Disclosure and Favoritism in Sequential Elimination Contests*

Qiang Fu Zenan Wu

March 14, 2021

*Fu: Department of Strategy and Policy, National University of Singapore, 15 Kent Ridge Drive, Singapore, 119245, bizfq@nus.edu.sg. Wu: Corresponding author. School of Economics, Peking University, Beijing, China, 100871, zenan@pku.edu.cn. We are grateful to Coeditor Leslie Marx and two anonymous referees for detailed comments that significantly improved the article. We thank Masaki Aoyagi, Parimal Bag, Stefano Barbieri, Zhijun Chen, Luis Corchón, Hanming Fang, Oliver Gürtler, Hongyi Li, Jianpei Li, Jin Li, Jingfeng Lu, Kai Konrad, Arijit Mookherjee, Paul Schweinzer, Marco Serena, Ruqu Wang, Junjie Zhou, and seminar/conference participants at the National University of Singapore, University of International Business and Economics, Harbin Institute of Technology (Shenzhen), the 4th Annual Conference on Contest: Theory and Evidence, the 2018 China Meeting of the Econometric Society, the 9th Annual ZJU International Conference on Industrial Economics, and the 2020 Chinese Information Economics Association Annual Meeting for helpful discussions, suggestions, and comments. We thank Shanglyu Deng for excellent research assistance. Fu thanks the financial support from the Singapore Ministry of Education Tier-1 Academic Research Fund (R-313-000-139-115). Wu thanks the National Natural Science Foundation of China (No. 71803003) and the seed fund of the School of Economics, Peking University, for financial support. Any remaining errors are our own.

Abstract

We consider a two-stage contest, in which only a subset of contestants enters the finale. We explore the optimal policy for disclosing contestants' interim status after the preliminary round, i.e., their interim ranking and elimination decision. The optimum depends on the design objective. We fully characterize the conditions under which disclosure or concealment emerges as the optimum. We further allow the organizer to bias the competition based on finalists' interim rankings, which endogenizes the dynamic structure of the contest. Concealment outperforms in generating total effort, while disclosure prevails when maximizing the expected winner's total effort.

Keywords: Elimination Contest; Disclosure Scheme; Optimal Bias; Contest Design

JEL Classification Codes: C72; L22

1 Introduction

Many contest-like competitive events proceed through multiple phases. Ultimate success demands on more than a single stroke of effort; instead, it requires continuous input. Two phenomena are widespread in these scenarios. First, contenders are often eliminated successively along a hierarchical ladder. Consider, for instance, the bid for the 2021 Summer Olympics: Six cities submitted proposals and three were slated for candidacy. In the final round, Tokyo beat Madrid and Istanbul as the grand winner. Organizational hierarchies provide another close analogy (Rosen, 1986). Twenty-three candidates were initially considered in the race to succeed Jack Welch at GE, and eight survived the first round of screening. The slate was further narrowed to three, and Jeff Immelt was the chosen successor. Various architectural competitions and grant calls are similarly organized.

Second, interim outcomes in the series may generate spillover to surviving contenders' relative standing in future stages. In track events, for instance, the best performers in qualification rounds are assigned middle lanes in heats and finals. This assignment rule rewards early performance, because middle lanes allow the runners to easily observe their competitors. In the CEO succession process, firms often appoint their leading candidate president or chief operating officer (COO). The candidate thus obtains an advantage in securing resources and developing managerial skills. Alternatively, in a grant call, an applicant may receive more detailed and constructive advice on revising his proposal when reviewers deem the submission to be more promising, which enables more productive revision and more careful and serious review after resubmission. The differentiated treatment given to contenders—which depends on their early performance and biases the subsequent competition—is often deliberately chosen by the administrators of the competitive events.

These observations compel us to explore two questions for optimal contest design in a two-stage sequential-elimination contest: (i) the optimal disclosure of contestants' interim status—i.e., whether one still remains in the race or has been eliminated and/or how he has been ranked relative to the others—prior to late-stage competition; and (ii) the optimal biases imposed on late-stage contestants based on their interim ranking.

Whether to disclose the interim outcome of a sequential elimination contest has subtle effects on contestants' incentives, because the information allows contestants to assess their winning odds more precisely. This ultimately determines the perceived marginal return of efforts. Previous studies of elimination contests have traditionally assumed that contestants are perfectly informed of where they stand before sinking their bids (e.g., Gradstein and Konrad, 1999; Moldovanu and Sela, 2006; and Fu and Lu, 2012a). We first consider a simple baseline model, with $N \geq 3$ symmetric contestants participating in the first stage, $M < N$ surviving to the second stage, and one being picked as the ultimate winner. An organizer either discloses the interim elimination decision to contestants or conceals it; she decides on the disclosure scheme and announces it publicly prior to the competition. The choice of biases (differential treatments) imposed on finalists is abstracted away.

Suppose that the announcement is withheld. A contestant, in a late stage, is prevented from learning his status. The uncertainty deceives actual losers into continuing their effort supply, while attenuating the incentive of actual survivors. In turn, this uncertainty in the late stage affects contestants' early incentives to bid for advancement. The optimum must address all of these concerns. The trade-off can be witnessed in the debate regarding firms' succession planning. Despite the plethora of high-profile CEO succession horse races that unfold in the spotlight—e.g., those at GE, P&G, Johnson & Johnson and, recently, MetLife[1]—succession in many organizations takes place quietly: The preference has been to keep the process secret simply to "avoid sapping the motivation of those who aren't on the fast track" (Conger and Fulmer, 2003).[2]

Opacity causes fundamental changes in the nature of strategic interactions in the competition. Contestants are uninformed of their status, which forces them to remain active throughout the contest. Despite the temporal sequence, the dynamic strategic linkage between stages dissolves, and contestants behave as if they commit to their first- and second-stage efforts altogether. The dynamic competition is converted into a (strategically equivalent) static contest that requires efforts in two dimensions, and renders early and late efforts complementary to each other: A unilateral increase in one's second-stage effort improves his conditional probability of winning the prize, which in turn amplifies the marginal return to early effort and encourages early bidding for advancement. In contrast, under transparency, late effort is chosen contingently only after learning one's survival; a contestant, when making his early effort decision, would factor in the future rent dissipation caused by his (contingent) late effort, which, recursively, diminishes his continuation value in the dynamics and discourages his early bidding. The sharp differences caused by disclosure schemes yield interesting and subtle implications for contest design.

