \geq \phi + \gamma, \tag{13}$$

and

$$V - k(\boldsymbol{p})\phi = b, \tag{14}$$

where $\mathcal{N}_+(\boldsymbol{p})$ and $k(\boldsymbol{p})$ are defined in (10) and (11), respectively. Constraint (12) simply requires that firms' winning probabilities be nonnegative and sum to one; (13) is the participation constraint for an active firm, which can be implied by active firms' first-order conditions (9); and (14) ensures budget balance, which requires that the prize be sufficiently funded by the sponsor's initial prize purse b and the revenues of entry fees, $k(\boldsymbol{p})\phi$, collected from the $k(\boldsymbol{p})$ entrants.

Note that constraint (13) must bind in the optimal R&D contest. The sponsor can otherwise increase V and ϕ simultaneously—while holding fixed $\boldsymbol{p} \equiv (p_1, \ldots, p_n)$—which improves her payoff without violating constraints (13) and (14). As a result, for a given profile of equilibrium winning probabilities $\boldsymbol{p} \equiv (p_1, \ldots, p_n)$ the sponsor intends to induce, she sets the entry fee such that

$$\phi = \min_{i\in\mathcal{N}_+(\boldsymbol{p})} \Big\{ p_i \left[1 - r(1-p_i)\right] \Big\} \times V - \gamma. \tag{15}$$

Combining (14) and (15) yields

$$V = \frac{b - k(\boldsymbol{p})\gamma}{1 - k(\boldsymbol{p}) \min_{i\in\mathcal{N}_+(\boldsymbol{p})} \Big\{ p_i \left[1 - r(1-p_i)\right] \Big\}}, \tag{16}$$

and

$$\phi = \frac{b \min_{i\in\mathcal{N}_+(\boldsymbol{p})} \Big\{ p_i \left[1 - r(1-p_i)\right] \Big\} - \gamma}{1 - k(\boldsymbol{p}) \min_{i\in\mathcal{N}_+(\boldsymbol{p})} \Big\{ p_i \left[1 - r(1-p_i)\right] \Big\}}. \tag{17}$$

For a given cost profile $\boldsymbol{c} \equiv (c_1, \ldots, c_n)$, the sponsor's optimization problem can be simplified as the following:

$$\max_{\boldsymbol{p}\in\Delta^{n-1},\, k(\boldsymbol{p})\geq 2} \mathcal{W}(\boldsymbol{p}, \boldsymbol{c}) := \left(\sum_{i\in\mathcal{N}} \frac{r p_i(1-p_i)}{c_i} \right) \times \frac{b - k(\boldsymbol{p})\gamma}{1 - k(\boldsymbol{p}) \min_{i\in\mathcal{N}_+(\boldsymbol{p})} \Big\{ p_i \left[1 - r(1-p_i)\right] \Big\}},$$

where Δ^{n-1} is an $(n-1)$-dimensional simplex as defined by (12).

To prove the proposition, it suffices to show that

$$\mathcal{W}(\boldsymbol{p}, \boldsymbol{c}) \leq \left(\frac{b}{2} - \gamma \right) \times \left(\frac{1}{c_1} + \frac{1}{c_2} \right), \tag{18}$$

with the equality holding if, and only if, $p_1 = p_2 = \frac{1}{2}$ and $p_3 = \cdots = p_n = 0$.

Note that

$$\begin{aligned}
\mathcal{W}(\boldsymbol{p}, \boldsymbol{c}) &\equiv \left(\sum_{i\in\mathcal{N}} \frac{rp_i(1-p_i)}{c_i}\right) \times \frac{b - k(\boldsymbol{p})\gamma}{1 - k(\boldsymbol{p}) \times \min_{i\in\mathcal{N}_+(\boldsymbol{p})} \Big\{p_i\big[1 - r(1-p_i)\big]\Big\}} \\
&\leq \left(\sum_{i\in\mathcal{N}} \frac{rp_i(1-p_i)}{c_i}\right) \times \frac{b - k(\boldsymbol{p})\gamma}{1 - \sum_{i\in\mathcal{N}_+(\boldsymbol{p})} \Big\{p_i\big[1 - r(1-p_i)\big]\Big\}} \\
&= \left(\sum_{i\in\mathcal{N}} \frac{p_i(1-p_i)}{c_i}\right) \times \frac{b - k(\boldsymbol{p})\gamma}{\sum_{i\in\mathcal{N}_+(\boldsymbol{p})} \big[p_i(1-p_i)\big]}.
\end{aligned} \tag{19}$$

Let $w_i := \frac{p_i(1-p_i)}{\sum_{j\in\mathcal{N}_+(\boldsymbol{p})}[p_j(1-p_j)]}$ for all $i \in \mathcal{N}$. It follows immediately that $\sum_{i\in\mathcal{N}} w_i = 1$ and

$$\begin{aligned}
w_1 = \frac{p_1(1-p_1)}{\sum_{j\in\mathcal{N}_+(\boldsymbol{p})}\big[p_j(1-p_j)\big]} &= \frac{p_1(1-p_1)}{p_1(1-p_1) + \sum_{j\in\mathcal{N}_+(\boldsymbol{p})\backslash\{1\}}\big[p_j(1-p_j)\big]} \\
&\leq \frac{p_1(1-p_1)}{p_1(1-p_1) + \sum_{j\in\mathcal{N}_+(\boldsymbol{p})\backslash\{1\}}\big(p_j p_1\big)} \\
&= \frac{p_1(1-p_1)}{2p_1(1-p_1)} = \frac{1}{2},
\end{aligned} \tag{20}$$

with the equality holding if, and only if, $k(\boldsymbol{p}) = 2$. Further, we have that

$$\begin{aligned}
\left(\sum_{i\in\mathcal{N}} \frac{p_i(1-p_i)}{c_i}\right) \times \frac{b - k(\boldsymbol{p})\gamma}{\sum_{i\in\mathcal{N}_+(\boldsymbol{p})} p_i(1-p_i)} &= \big[b - k(\boldsymbol{p})\gamma\big] \times \left(\frac{w_1}{c_1} + \sum_{i\in\mathcal{N}\backslash\{1\}} \frac{w_i}{c_i}\right) \\
&\leq \big[b - k(\boldsymbol{p})\gamma\big] \times \left(\frac{w_1}{c_1} + \sum_{i\in\mathcal{N}\backslash\{1\}} \frac{w_i}{c_2}\right) \\
&= \big[b - k(\boldsymbol{p})\gamma\big] \times \left(\frac{w_1}{c_1} + \frac{1-w_1}{c_2}\right) \\
&\leq \left(\frac{b}{2} - \gamma\right) \times \left(\frac{1}{c_1} + \frac{1}{c_2}\right),
\end{aligned} \tag{21}$$

where the first inequality follows from $c_2 \leq \cdots \leq c_n$; the second equality follows from $\sum_{i\in\mathcal{N}} w_i = 1$; and the second inequality follows from $k(\boldsymbol{p}) \geq 2$, $w_1 \leq \frac{1}{2}$, and $c_1 \leq c_2$.

Combining (19) and (21) yields (18), with the equality holding if, and only if, $p_1 = p_2 = \frac{1}{2}$ and $p_3 = \cdots = p_n = 0$. From $\boldsymbol{p}^* \equiv (p_1^*, \ldots, p_n^*) = (\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0)$ and (17), we can obtain the optimally designed entry fee as follows:

$$\phi = \frac{b(2-r) - 4\gamma}{2r},$$

which is positive if $b > \underline{b}^* \equiv \frac{4\gamma}{2-r}$. This concludes the proof. ∎

Proof of Proposition 2

Proof. By Lemma 1 and Equation (9), we can rewrite $Z^\star$ defined in (3) as the following:

$$Z^\star \equiv \sum_{i\in\mathcal{N}} \alpha_i \cdot x_i^r = \sum_{i\in\mathcal{N}} \left[\alpha_i p_i^r (1-p_i)^r \frac{(rV)^r}{c_i^r}\right] = \frac{(rV)^r}{\eta(\boldsymbol{p})}, \tag{22}$$

where $\eta(\boldsymbol{p})$ is defined in Lemma 1. Combining (16) and (22), the optimization problem can be simplified as follows:

$$\max_{\boldsymbol{p}\in\Delta^{n-1},\, k(\boldsymbol{p})\geq 2} \mathcal{M}(\boldsymbol{p},\boldsymbol{c}) := \frac{\left[b-k(\boldsymbol{p})\gamma\right]^r}{\left(\sum_{i\in\mathcal{N}_+(\boldsymbol{p})} \frac{c_i^r p_i^{1-r}}{(1-p_i)^r}\right)\left(1-k(\boldsymbol{p})\min_{i\in\mathcal{N}_+(\boldsymbol{p})}\left\{p_i\left[1-r(1-p_i)\right]\right\}\right)^r}. \tag{23}$$

From the rearrangement inequality, we can show that $p_1 \geq \cdots \geq p_n$ in the optimal research contest. Next, we show that $\mathcal{N}_+(\boldsymbol{p}) = \{1,2\}$. It is evident that $k(\boldsymbol{p}) \geq 2$ in the optimum, which in turn implies that $b - k\gamma \leq b - 2\gamma$.