We show, in the baseline setting, that the optimal disclosure scheme depends on the organizer's objective. Two alternatives are considered: (i) maximization of total effort in the overall contest and (ii) maximization of the expected winner's total effort along the ladder—i.e., the sum of the winner's efforts in the early and late stages.[3] Opacity retains a larger number of active contestants in the late stage, while leading each to bid less than a surviving contestant would under transparency. Moreover, each contestant bids less in the early stage

[1]Eli Lilly & Co. even allows its employees to access their rankings in the succession planning system.

[2]The implication of our analysis should be interpreted with caution for the specific context of CEO succession races. Our model primarily focuses on a moral hazard problem and the incentive effect of a disclosure scheme. However, firms may select CEOs based on alternative criteria other than efforts or performance—e.g., leadership style, match with the corporate culture, etc.

[3]In a principal-agent setting, Levitt (1995) shows that the optimal incentive scheme depends on whether the principal's payoff is determined by all agents' total output or the best output.

under transparency: As stated above, anticipated future (contingent) effort discounts the marginal return to contestants' early efforts. The optimal information disclosure scheme thus hinges on the three-way trade-off. We show that opacity leads to higher total effort in the overall contest.

The answer is less than explicit when the organizer maximizes the expected winner's (individual) total effort. A tension arises for each contestant: One bids less in the early stage under transparency than under opacity, while he bids more once he manages to enter the finale. A sufficient and necessary condition is provided to characterize the optimum, which depends sensitively on the structure of the contest and the contest technology. First, transparency is more likely to prevail when the contest is noisier—i.e., when a higher effort is less likely to lead to a win. Second, transparency is more likely to prevail when the contest involves a large number of participants but admits a smaller number of finalists—i.e., when the contest involves a tougher elimination process.

We further generalize the model to allow the organizer to bias the late-stage competition. She assigns a weight to each contestant's second-stage effort entry based on his interim ranking. This flexibility creates asymmetry among ex ante symmetric contestants. The baseline model can be viewed as a special case, since outright elimination is equivalent to assigning an excessively small weight, by which the contestant's winning odds are reduced to zero. The organizer announces the disclosure scheme and the rank-based weighting rule altogether. Under transparency, contestants are informed not only of their status in the competition, but also the weight assigned to them in the second-stage competition. Under opacity, contestants learn neither. The aforementioned key appointment for the frontrunner in a CEO succession race conspicuously announces candidates' status. However, differentiated treatment can be exercised in quieter and subtler ways. For instance, a preferred candidate may not be appointed to a key position explicitly, but can arguably receive more attention, support, and mentorship in the evaluation and development process.[4] In a grant call, reviewers' and judging panels' preferences are typically kept secret from applicants.[5]

We characterize the optimal rank-based bias rule for a two-stage contest under both transparency and opacity. First, the comparison between transparency and opacity is straightforward when the contest can be biased: Opacity generates higher total effort for the overall contest, while transparency leads to a higher expected winner's total effort. Second, asymmetric competition always arises in the finale, and the optimal contest rewards superior early performance by assigning larger weights for higher ranks. Subtle trade-offs arise in the

[4]For instance, incumbent CEOs and senior board members at GE and Xerox have been known to expend effort on grooming leading candidates. Anne M. Mulcahy, the former CEO of IdeaXerox, devoted a decade to grooming Ursula Burns as her preferred successor (see Mulcahy, 2010).

[5] See, e.g., the Australian Research Council's Discovery Program (`https://www.grants.gov.au/?event=public.FO.show&FOUUID=8194D407-CEA7-D8C5-793FD73E8CCFC0BD`), which does not disclose applicants' ranking when inviting an applicant to respond to external reviewers' assessment of their proposal.

dynamic setting when the organizer imposes biases on the contest, which will be detailed in Section 4.

The rest of the paper proceeds as follows. In the remainder of this section, we provide a brief review of the relevant literature. We next set up the baseline model in Section 2 and conduct the analysis in Section 3. In Section 4, we allow the organizer to impose identity-dependent treatment on the finalists based on their interim rankings. In Section 5, we discuss extensions of the model. In Section 6, we conclude. Equilibrium characterization of the baseline model and proofs of our main results are collected in Appendices A and B, respectively.

Contributions and Link to the Literature Our contributions to the contest literature are twofold. To the best of our knowledge, we are the first to explore the natural question of the optimal disclosure scheme in sequential-elimination contests. We identify the contexts in which transparency or opacity may emerge as the optimum. In addition, our analysis in the extended model contributes to the long-standing discussion of optimally biased contests in a dynamic setting.

The literature on sequential-elimination contests dates back to Rosen (1986). Rosen (1986), Gradstein and Konrad (1999), Moldovanu and Sela (2006), and Brown and Minor (2014) all assume that in the preliminary stage of an elimination contest, contestants are split into multiple subcontests; one survives in each subcontest and advances to the next stage, and all survivors are split and matched for the next round of elimination. In contrast, Fu and Lu (2012a) consider a "pooling" approach by which all remaining contestants compete in each stage. Fu and Lu adopt multi-winner nested Tullock contests, because the competition in each stage is inherently a multi-prize contest. We follow Fu and Lu's modeling approach in a two-stage setting, because the pooling structure more closely fits the hierarchical competitions we study in this paper—e.g., grant calls, bids for the Olympics, succession races, etc. Arve and Chiappinelli (2019) study a three-player two-stage elimination contest and assume that one player is subject to a budget constraint. All of these studies assume that the elimination decision is publicly known, while in our study its observability is the primary design variable. Zhang and Wang (2009) study a two-stage contest, in which four players are matched into two paired competitions in the preliminary, with the two winners advancing to the finale; an organizer decides whether to disclose contestants' early bids. In contrast to our study, they assume private types and focus on contestants' strategic inference of opponents' types, while keeping the elimination decision transparent and the hierarchical structure fixed.

Our paper is conceptually linked to the literature on interim feedback and information disclosure in dynamic contests (Gershkov and Perry, 2009; Aoyagi, 2010; Ederer, 2010; Gürtler and Harbring, 2010; Goltsman and Mukherjee, 2011), but stands in sharp contrast in terms of both focus and model settings. These studies typically consider two-player two-period contests, with the winner being determined by players' overall performance summed up over the two periods; they then explore whether a contest organizer should reveal intermediate performance. In contrast, we focus on disclosure of the elimination decision and the biases that depend on interim rankings; this naturally requires a model that involves sequential elimination and three or more players.[6]

An extensive literature examines the optimal biases in contests that manipulate the balance of the competition. This includes Che and Gale (2003); Konrad (2002); Nti (2004); Fu (2006); Franke (2012); Franke, Kanzow, Leininger, and Schwartz (2013, 2014); Kirkegaard (2012); and Fu and Wu (2020), among others.[7] All of these studies occur in static contest settings, while ours focuses on hierarchical competitions in dynamics. In a best-of-three contest between two ex ante identical contestants, Barbieri and Serena (2018) show that the contest organizer can bias the competition to improve its performance. Meyer (1992) considers a two-period contest between two individual contestants, and shows that it is optimal to set a bias in favor of the interim frontrunner.[8] These studies assume full disclosure of interim outcomes, while the disclosure scheme is the primary interest of our study. In addition, their settings do not involve elimination in hierarchy. Cohen, Maor, and Sela (2018) study a two-stage elimination contest similar to ours, in which the organizer can choose to favor the top-ranked finalist. They focus on optimal favoritism, but assume that contestants are fully informed of the interim outcome.

2 Baseline Model

There are $N \geq 3$ risk-neutral identical contestants involved in a two-stage contest. In the first stage, all contestants participate, and $M \in \{2, \ldots, N-1\}$ avoid elimination and are qualified for the finale.[9] In the second stage, a winner is selected from the M qualified finalists as the recipient of a prize $\mathcal{V} > 0$. Without loss of generality, we normalize $\mathcal{V} = 1$.

[6] Lizzeri, Meyer, and Persico (2002) consider the optimal interim performance feedback policy in a dynamic moral hazard model, in which the agent cannot observe the outcome of his effort. The principal decides whether to disclose the agent's performance before he chooses his second-period effort. Lizzeri et al. consider a setting with a single agent, while we consider competitions among multiple players.

[7] The majority of these studies embrace the notion of leveling the playing field, but a handful of studies identify the contexts in which this conventional wisdom fails. See, for instance, Chen (2016); Drugov and Ryvkin (2017); and Fu and Wu (2020).

[8] The majority of these studies assume that biases are set strategically to address the moral hazard problem. In contrast, Meyer (1991) views a contest as a bounded-rational learning process, and shows that biases could facilitate learning.

[9] The game effectively reduces to a static winner-take-all contest when $M = 1$ or N. We discuss this possibility in Section 5.2.2.

2.1 Winner-selection Mechanism

We adopt a multi-winner nested Tullock contest (Clark and Riis, 1996, 1998) to model the prevailing winner-selection mechanism. Let $\mathcal{C}(\overline{N}, \underline{N})$ denote a static contest with $\overline{N} \geq 2$ participants and $\underline{N} \in \{1, \ldots, \overline{N}\}$ prizes to be given away. Clark and Riis (1996, 1998) conveniently depict the selection mechanism as a sequential lottery process that consists of a series of independent draws. Let contestants be indexed by $i \in \{1, \ldots, \overline{N}\}$. They simultaneously submit effort entry $e^i \geq 0$. Once a contestant is picked as the recipient of a prize, he is immediately removed from the pool of contestants eligible for the rest of the prizes, since each contestant is eligible for at most one prize. Let Ω^m be the set of contestants who remain eligible for the mth-draw and $\boldsymbol{e}^{\Omega^m}$ be the effort profile of all contestants in the set Ω^m, with $m \in \{1, \ldots, \underline{N}\}$. Then the probability of a contestant i's receiving the mth prize *conditional* on him not having been picked in the previous $m-1$ draws is given by

$$p_m^i\left(\boldsymbol{e}^{\Omega^m}; \Omega^m\right) = \begin{cases} \dfrac{(e^i)^r}{\sum_{j \in \Omega^m}(e^j)^r}, & \text{if } \boldsymbol{e}^{\Omega^m} \neq (0, \ldots, 0), \\ \dfrac{1}{\overline{N} - m + 1}, & \text{if } \boldsymbol{e}^{\Omega^m} = (0, \ldots, 0), \end{cases} \tag{1}$$

where r indicates the discriminatory power of the contest technology. The process continues until all $\underline{N}$ winners have been drawn. The usual winner-take-all Tullock contest is a special case of the model, with $\underline{N} = 1$.

Despite the convenient analogy to a sequential lottery process, Fu and Lu (2012b) show that the multi-winner nested Tullock contest is uniquely underpinned by a noisy ranking tournament with an additive noise following a type-I extreme-value distribution. Lu and Wang (2015) further provide an axiomatic foundation for the model. Fu, Wu, and Zhu (2020) formally establish equilibrium existence and uniqueness in this contest game under plausible conditions.

2.2 A Two-stage Elimination Contest

We consider symmetric competitions among homogeneous contestants.[10] As stated above, the contest proceeds in two stages. In each stage, contestants simultaneously submit their

[10] We assume homogeneous contestants for the sake of tractability. The first-stage competition is modeled as a multi-prize contest, which allows for tractable analysis only in symmetric settings. Our model thus depicts contexts in which contestants do not differ substantially from each other in terms of abilities. Consider, for instance, high-profile architectural competitions in which only prestigious design firms can survive prescreening. Alternatively, imagine the succession races in major listed companies: All shortlisted candidates in advanced stages are slated from a large talent pool, and the variance in their abilities is presumably smaller than that in the entire organization. For instance, consider the succession race at GE to replace Jack Welch. According to Welch, he was prompted by his "nose and gut" to choose Jeff Immelt because all of the candidates were "equally capable" (Welch and Byrne, 2003).

effort entries and the effort incurs a unity marginal cost.[11] The first stage selects M finalists and eliminates the rest, and one of the finalists is selected in the second-stage competition to receive a single prize of value $\mathcal{V} = 1$. The contest mechanism described in Section 2.1 is adopted for winner selection. That is, the competition in the elimination stage is a multi-winner nested Tullock contest with $\overline{N} = N$ participants; $\underline{N} = M$ winners receive an *identical* prize that entitles them to advance. The finale is a winner-take-all Tullock contest among the M survivors, i.e., $\overline{N} = M$ and $\underline{N} = 1$.

We focus on a symmetric pure-strategy equilibrium, with contestants, in each stage, placing the same amount of bid. As is well known in the contest literature, pure-strategy bidding dissolves when the contest is excessively discriminatory—i.e., when r is sufficiently large—and mixed-strategy bidding equilibria are often elusive,[12] especially in our dynamic setting (see, e.g., Fu and Lu, 2012a). We focus on the case of moderate r, which ensures the existence and uniqueness of a symmetric pure-strategy bidding equilibrium.[13]

In particular, define the function $f(N, M) := \sum_{g=0}^{M-1} \frac{1}{N-g}$ and the cutoff $\overline{r}(N, M) := \min\left\{1, \frac{M}{(M-1)+(N-M)f(N,M)}\right\}$. The following regularity condition is imposed throughout Section 3.