Fixing an arbitrary equilibrium winning probability profile $\boldsymbol{p} = (p_1,\ldots,p_n)$, with $p_1 \geq \cdots \geq p_n$ and $p_3 > 0$, we construct $\boldsymbol{p}^\dagger := (p_1^\dagger,\ldots,p_n^\dagger)$ as follows:

$$p_i^\dagger = \begin{cases} \max\left\{p_1, 1/2\right\}, & \text{for } i=1, \\ \min\left\{1-p_1, 1/2\right\}, & \text{for } i=2, \\ 0, & \text{for } i\geq 3. \end{cases}$$

It is straightforward to verify that $p_1 \leq p_1^\dagger$, $p_2 < p_2^\dagger$, and $p_i \leq p_2^\dagger \leq 1/2 \leq p_1^\dagger$ for all $i \in \{3,\ldots,n\}$, from which we can obtain that

$$\frac{c_1^r p_1^{1-r}}{(1-p_1)^r} = p_1 \frac{c_1^r}{\left[p_1(1-p_1)\right]^r} \geq p_1 \frac{c_1^r}{\left[p_1^\dagger(1-p_1^\dagger)\right]^r} = p_1 \frac{c_1^r}{\left(p_1^\dagger p_2^\dagger\right)^r}, \tag{24}$$

$$\frac{c_2^r p_2^{1-r}}{(1-p_2)^r} = p_2 \frac{c_2^r}{\left[p_2(1-p_2)\right]^r} > p_2 \frac{c_2^r}{\left[p_2^\dagger(1-p_2^\dagger)\right]^r} = p_2 \frac{c_2^r}{\left(p_1^\dagger p_2^\dagger\right)^r}, \tag{25}$$

and

$$\frac{c_i^r p_i^{1-r}}{(1-p_i)^r} = p_i \frac{c_i^r}{\left[p_i(1-p_i)\right]^r} \geq p_i \frac{c_2^r}{\left[p_2^\dagger(1-p_2^\dagger)\right]^r} = p_i \frac{c_2^r}{\left(p_1^\dagger p_2^\dagger\right)^r}, \text{ for } i\geq 3. \tag{26}$$

Therefore, we have that

$$\begin{aligned}\sum_{i\in\mathcal{N}_+(\boldsymbol{p})} \frac{c_i^r p_i^{1-r}}{(1-p_i)^r} =& \frac{c_1^r p_1^{1-r}}{(1-p_1)^r} + \sum_{i\in\mathcal{N}_+(\boldsymbol{p})\setminus\{1\}} \frac{c_i^r p_i^{1-r}}{(1-p_i)^r} \\ >& p_1 \frac{c_1^r}{\left(p_1^\dagger p_2^\dagger\right)^r} + \sum_{i\in\mathcal{N}_+(\boldsymbol{p})\setminus\{1\}} p_i \frac{c_2^r}{\left(p_1^\dagger p_2^\dagger\right)^r}\end{aligned}$$

$$
\begin{aligned}
&= p_1 \frac{c_1^r}{\left(p_1^\dagger p_2^\dagger\right)^r} + (1-p_1)\frac{c_2^r}{\left(p_1^\dagger p_2^\dagger\right)^r} \\
&\geq p_1^\dagger \frac{c_1^r}{\left(p_1^\dagger p_2^\dagger\right)^r} + p_2^\dagger \frac{c_2^r}{\left(p_1^\dagger p_2^\dagger\right)^r} \\
&= \frac{c_1^r \left(p_1^\dagger\right)^{1-r}}{\left(p_2^\dagger\right)^r} + \frac{c_2^r \left(p_2^\dagger\right)^{1-r}}{\left(p_1^\dagger\right)^r} = \sum_{i\in\mathcal{N}_+(\boldsymbol{p}^\dagger)} \frac{c_i^r \left(p_i^\dagger\right)^{1-r}}{\left(1-p_i^\dagger\right)^r},
\end{aligned} \tag{27}
$$

where the first inequality follows from (24), (25), and (26) and the second inequality follows from $c_1 \leq c_2$ and $p_1 \leq p_1^\dagger$.

Next, note that $1 - p_{k(\boldsymbol{p})} \geq 1 - p_2^\dagger$ and

$$
\begin{aligned}
k(\boldsymbol{p})p_{k(\boldsymbol{p})} &\leq \min\left\{1, \frac{k(\boldsymbol{p})}{k(\boldsymbol{p})-1} \times \sum_{i\in\mathcal{N}\setminus\{1\}} p_i\right\} \\
&= \min\left\{1, \frac{k(\boldsymbol{p})}{k(\boldsymbol{p})-1}(1-p_1)\right\} \leq \min\left\{1, 2(1-p_1)\right\} = 2p_2^\dagger,
\end{aligned}
$$

from which we can conclude that

$$
\begin{aligned}
1 - k(\boldsymbol{p}) \times \min_{i\in\mathcal{N}_+(\boldsymbol{p})}\left\{p_i\left[1 - r(1-p_i)\right]\right\} &= 1 - k(\boldsymbol{p})p_{k(\boldsymbol{p})}\left[1 - r\left(1 - p_{k(\boldsymbol{p})}\right)\right] \\
&\geq 1 - 2p_2^\dagger\left[1 - r\left(1 - p_2^\dagger\right)\right] \\
&= 1 - k(\boldsymbol{p}^\dagger) \times \min_{i\in\mathcal{N}_+(\boldsymbol{p}^\dagger)}\left\{p_i^\dagger\left[1 - r\left(1 - p_i^\dagger\right)\right]\right\}.
\end{aligned} \tag{28}
$$