Assumption 1 $r \in \left(0, \overline{r}(N, M)\right]$.

We require that discriminatory parameter r be contained by the upper bound $\overline{r}(N, M)$, i.e., $r \leq 1$ and $r \leq \frac{M}{(M-1)+(N-M)f(N,M)}$. The first condition guarantees that the winning probability specified in Equation (1) is concave in a contestant's effort and thus ensures that a symmetric pure-strategy equilibrium exists for the case of transparency. As will become clear later, the latter condition ensures that a symmetric pure-strategy equilibrium exists under opacity. Consider a case of $N = 6$. The cutoff $\overline{r}(N, M)$ takes the value of 0.8108, 0.7792, 0.8163, and 0.9174, respectively, for $M = 2, 3, 4$, and 5.

2.3 Disclosure Schemes and Equilibrium Preliminaries

The contest is governed by an organizer who chooses the disclosure scheme for the contest from two alternatives: full disclosure (*transparency*) and no disclosure (*opacity*).[14] Under the former, contestants are informed of their rankings upon completing the first-stage competition, and thereby learn whether they have survived the elimination. Under the latter, rankings in the first-stage competition are kept secret, so that contestants are uncertain

[11] We consider convex cost functions in Section 5.1.1, and the main results are robust.

[12] See Alcalde and Dahm (2010) and Ewerhart (2015, 2017).

[13] It is implicitly assumed that the discriminatory powers of the contest technology across the two stages are identical. We will relax this assumption in Section 5.1.2.

[14] In Online Appendix A, we allow for partial disclosure and show that this is suboptimal.

about their status. The organizer commits to and publicly announces the disclosure policy prior to the contest.

Under transparency, the contest renders a standard two-stage complete-information sequential game. Let (e_1, e_2) denote the symmetric equilibrium of the contest game, where e_t, $t \in \{1, 2\}$ is a contestant's effort in stage t. The game can be solved by backward induction.

In the first stage, each of the N contestants exerts effort to advance, i.e., competing for one of the M tickets to the finale. Each ticket is worth $\frac{1}{M} - e_2$ to a contestant in stage 1: He anticipates symmetric equilibrium play in the second stage, so each finalist stands a chance of $\frac{1}{M}$ to win the prize of unity value and exerts an effort e_2. Let $P_m(e_1', e_1)$ be the ex ante probability of a contestant's being ranked in the mth place, $m \in \{1, \ldots, N\}$, when he bids e_1' and all others exert an effort e_1. We can obtain that

$$P_m(e_1', e_1) \equiv \frac{(N-1)!}{(N-m)!} \times \left(\prod_{j=1}^{m-1} \frac{(e_1)^r}{(N-j)(e_1)^r + (e_1')^r} \right) \times \frac{(e_1')^r}{(N-m)(e_1)^r + (e_1')^r}.$$

Each contestant chooses his effort e_1' for the following maximization problem:

$$\max_{\{e_1'\}} \left[\sum_{m=1}^{M} P_m(e_1', e_1) \right] \times \left(\frac{1}{M} - e_2 \right) - e_1', \tag{2}$$

where the sum $\sum_{m=1}^{M} P_m(e_1', e_1)$ is the probability of his advancing to the finale.

Let $(\hat{e}_1, \hat{e}_2)$ denote the symmetric equilibrium of the contest game under opacity, where $\hat{e}_t$ is a contestant's effort in stage $t \in \{1, 2\}$. Opacity leads to a drastically different strategic problem than transparency. Under transparency, the choice of early effort e_1 factors in one's own future effort e_2 if he survives the elimination, which is seen in the continuation value $\frac{1}{M} - e_2$. In contrast, under opacity, one's late-stage effort, $\hat{e}_2$, is no longer contingent on the interim outcome because a contestant is uninformed of his own status. The dynamic linkage between stages thus dissolves despite the temporal sequence: A contestant's choice of the effort pair $(\hat{e}_1, \hat{e}_2)$ is strategically no different from that in a static game, in which his strategy involves two simultaneous actions. Suppose that all others use the same effort strategy $(\hat{e}_1, \hat{e}_2)$; a contestant chooses $(\hat{e}_1', \hat{e}_2')$ for the following maximization problem:

$$\max_{\{\hat{e}_1', \hat{e}_2'\}} \left[\sum_{m=1}^{M} P_m(\hat{e}_1', \hat{e}_1) \right] \times \frac{(\hat{e}_2')^r}{(\hat{e}_2')^r + (M-1)(\hat{e}_2)^r} - \hat{e}_1' - \hat{e}_2', \tag{3}$$

where $\frac{(\hat{e}_2')^r}{(\hat{e}_2')^r + (M-1)(\hat{e}_2)^r}$ is the conditional probability of one's winning the second-stage competition provided that he indeed survives.[15]

For brevity, we relegate the detailed equilibrium analysis to Appendix A and present contestants' equilibrium efforts in the following table.

	First-stage Individual Effort	Second-stage Individual Effort
Transparency	$e_1 = \frac{r\left[M-(M-1)r\right]}{NM^2} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right)$	$e_2 = \frac{(M-1)r}{M^2}$
Opacity	$\hat{e}_1 = \frac{r}{NM} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right)$	$\hat{e}_2 = \frac{(M-1)r}{NM}$

Table 1: Equilibrium Individual Effort under Different Disclosure Schemes.

2.4 Objectives for Contest Design

We consider contest design—i.e., the choice of disclosure policy—for two objectives: (i) maximization of total effort in the contest and (ii) maximization of the expected winner's total effort in the contest. The former objective has been conventionally adopted in the contest literature. The latter, however, is gaining increasing attention (see, for example, Baye and Hoppe, 2003; Serena, 2017), given its relevance in a broad array of contexts.[16] Consider an architecture competition, in which only the quality of the selected design accrues to the buyer's benefit.

The equilibrium total effort under transparency and that under opacity, denoted by $TE^T(N,M,r)$ and $TE^O(N,M,r)$, respectively, are

$$TE^T(N,M,r) := Ne_1 + Me_2, \text{ and } TE^O(N,M,r) := N\hat{e}_1 + N\hat{e}_2.$$

Similarly, we denote by $WE^T(N,M,r)$ and $WE^O(N,M,r)$ the expected winner's total effort under transparency and that under opacity, respectively, with

$$WE^T(N,M,r) := e_1 + e_2, \text{ and } WE^O(N,M,r) := \hat{e}_1 + \hat{e}_2.$$

[15]The opaque contest model strategically resembles the setting of Lu, Shen, and Wang (2019). They consider a multi-battle contest that allows for performance bundling: Two contestants meet each other in a series of competitions, and one receives the prize only if he prevails in all component battles. The model is also related to contests with multi-dimensional actions, which include Arbatskaya and Mialon (2010, 2012); Melkonyan (2013); and Lagerlöf (2020). In our model, a contestant's early and late efforts enter two separate winner-selection processes—i.e., competitions in the early and late stages—while in theirs, multiple actions are converted into a single output through a production function, and the composite output enters a single winner–selection process.

[16]The second objective is parallel to the widely adopted objective of maximizing the expected highest effort in the perfectly discriminating contest, or equivalently, an all-pay auction (e.g., Moldovanu and Sela, 2006).

3 Analysis of the Baseline Model

In this section, we compare the contest's performance—i.e., the total effort and expected winner's total effort—under different disclosure schemes. Before we present the result, we briefly discuss the strategic implications caused by the different disclosure schemes for the contest game, which will pave the way for our discussion of the main result.

From the maximization problem (2) under transparency, a contestant in the first stage factors his future effort cost—i.e., e_2—into his decision on his early bid, which erodes his continuation value and therefore diminishes the marginal return to e_1. The tension between early and late efforts, however, does not take place under opacity: As explained above, the dynamic contest is strategically equivalent to a static two-action decision problem. The maximization problem (3) under opacity implies that a contestant's efforts in the two stages, $\hat{e}_1$ and $\hat{e}_2$, are *complementary* to each other: Holding other things equal, a higher $\hat{e}_1$ improves the chance of surviving elimination, which in turn increases the marginal return to $\hat{e}_2$, since the late effort is more likely to count for his ultimate win; a higher $\hat{e}_2$ does the same to the marginal return to $\hat{e}_1$, because it is expected to increase his probability of winning the prize, which in turn incentivizes him to vie to advance in the first place. Such positive reflexive interactions are in sharp contrast to those under transparency.

Based on the equilibrium results in Table 1, we can compute the equilibrium total effort and the expected winner's total effort under each disclosure scheme. Let $r^\dagger := \frac{1}{f(N,M)} \in (0, \overline{r}(N,M)]$. A comparison leads to the following.

Proposition 1 ***(Optimal Disclosure Scheme)*** *Suppose that Assumption 1 is satisfied, and consider an N-M two-stage elimination contest. Then:*

(i) Opacity always generates a higher total effort than transparency, i.e., $TE^O(N,M,r) > TE^T(N,M,r)$.

(ii) There exists a threshold $M^\dagger \in \left\{\lfloor \frac{N}{2} \rfloor, \ldots, N-2\right\}$ such that

(a) for $2 \leq M \leq M^\dagger$, transparency generates a higher expected winner's total effort than opacity, i.e., $WE^T(N,M,r) > WE^O(N,M,r)$;

(b) for $M^\dagger < M \leq N-1$, transparency generates a higher expected winner's total effort than opacity—i.e., $WE^T(N,M,r) > WE^O(N,M,r)$—if and only if $r < r^\dagger$.

Proposition 1 shows that opacity always prevails when the organizer aims to maximize total effort. To better understand the result and interpret it in depth, it is important to note that a switch from opacity to transparency triggers three effects. First, it resolves the uncertainty and leads each survivor to bid more in the second stage. Second, it reduces the number of active contestants in the second stage from N to M. Third, it alters contestants' incentive to bid for advancement to the finale.

The first effect leads to the observation of $e_2 > \hat{e}_2$, i.e., higher individual effort under transparency. Recall by Table 1 that $e_2 = \frac{(M-1)r}{M^2}$ and $\hat{e}_2 = \frac{(M-1)r}{NM}$, which implies that $Me_2 = N\hat{e}_2$: The first two effects cancel each other out, and the total effort generated in the second stage of the competition is constant across the two scenarios. As a result, the optimum that maximizes total effort in the contest depends solely on the comparison of equilibrium efforts in the first stage under different disclosure schemes.[17]

The discussion laid out in the beginning of the section and a comparison between (2) and (3) reveal the answer. Under transparency, a contestant in the first stage takes into account his future effort cost—i.e., e_2—when choosing e_1: By (2), the ticket to the finale is worth $\frac{1}{M} - e_2$. Under opacity, a contestant is concerned only about the probability of winning the finale conditional on his surviving the elimination stage when choosing $\hat{e}_1$, as in a static contest with two simultaneous actions. He expects a conditional winning probability $\frac{1}{M}$, which implies that the contestant expects a higher marginal return to $\hat{e}_1$, as $\hat{e}_2$ does not reduce the marginal return to early effort, as shown by (3). This incentivizes contestants to bid more for advancement under opacity—i.e., $e_1 < \hat{e}_1$—and leads to the prediction of Proposition 1(i).

The optimum that maximizes the expected winner's total effort, however, is less than explicit because $e_1 < \hat{e}_1$ and $e_2 > \hat{e}_2$. It ultimately depends on the specific contest environment (N, M, r). Recall that $f(N, M) \equiv \sum_{g=0}^{M-1} \frac{1}{N-g}$. Examining the conditions in Proposition 1(ii.b), we have that the cutoff $r^\dagger \equiv \frac{1}{f(N,M)}$ strictly decreases in M and increases in N. This observation, together with the condition established in Proposition 1(ii.a), implies that transparency is more likely to prevail when M is relatively small, i.e., when the elimination process is more selective. A finalist tends to bid more when M is small, as he expects a higher chance of winning the prize. A larger loss in an individual bid in the finale would result if opacity were present. To see this, note that $\hat{e}_2 = \frac{M}{N} e_2$ from Table 1, which discounts e_2 by $\frac{M}{N}$: e_2 strictly decreases with M, and the discount is also more severe when M is smaller. Further, a smaller M implies less competition in the finale, which amplifies the value for advancement, thereby boosting the first-stage effort.

Transparency is more likely to prevail when N is larger. First, a larger contest dilutes each individual's incentive, which implies a smaller bid in the first stage regardless of the disclosure scheme. As a result, the loss in early effort caused by transparency, i.e., $\hat{e}_1 - e_1$, tends to be diminished. Second, as shown above, $\hat{e}_2$ discounts e_2 by $\frac{M}{N}$: The larger the N, the more severe the discount. This implies that opacity causes a larger loss of second-stage

[17]The observation of $Me_2 = N\hat{e}_2$ relies on the linear effort cost function. In Section 5.1.1, we allow for a strictly convex effort cost function and show that our main findings remain qualitatively robust although the first two effects do not perfectly cancel out, as in the baseline setting.

effort. Both effects favor transparency in the case of a large N.