Combining (27) and (28) yields $\mathcal{M}(\boldsymbol{p}, \boldsymbol{c}) < \mathcal{M}(\boldsymbol{p}^\dagger, \boldsymbol{c})$, which implies that $\mathcal{N}_+(\boldsymbol{p}) = \{1, 2\}$ in the optimally designed contest. Therefore, the sponsor's optimization problem (23) boils down to

$$
\min_{p_1+p_2=1,\, p_1\geq p_2>0} \left(\frac{c_1^r p_1^{1-r}}{p_2^r} + \frac{c_2^r p_2^{1-r}}{p_1^r}\right) \times \left[1 - 2p_2(1-rp_1)\right]^r .
$$

Substituting the solution to the above optimization problem—which we denote by $(p_1^\star, p_2^\star)$—into (17), we can derive the corresponding entry fee as follows:

$$
\phi^\star = \frac{bp_2^\star\left(1 - rp_1^\star\right) - \gamma}{1 - 2p_2^\star\left(1 - rp_1^\star\right)}.
$$

The entry fee is positive if $b > \underline{b}^\star \equiv \frac{\gamma}{p_2^\star(1-rp_1^\star)}$. This concludes the proof. ■

Proof of Corollary 1

Proof. It is useful to state an intermediate result.

Lemma 2 *Consider the following optimization problem:*

$$\min_{p_1+p_2=1,\, p_1\geq p_2>0} \left(\frac{c_1^\dagger p_1^{1-r}}{p_2^r} + \frac{c_2^\dagger p_2^{1-r}}{p_1^r} \right),$$

where $c_i^\dagger := (c_i)^r$, *with* $i \in \{1, 2\}$. *Denote the solution by* $\widetilde{\boldsymbol{p}}^\star := (\widetilde{p}_1^\star, \widetilde{p}_2^\star)$ *and the corresponding resource allocation rule derived from Lemma 1 by* $\widetilde{\boldsymbol{\alpha}}^\star := (\widetilde{\alpha}_1^\star, \widetilde{\alpha}_2^\star)$. *Then* $\widetilde{\alpha}_1^\star \gtreqless \widetilde{\alpha}_2^\star$ *if and only if* $r \lesseqgtr \frac{1}{2}$.

Proof. Taking the logarithm of the objective function in the lemma yields

$$\psi(p_2, r) := \log \left(\frac{c_1^\dagger p_1^{1-r}}{p_2^r} + \frac{c_2^\dagger p_2^{1-r}}{p_1^r} \right) = \log \left(c_1^\dagger (1-p_2) + c_2^\dagger p_2 \right) - r \log \left(p_2(1-p_2) \right).$$

Carrying out the algebra, we can obtain that

$$\frac{\partial^2 \psi}{\partial p_2 \partial r} = -\frac{1-2p_2}{p_2(1-p_2)} < 0.$$

Therefore, $\psi(p_2, r)$ is submodular in (p_2, r). By Topkis's theorem, $\widetilde{p}_2^\star$ is increasing in r, which in turn implies that

$$\frac{\widetilde{\alpha}_1^\star}{\widetilde{\alpha}_2^\star} = \frac{(c_1)^r \left(\widetilde{p}_1^\star\right)^{1-r} / \left(1-\widetilde{p}_1^\star\right)^r}{(c_2)^r \left(\widetilde{p}_2^\star\right)^{1-r} / \left(1-\widetilde{p}_2^\star\right)^r} = \frac{c_1^\dagger \widetilde{p}_1^\star}{c_2^\dagger \widetilde{p}_2^\star} = \frac{c_1^\dagger \left(1-\widetilde{p}_2^\star\right)}{c_2^\dagger \widetilde{p}_2^\star}$$

is decreasing in r.

Therefore, to prove the lemma, it suffices to show that $\widetilde{\alpha}_1^\star = \widetilde{\alpha}_2^\star$ when $r = \frac{1}{2}$. In this case, the optimization problem can be written as

$$\min_{p_1+p_2=1,\, p_1\geq p_2>0} c_1^\dagger \sqrt{\frac{p_1}{p_2}} + c_2^\dagger \sqrt{\frac{p_2}{p_1}}.$$