Finally, the condition of $r < r^{\dagger}$ implies that transparency is more likely to prevail when the contest is less discriminatory, i.e., with a smaller r. To understand the logic, we present the following result.

Remark 1 *Suppose that Assumption 1 is satisfied, and consider an N-M two-stage elimination contest. Then:*

(i) In the symmetric subgame perfect equilibrium in pure strategies under transparency, a finalist's effort e_2 strictly increases with r—i.e., $\frac{de_2}{dr} > 0$—while one's stage-1 effort e_1 increases with r if and only if r is sufficiently small, i.e., $\frac{de_1}{dr} \gtreqless 0$ if and only if $r \lesseqgtr \frac{M}{2(M-1)}$.

(ii) In the symmetric pure-strategy equilibrium under opacity, a contestant's efforts in both stages strictly increase with r, i.e., $\frac{d\hat{e}_1}{dr}, \frac{d\hat{e}_2}{dr} > 0$.

Remark 1 reveals the contrasting roles played by r on contestants' effort strategies under transparency and opacity. Under transparency, a larger r may have opposite effects on early and late bids: e_2 always increases with r, whereas e_1 decreases with it when r is large. In contrast, under opacity, it uniformly increases both $\hat{e}_1$ and $\hat{e}_2$. An increase in r thus aggravates the aforementioned tension between early and late efforts under transparency and tends to favor opacity in the comparison.

Conventional wisdom in the contest literature predicts that equilibrium effort in contest, increases with r. However, two competing forces emerge under transparency when r enlarges, which gives rise to ambiguity with respect to $\frac{de_1}{dr}$, as illustrated in Remark 1(i). First, it magnifies the marginal return of effort and strengthens contestants' incentives. Second, a larger r increases rent dissipation in the second stage—i.e., an increase in e_2—which diminishes the value of participating in the finale and discourages contestants in the first stage from striving to advance. The latter effect outweighs the direct effect when r is sufficiently large, in which case the intense competition in the finale causes excessive rent dissipation, thereby diminishing early incentive. The conflict between early and late incentives fades away under opacity. One's late effort, $\hat{e}_2$, does not depend on the elimination outcome and is complementary to $\hat{e}_1$. As a result they both strictly increase with r, as in a static contest.

4 Optimal Disclosure Scheme with Endogenous Biases

We now allow the organizer to bias the second-stage competition by imposing identity-dependent treatment on finalists based on their interim rankings.

Specifically, the organizer places different weights on contestants' effort entries. Let the set of N contestants be indexed by $m \in \{1, \ldots, N\}$, which denotes their ranks in the first-stage competition. For a given stage-2 effort profile $\boldsymbol{e}_2 \equiv (e_2^1, \ldots, e_2^N)$, where $e_2^m, m \in \{1, \ldots, N\}$ is the stage-2 effort of a contestant ranked in the mth place in stage 1. A contestant wins the prize with a probability

$$q_m(\boldsymbol{e}_2, \boldsymbol{\delta}) := \begin{cases} \dfrac{(\delta_m e_2^m)^r}{\sum_{j=1}^N (\delta_j e_2^j)^r}, & \text{if } \boldsymbol{e}_2 \neq (0, \ldots, 0), \\ \dfrac{(\delta_m)^r}{\sum_{j=1}^N (\delta_j)^r}, & \text{if } \boldsymbol{e}_2 = (0, \ldots, 0), \end{cases} \tag{4}$$

where $\delta_m \geq 0$ is the weight assigned to a contestant based on his ranking. A contestant's stage-2 winning probability $q_m(\boldsymbol{e}_2, \boldsymbol{\delta})$ depends on both the effort profile $\boldsymbol{e}_2 \equiv (e_2^1, \ldots, e_2^N)$ and the bias rule $\boldsymbol{\delta} \equiv (\delta_1, \ldots, \delta_N) \in \mathbb{R}_+^N \setminus \{(0, \ldots, 0)\}$. We no longer assume outright elimination when $\boldsymbol{\delta}$ is the organizer's choice: She may equivalently assign an excessively small weight to a contestant, which effectively disqualifies him for the finale.[18,19]

The organizer commits to and publicly announces the rank-based bias rule $\boldsymbol{\delta} \equiv (\delta_1, \ldots, \delta_N)$ prior to the contest, together with the disclosure policy. Under transparency, the disclosure of interim results allows all contestants to learn whether they have survived the elimination process and how contestants will be relatively positioned in the finale if they remain in the race. Consider the aforementioned example of track events: Lane assignment publicly reveals athletes' relative positions in the finale. However, the discriminatory scheme can be implemented less conspicuously, which corresponds to concealment. For instance, a promising grant applicant can presumably receive more advice from reviewers before revising, and his resubmission will also be evaluated more seriously. When proposals' rankings are kept secret—e.g., in the Australian Research Council's Discovery Program (see Footnote 5)—applicants cannot precisely infer their relative standing and how their resubmission will be processed, even though they would expect reviewers and judging panels to have developed preferences after preliminary reviews.

The additional flexibility in contest design leads to a trade-off under transparency. On the one hand, a biased competition upsets the balance in the finale, which reduces late effort supply. On the other hand, a biased second-stage competition may boost early effort supply. Recall the aforementioned trade-off between early and late efforts under transparency:

[18] To be more specific, the baseline N-M two-stage elimination contest corresponds to $\boldsymbol{\delta} \equiv (\delta_1, \ldots, \delta_M, \delta_{M+1}, \ldots, \delta_N) = (1, \ldots, 1, 0, \ldots, 0)$ in the extended setting.

[19] Elimination may emerge endogenously in the optimum, but is not explicitly assumed in Section 4. Analysis of the baseline model requires $N \geq 3$, while the analysis of the extended model does not exclude the case of $N = 2$ when examining the optimal biases based on interim results. Our model is thus related to the study of Meyer (1992), which analyzes this issue for $N = 2$ under transparency.

A less competitive finale amplifies the continuation value of advancement, which compels contestants to bid harder in the first stage to survive elimination. Such a trade-off, however, is absent under opacity. Contestants remain symmetric throughout the race, as the interim ranking is kept secret from them. An additional positive effect nevertheless looms large, regardless of the disclosure scheme: Suppose that the organizer favors frontrunners in the finale. This would incentivize contestants in the early stage to vie for higher ranks in order to secure a headstart, instead of merely pursuing advancement.