From the AM-GM inequality, we have that

$$c_1^\dagger \sqrt{\frac{p_1}{p_2}} + c_2^\dagger \sqrt{\frac{p_2}{p_1}} \geq 2\sqrt{c_1^\dagger c_2^\dagger},$$

with the equality holding if, and only if, $c_1^\dagger \sqrt{\frac{\widetilde{p}_1^\star}{\widetilde{p}_2^\star}} = c_2^\dagger \sqrt{\frac{\widetilde{p}_2^\star}{\widetilde{p}_1^\star}}$, from which we can conclude that

$$\frac{\widetilde{\alpha}_1^\star}{\widetilde{\alpha}_2^\star} = \frac{c_1^\dagger \widetilde{p}_1^\star}{c_2^\dagger \widetilde{p}_2^\star} = 1.$$

This concludes the proof. ■

Now we can prove part (i) of the corollary. Consider the following function:

$$\zeta(p_2;\theta) := \log\left(\frac{c_1^\dagger p_1^{1-r}}{p_2^r} + \frac{c_2^\dagger p_2^{1-r}}{p_1^r}\right) + \theta r \log\big(1 - 2p_2(1 - rp_1)\big)\,, \text{ with } \theta \in [0,1].$$

It is evident that minimizing $\zeta(p_2;\theta)$ is equivalent to minimizing the objective function (5) when $\theta = 1$. Similarly, minimizing $\zeta(p_2;\theta)$ is equivalent to minimizing the objective function stated in Lemma 2 when $\theta = 0$.

Moreover, carrying out the algebra, we can obtain that

$$\frac{\partial^2 \zeta}{\partial p_2 \partial \theta} = \frac{2r^2\,(1-2p_2) - 2r}{1 - 2p_2\left[1 - r\,(1-p_2)\right]} < 0.$$

Therefore, $\zeta(p_2;\theta)$ is submodular in (p_2,θ). Again, by Topkis's theorem, we have that $p_2^\star > \widetilde{p}_2^\star$; together with Lemma 2, we can conclude that $\alpha_1^\star < \alpha_2^\star$ for $r \geq 1/2$.

Next, we prove part (ii) of the corollary. Recall $\alpha_2/\alpha_1 = (c_2^\dagger p_2)/(c_1^\dagger p_1)$. Define $c := c_2^\dagger/c_1^\dagger$ and $\alpha := \alpha_2/\alpha_1$. It follows immediately that $c > 1$ and $c = \alpha \times (p_1/p_2) \geq \alpha$. The logarithm of the objective function (5)—or equivalently, $\zeta(p_2;1)$—can be viewed as a function of α and expressed as

$$\eta(\alpha,c) := \zeta(p_2;1) = \log(1+\alpha) - \log(c+\alpha) + r\log\left(2r - \frac{\alpha}{c} + \frac{c}{\alpha}\right).$$

Fixing $c > 1$, the optimization problem stated in Proposition 2 boils down to one in which the sponsor chooses $\alpha \in (0,c]$ to minimize $\eta(\alpha,c)$.

The proof consists of three steps. In the first step, we show that $\alpha^\star := \alpha_2^\star/\alpha_1^\star$ is strictly decreasing in c. In the second step, we show that $\alpha^\star = c > 1$ when c is sufficiently close to 1. Last, we show that $\alpha^\star \leq 1$ when c is sufficiently large. All together, the three steps imply that there exists a threshold $\bar{c}$ such that $\alpha^\star \equiv \alpha_2^\star/\alpha_1^\star > 1$ if $c < \bar{c}$ and $\alpha^\star \equiv \alpha_2^\star/\alpha_1^\star < 1$ if $c > \bar{c}$.

Step I

For $\alpha > 0$, we can obtain that

$$\begin{aligned}\frac{\partial^2 \eta}{\partial \alpha \partial c} =& \frac{c^4(1-2r^2) + 4c^3\alpha r(2-r) - 2c^2\alpha^2(1-4r-2r^2) + 4c\alpha^3 r^2 + \alpha^4(1+2r^2)}{(c+\alpha)^2\left(c^2 + 2rc\alpha - \alpha^2\right)^2} \\ \geq& \frac{2c^2\alpha^2(\sqrt{1-4r^4} + 4r + 2r^2 - 1)}{(c+\alpha)^2\left(c^2+2rc\alpha-\alpha^2\right)^2} > 0,\end{aligned}$$

where the first inequality follows from the AM-GM inequality and the second inequality follows from $0 < r < 1/2$. Therefore, $\eta(\alpha,c)$ is supermodular in (α,c). By Topkis's theorem, $\alpha^\star$ is strictly decreasing in c.

Step II

Note that

$$\eta(\alpha, c) - \eta(c, c) = r \log\left(1 + \frac{(c+\alpha)(c-\alpha)}{2rc\alpha}\right) - \log\left(1 + \frac{(c-\alpha)(c-1)}{2c(\alpha+1)}\right).$$

Next, we show that $\eta(\alpha, c) > \eta(c, c)$ for all $\alpha \in (0, c)$ when $c < 2^r$. Consider the following two cases depending on $\frac{(c+\alpha)(c-\alpha)}{2rc\alpha}$ relative to 1.

Case I: $\frac{(c+\alpha)(c-\alpha)}{2rc\alpha} > 1$. Then we have that

$$\eta(\alpha, c) - \eta(c, c) > r \log 2 - \log\left(1 + \frac{(c-\alpha)(c-1)}{2c(\alpha+1)}\right) > r \log 2 - \log(c) > 0,$$

where the second inequality follows from the fact that $\frac{(c-\alpha)(c-1)}{2c(\alpha+1)} < \frac{c(c-1)}{2c} < c - 1$.

Case II: $\frac{(c+\alpha)(c-\alpha)}{2rc\alpha} \leq 1$. Then we have that

$$\begin{aligned}\eta(\alpha, c) - \eta(c, c) \geq & r \times \frac{(c+\alpha)(c-\alpha)}{4rc\alpha} - \log\left(1 + \frac{(c-\alpha)(c-1)}{2c(\alpha+1)}\right) \\ \geq & \frac{(c+\alpha)(c-\alpha)}{4c\alpha} - \frac{(c-\alpha)(c-1)}{2c(\alpha+1)} \\ = & \frac{\left[c + (3-c)\alpha + \alpha^2\right](c-\alpha)}{4c\alpha(\alpha+1)} > 0,\end{aligned}$$

where the first inequality follows from the fact that $\log(1+x) \geq \frac{x}{2}$ for every $x \in [0, 1]$; the second inequality follows from the fact that $\log(1+x) \geq x$ for every $x > 0$; and the third inequality follows from $c < 2^r < 3$.

Step III

Carrying out the algebra, we have that

$$\eta(\alpha, c) - \eta(1, c) = \log\left(1 + \frac{(c-1)(\alpha-1)}{2(c+\alpha)}\right) - r \log\left(1 + \frac{(\alpha-1)(\frac{c}{\alpha} + \frac{1}{c})}{2r + \frac{c}{\alpha} - \frac{\alpha}{c}}\right).$$

Next, we show that fixing $0 < r < 1/2$, $\eta(\alpha, c) - \eta(1, c) > 0$ for every $\alpha \in [1, c)$ when c is sufficiently large.

Note that fixing r, there exists a threshold δ such that $\log(1 + \frac{rx}{3}) > r \log(1+x)$ for every $x > \delta$. Consider the following two cases depending on α relative to 2δ.

Case I: $\alpha < 2\delta$. Then we have that

$$\begin{aligned}\eta(\alpha, c) - \eta(1, c) = & \log\left(1 + \frac{(c-1)(\alpha-1)}{2(c+\alpha)}\right) - r \log\left(1 + \frac{(\alpha-1)(\frac{c}{\alpha} + \frac{1}{c})}{2r + \frac{c}{\alpha} - \frac{\alpha}{c}}\right) \\ > & \log\left(1 + \frac{(c-1)(\alpha-1)}{2(c+\alpha)}\right) - \log\left(1 + \frac{r(\alpha-1)(\frac{c}{\alpha} + \frac{1}{c})}{2r + \frac{c}{\alpha} - \frac{\alpha}{c}}\right)\end{aligned}$$

$$
\begin{aligned}
&= \log\left(1+\frac{(c-1)(\alpha-1)}{2(c+\alpha)}\right) - \log\left(1+\frac{r(\alpha-1)(c+\frac{\alpha}{c})}{2\alpha r+c-\frac{\alpha^2}{c}}\right)\\
&> \log\left(1+\frac{(c-1)(\alpha-1)}{2(c+2\delta)}\right) - \log\left(1+\frac{r(\alpha-1)(c+\frac{2\delta}{c})}{2r+c-\frac{4\delta^2}{c}}\right),
\end{aligned}
$$

where the first inequality follows from Bernoulli's inequality, and the second inequality follows from $1 \le \alpha < 2\delta$.

Next, note that

$$
\lim_{c\to\infty} \frac{(c-1)}{2(c+2\delta)} = \frac{1}{2} > r = \lim_{c\to\infty} \frac{r(c+\frac{2\delta}{c})}{2r+c-\frac{4\delta^2}{c}}.
$$

Therefore, there exists $\underline{c}_1$ such that

$$
\frac{(c-1)}{2(c+2\delta)} > \frac{r(c+\frac{2\delta}{c})}{2r+c-\frac{4\delta^2}{c}}, \text{ for every } c > \underline{c}_1,
$$

which in turn implies that $\eta(\alpha, c) > \eta(1, c)$ for every $c > \underline{c}_1$.