In the rest of this section, we characterize the optimal contest under transparency and opacity. We then compare the resulting performance under different disclosure schemes to find the optimum. Again, we focus on the scenario in which a symmetric pure-strategy equilibrium exists.[20] We follow the tradition in the literature and assume a moderate discriminatory parameter.

Assumption 2 $r \in (0, 1]$.

The requirement of $r \leq 1$ guarantees that the winning probability specified in Equation (4) is concave in a contestant's effort for any contest bias rule $\boldsymbol{\delta} \in \mathbb{R}_+^N \setminus \{(0, \ldots, 0)\}$. This assumption ensures the existence of a symmetric pure-strategy equilibrium for the case of transparency. The case of opacity involves more complications when r is sufficiently large due to the aforementioned complementarity between first-stage and second-stage efforts. Instead of imposing a more stringent upper bound on r as in Assumption 1, we assume for simplicity that the contest organizer selects the bias rule from those that induce a symmetric pure-strategy equilibrium.[21]

4.1 Optimal Bias Rule under Transparency

We first explore the optimum under transparency. It is noteworthy that a conventional optimization approach loses its bite in our setting, because a closed-form solution to the equilibrium in an asymmetric Tullock contest is unavailable. We adopt the indirect approach suggested by Fu and Wu (2020) and Deng, Fu, and Wu (2021) to tackle this problem without solving for the equilibrium explicitly. It first reformulates the contest organizer's objective as a function of contestants' equilibrium winning probabilities only and treats the equilibrium winning probability distribution as the design variable. It then solves for the equilibrium winning probability distribution that maximizes the reformulated objective function. Finally, it obtains a bias rule that induces the desirable winning probability distribution in equilibrium, which closes the loop.

[20] Under transparency, contestants can be made asymmetric in the second stage. Symmetric equilibria require that they place the same bid in the first stage.

[21] We discuss in Online Appendix B the case in which the restrictions are removed and demonstrate that our main predictions remain largely intact.

Denote the symmetric equilibrium by $\langle e_1, (e_2^1, \ldots, e_2^N)\rangle$, where e_1 is contestants' stage-1 equilibrium effort and e_2^m, with $m \in \{1, \ldots, N\}$, is the stage-2 equilibrium effort of the mth-ranked contestant under an arbitrary bias rule $\boldsymbol{\delta} \equiv (\delta_1, \ldots, \delta_N)$. By the first-order conditions, we can rewrite the stage-2 equilibrium effort e_2^m as

$$e_2^m = q_m(\boldsymbol{e_2}, \boldsymbol{\delta}) \times \left[1 - q_m(\boldsymbol{e_2}, \boldsymbol{\delta})\right] r, \text{ for } m \in \{1, \ldots, N\}.$$

Similarly, the stage-1 equilibrium effort e_1 can be derived as

$$e_1 = \frac{r}{N} \times \left\{ \sum_{m=1}^{N} \alpha_m \times \left[(1-r) q_m(\boldsymbol{e_2}, \boldsymbol{\delta}) + r \left[q_m(\boldsymbol{e_2}, \boldsymbol{\delta})\right]^2\right] \right\}.$$ [22]

The total effort of the contest can accordingly be written as

$$TE^{RT}(\boldsymbol{q}) \equiv Ne_1 + \sum_{m=1}^{N} e_2^m = r \times \left[\sum_{m=1}^{N} \left\{ \alpha_m \left[(1-r) q_m + r (q_m)^2\right] \right\} + \sum_{m=1}^{N} \left[q_m (1 - q_m)\right] \right].$$

Similarly, the expected winner's total effort is expressed as

$$WE^{RT}(\boldsymbol{q}) \equiv e_1 + \sum_{m=1}^{N} \left[q_m e_2^m\right] = r \times \left[\frac{1}{N} \times \sum_{m=1}^{N} \left\{ \alpha_m \left[(1-r) q_m + r (q_m)^2\right] \right\} + \sum_{m=1}^{N} \left[(q_m)^2 (1 - q_m)\right] \right].$$

Both $TE^{RT}(\boldsymbol{q})$ and $WE^{RT}(\boldsymbol{q})$ appear to be functions of the equilibrium winning probability distribution $\boldsymbol{q}$. We allow the organizer to choose $\boldsymbol{q} \equiv (q_1, \ldots, q_N) \in \Delta^{N-1}$ to maximize the reformulated objective functions $TE^{RT}(\boldsymbol{q})$ and $WE^{RT}(\boldsymbol{q})$.

For notational convenience, define μ_n and κ as follows:

$$\mu_n := \frac{(n-2) + \sum_{m=1}^{n} \frac{\alpha_m}{1-\alpha_m r}}{\sum_{m=1}^{n} \frac{1}{1-\alpha_m r}}, \text{ for } n \in \{2, \ldots, N\} \tag{5}$$

and

$$\kappa := \max \left\{ n = 2, \ldots, N \,\middle|\, 1 + (1-r)\alpha_n > \mu_n \right\}, \tag{6}$$

where α_m is defined as

$$\alpha_m := 1 - \sum_{g=0}^{m-1} \frac{1}{N-g}, \text{ for } m \in \{1, \ldots, N\}. \tag{7}$$

[22] The derivation of e_1 and e_2^m can be found in the proof of Lemma 1 in Appendix B.

The following result ensues.

Lemma 1 ***(Optimal Contest under Transparency)*** *Suppose that Assumption 2 is satisfied, and consider the contest design problem under transparency for $N \geq 3$. Then:*

(i) A unique equilibrium winning probability distribution $\boldsymbol{q}^* \equiv (q_1^*, \ldots, q_N^*)$ maximizes total effort $TE^{RT}(\boldsymbol{q})$. The optimum requires that $\boldsymbol{q}_m^* = \frac{1}{2} + \frac{1}{2}\frac{\alpha_m - \mu_\kappa}{1-\alpha_m r}$ for $m \leq \kappa$, and $\boldsymbol{q}_m^* = 0$ for $m \geq \kappa + 1$. That is, κ contestants remain active in the optimum. Moreover, $\boldsymbol{q}^*$ can be induced by a bias rule $\boldsymbol{\delta}^* \equiv (\delta_1^*, \ldots, \delta_N^*)$, with $\delta_m^* = \frac{1}{1-q_m^*}(q_m^*)^{\frac{1-r}{r}}$ for $m \leq \kappa$ and $\delta_m^* = 0$ for $m \geq \kappa + 1$.