Case II: $\boldsymbol{\alpha \ge 2\delta}$. Recall that $\log(1+\frac{rx}{3}) > r\log(1+x)$ for every $x > \delta$. Further, we have that $\frac{(\frac{c}{\alpha}+\frac{1}{c})}{2r+\frac{c}{\alpha}-\frac{\alpha}{c}} > \frac{\frac{c}{\alpha}}{1+\frac{c}{\alpha}} > \frac{1}{2}$, which in turn implies that

$$
1+\frac{(\alpha-1)(\frac{c}{\alpha}+\frac{1}{c})}{2r+\frac{c}{\alpha}-\frac{\alpha}{c}} > 1+\frac{\alpha-1}{2} > \delta.
$$

Therefore, we can obtain that

$$
\begin{aligned}
\eta(\alpha, c) - \eta(1, c) &= \log\left(1+\frac{(c-1)(\alpha-1)}{2(c+\alpha)}\right) - r\log\left(1+\frac{(\alpha-1)(\frac{c}{\alpha}+\frac{1}{c})}{2r+\frac{c}{\alpha}-\frac{\alpha}{c}}\right)\\
&> \log\left(1+\frac{(c-1)(\alpha-1)}{2(c+\alpha)}\right) - \log\left(1+\frac{r(\alpha-1)(\frac{c}{\alpha}+\frac{1}{c})}{3(2r+\frac{c}{\alpha}-\frac{\alpha}{c})}\right).
\end{aligned}
$$

It suffices to show that

$$
\frac{r(\frac{c}{\alpha}+\frac{1}{c})}{3(2r+\frac{c}{\alpha}-\frac{\alpha}{c})} < \frac{c-1}{2(c+\alpha)},
$$

for every $\alpha \in [2\delta, c)$ when c is sufficiently large. Note that

$$
\frac{\frac{c}{\alpha}+\frac{1}{c}}{2r+\frac{c}{\alpha}-\frac{\alpha}{c}} - \frac{1+\frac{1}{c}}{2r} = -\frac{(c-\alpha)\left[(1+\frac{1}{c})(1+\frac{\alpha}{c})-2r\right]}{2r\alpha\left(2r+\frac{c}{\alpha}-\frac{\alpha}{c}\right)} < 0. \tag{29}
$$

Therefore, we can obtain that

$$
\frac{r(\frac{c}{\alpha}+\frac{1}{c})}{3\left(2r+\frac{c}{\alpha}-\frac{\alpha}{c}\right)} < \frac{1+\frac{1}{c}}{6} < \frac{c-1}{4c} < \frac{c-1}{2\left(c+\alpha\right)},
$$

where the first inequality follows from (29); the second inequality holds for $c > 5$; and the third inequality follows from $\alpha < c$ and $c > 1$.

In summary, if $c > \max\{\underline{c}_1, 5\}$, then $\eta(\alpha, c) - \eta(1, c) > 0$ for each $\alpha \in (1, c)$, which in turn implies that $\alpha^\star \leq 1$. This completes the proof. ■

Proof of Proposition 3

Proof. Similar to the analysis in the proof of Proposition 1, the optimal contest design problem can be reformulated as follows: The sponsor chooses $(V, \boldsymbol{\phi}, \boldsymbol{p})$ to maximize (2), subject to constraints (12),

$$p_i\left[1 - r(1 - p_i)\right] \times V \geq \phi_i + \gamma, \text{ for all } i \in \mathcal{N}_+(\boldsymbol{p}), \tag{30}$$

and

$$V - \sum\nolimits_{i \in \mathcal{N}_+(\boldsymbol{p})} \phi_i = b. \tag{31}$$

Constraint (30) provides participation constraints for firms. Constraint (31) requires a binding budget constraint. The optimization problem can be further simplified as follows:

$$\max_{\boldsymbol{p} \in \Delta^{n-1},\, k(\boldsymbol{p}) \geq 2} \left(\sum_{i \in \mathcal{N}} \frac{p_i(1 - p_i)}{c_i}\right) \times \frac{b - k(\boldsymbol{p})\gamma}{\sum_{i \in \mathcal{N}_+(\boldsymbol{p})} \left\{p_i(1 - p_i)\right\}}.$$

Denote by $\widehat{\boldsymbol{p}}^* \equiv (\widehat{p}_1^*, \ldots, \widehat{p}_n^*)$ the equilibrium winning probabilities in the optimum with discriminatory entry fees. Recall that we have shown (21) in the proof of Proposition 1, from which we can conclude that the maximum can be reached by an arbitrary profile of equilibrium winning probabilities $\boldsymbol{p} \in \Delta^{n-1}$ such that $\mathcal{N}_+(\boldsymbol{p}) = \{1, 2\}$. Therefore, we must have that $\mathcal{N}_+(\widehat{\boldsymbol{p}}^*) = \{1, 2\}$.

Next, we solve for $\widehat{V}^*$ and $\widehat{\boldsymbol{\phi}}^*$. Because (30) must bind for all active firms, we have that

$$\widehat{\phi}_i^* = \widehat{p}_i^*\left[1 - r(1 - \widehat{p}_i^*)\right] \times \widehat{V}^* - \gamma, \text{ for } i \in \{1, 2\}. \tag{32}$$

Plugging (32) into (31) yields that

$$\widehat{V}^* = b + \left[\widehat{p}_1^*(1 - r\widehat{p}_2^*) \times \widehat{V}^* - \gamma\right] + \left[\widehat{p}_2^*(1 - r\widehat{p}_1^*) \times \widehat{V}^* - \gamma\right],$$

which in turn implies that

$$\widehat{V}^* = \frac{b - 2\gamma}{1 - \widehat{p}_1^*(1 - r\widehat{p}_2^*) - \widehat{p}_2^*(1 - r\widehat{p}_1^*)} = \frac{b - 2\gamma}{2r\widehat{p}_1^*\widehat{p}_2^*}. \tag{33}$$

Substituting (33) into (32) yields that

$$(\widehat{\phi}_1^*, \widehat{\phi}_2^*) = \left(\widehat{p}_1^*(1 - r\widehat{p}_2^*) \times \widehat{V}^* - \gamma, \widehat{p}_2^*(1 - r\widehat{p}_1^*) \times \widehat{V}^* - \gamma\right)$$

$$= \left(\frac{b(1 - r\widehat{p}_2^*) - 2\gamma}{2r\widehat{p}_2^*}, \frac{b(1 - r\widehat{p}_1^*) - 2\gamma}{2r\widehat{p}_1^*} \right).$$

It is straightforward to verify that there exists at least one tuple $(\widehat{p}_1^*, \widehat{p}_2^*)$, with $\widehat{p}_1^* \geq 0$, $\widehat{p}_2^* \geq 0$, and $\widehat{p}_1^* + \widehat{p}_2^* = 1$, such that $\widehat{\phi}_1^* \geq 0$ and $\widehat{\phi}_2^* \geq 0$ if $b > \underline{b}^* \equiv \frac{4\gamma}{2-r}$. This concludes the proof. ■

Proof of Proposition 4

Proof. The optimization problem can be simplified as follows:

$$\max_{\boldsymbol{p} \in \Delta^{n-1}, k(\boldsymbol{p}) \geq 2} \frac{b - k(\boldsymbol{p})\gamma}{\left(\sum_{i \in \mathcal{N}_+(\boldsymbol{p})} \frac{c_i^r p_i^{1-r}}{(1-p_i)^r} \right) \left\{ 1 - \sum_{i \in \mathcal{N}_+(\boldsymbol{p})} p_i \left[1 - r(1 - p_i) \right] \right\}^r}. \tag{34}$$

By the same argument as in establishing (20), we have that

$$1 - \sum_{i \in \mathcal{N}_+(\boldsymbol{p})} p_i \left[1 - r(1 - p_i) \right] = r \times \sum_{i \in \mathcal{N}_+(\boldsymbol{p})} \left[p_i (1 - p_i) \right] \geq 2p_1(1 - p_1)r.$$

Denote by $\widehat{\boldsymbol{p}}^\star \equiv (\widehat{p}_1^\star, \ldots, \widehat{p}_n^\star)$ the equilibrium winning probabilities in the quality-maximizing research contest with discriminatory entry fees. The above inequality, together with (27), implies that $\mathcal{N}_+(\widehat{\boldsymbol{p}}^\star) = \{1, 2\}$. The objective (34) can then be simplified as

$$\frac{b - 2\gamma}{(2r)^r \times \left[c_1^r p_1 + c_2^r (1 - p_1) \right]},$$

which strictly increases with p_1 if $c_1 < c_2$ and remains constant if $c_1 = c_2$. In the case in which $c_1 < c_2$, the constraint $\widehat{\phi}_2^\star \geq 0$ must bind, which gives (8) when $\frac{1}{r} - \frac{2\gamma}{rb} < 1$, or equivalently, when $b < \frac{2\gamma}{1-r}$. In the case in which $c_1 = c_2$, it is evident that any tuple (p_1, p_2) can achieve the maximum, given that entry fees for the two firms are nonnegative, and the analysis closely follows that of Proposition 3. This concludes the proof. ■

Proof of Corollary 2

Proof. It suffices to show that $p_1^\star < \widehat{p}_1^\star$. By (8), we have that $\widehat{p}_1^\star = \frac{1}{r} - \frac{2\gamma}{rb}$. By Assumption 1, (4) and (6), we can obtain that $\widehat{p}_1^\star > 1/2$ and $bp_2^\star(1 - rp_1^\star) \geq \gamma$. For the case in which $p_2^\star = \frac{1}{2}$, we have that $\widehat{p}_1^\star > \frac{1}{2} = p_1^\star$. For the case in which $p_2^\star < \frac{1}{2}$, we have that $\frac{b}{2}(1 - rp_1^\star) > bp_2^\star(1 - rp_1^\star) \geq \gamma$, which in turn implies that $p_1^\star < \frac{1}{r} - \frac{2\gamma}{rb} = \widehat{p}_1^\star$. This concludes the proof. ■