(ii) A unique equilibrium winning probability distribution $\boldsymbol{q}^{**} \equiv (q_1^{**}, \ldots, q_N^{**})$ maximizes the expected winner's total effort $WE^{RT}(\boldsymbol{q})$. The optimum requires that $q_1^{**} = \frac{1}{2} \times \left[1 + \frac{1}{N(N-1)-[2(N+\frac{1}{N})-5]r}\right]$; $q_2^{**} = 1 - q_1^{**}$; and $q_m^{**} = 0$ for $m \geq 3$. That is, only the two top-ranked contestants remain active in the finale. Moreover, $\boldsymbol{q}^{**}$ can be induced by a bias rule $\boldsymbol{\delta}^{**} \equiv (\delta_1^{**}, \ldots, \delta_N^{**})$, with $\delta_m^{**} = \frac{1}{1-q_m^{**}}(q_m^{**})^{\frac{1-r}{r}}$ for $m \leq 2$ and $\delta_m^{**} = 0$ for $m \geq 3$.

Lemma 1 provides the formulae to obtain the optimal bias rules $\boldsymbol{\delta}^*$ and $\boldsymbol{\delta}^{**}$ from $\boldsymbol{q}^*$ and $\boldsymbol{q}^{**}$, respectively. More importantly, we can directly calculate based on $\boldsymbol{q}^*$ and $\boldsymbol{q}^{**}$ the resultant total effort and the expected winner's total effort, $TE^{RT}(\boldsymbol{q}^*)$ and $WE^{RT}(\boldsymbol{q}^{**})$, which paves the way for further analysis.[23]

Ex post asymmetry always arises regardless of the prevailing design objective, in the sense that a higher-ranked active contestant is rewarded with a strictly larger weight δ_m^* or δ_m^{**}. To maximize total effort, the optimal contest eliminates $N - \kappa$ bottom-ranked contestants by placing zero weights on their effort entries and admits the rest into the finale. To maximize the expected winner's total effort, only two contestants are allowed to proceed to the finale in the optimum, i.e., $\delta_m^{**} = 0$ for $m \geq 3$. The logic is straightforward. This objective stresses strong individual incentive, which demands restrictions on the size of the finale to avoid diluting each finalist's performance. Indeed, the organizer must strike a balance between an individual contestant's early incentive and his contingent effort supply in the finale: An intense head-to-head competition in the finale could also hurt a contestant's incentive to vie for advancement. However, this conflict can be reconciled by creating asymmetry between finalists, i.e., setting $\delta_1^{**} > \delta_2^{**}$ to favor the top-ranked contestant in the finale. The asymmetry achieves two goals. First, it softens the head-to-head competition in the finale to reduce rent dissipation, in order to avoid excessive loss in the continuation value for contestants. Second, it increases the value for the top rank, which fuels first-stage competition.

[23] A three-player example is presented in Online Appendix C to illustrate this reformulated optimization problem.

4.2 Optimal Bias Rule under Opacity

Next, we characterize the optimum under opacity. Contestants remain symmetric throughout the game—even if they are assigned different weights in the second stage—since contestants are uninformed of their status.

The analysis largely follows in the same vein as that under transparency. Let $\hat{\boldsymbol{q}} \equiv (\hat{q}_1, \ldots, \hat{q}_N)$ be an equilibrium winning probability distribution. Define

$$\Gamma_N(\hat{\boldsymbol{q}}) := \sum_{m=1}^{N} [\alpha_m \hat{q}_m] + \sum_{m=1}^{N} \left[\hat{q}_m(1-\hat{q}_m)\right].$$

Let $\hat{\boldsymbol{q}}^* \equiv (\hat{q}_1^*, \ldots, \hat{q}_N^*)$ be the maximizer of $\Gamma_N(\hat{\boldsymbol{q}})$, subject to $\sum_{m=1}^{N} \hat{q}_m = 1$ and $\hat{q}_m \geq 0$, for $m \in \{1, \ldots, N\}$, since $\hat{\boldsymbol{q}}$ is a probability distribution. It can be verified that

$$\hat{q}_m^* = \begin{cases} \frac{1}{\hat{\kappa}} + \frac{\alpha_m}{2} - \frac{\sum_{s=1}^{\hat{\kappa}} \alpha_s}{2\hat{\kappa}}, & \text{for } m \in \{1, \ldots, \hat{\kappa}\}, \\ 0, & \text{for } m \in \mathcal{N} \setminus \{1, \ldots, \hat{\kappa}\}, \end{cases} \tag{8}$$

where $\hat{\kappa}$ is given by

$$\hat{\kappa} := \max\left\{ n = 2, \ldots, N \,\middle|\, \frac{1}{n} + \frac{\alpha_n}{2} - \frac{\sum_{s=1}^{n} \alpha_s}{2n} > 0 \right\}. \tag{9}$$

We are ready to fully characterize the optimum.

Lemma 2 ***(Optimal Contest under Opacity)*** *Suppose that Assumption 2 is satisfied, and consider the contest design problem under opacity for $N \geq 3$. Let the organizer optimize the contest over the set of bias rules that induce a symmetric pure-strategy equilibrium. Then:*

(i) For $r \in (0, \frac{1}{\Gamma_N(\hat{\boldsymbol{q}}^*)}]$, there exists a symmetric pure-strategy equilibrium for any bias rule $\hat{\boldsymbol{\delta}} \in \mathbb{R}_+^N \setminus \{(0, \ldots, 0)\}$. The optimal contest simultaneously maximizes the total effort in the overall contest and the expected winner's total effort. In the optimum, the winning probability of the mth-ranked contestant's winning the prize is $\hat{q}_m^*$, as specified in (8), and the optimum can be induced by a bias rule $\hat{\boldsymbol{\delta}}^* = \hat{\boldsymbol{q}}^*$. The maximum total effort and maximum expected winner's total effort are $r \times \Gamma_N(\hat{\boldsymbol{q}}^*)$ and $\frac{r}{N} \times \Gamma_N(\hat{\boldsymbol{q}}^*)$, respectively.

(ii) For $r \in (\frac{1}{\Gamma_N(\hat{\boldsymbol{q}}^*)}, 1]$, there exists a bias rule that induces a symmetric pure-strategy equilibrium, leading to a maximum total effort of 1 and the maximum expected winner's total effort of $\frac{1}{N}$.

We now briefly interpret the result. When the contest is sufficiently noisy, i.e., when r falls below $1/\Gamma_N(\hat{\boldsymbol{q}}^*)$, any bias rule induces a symmetric pure-strategy equilibrium. The optimum can be achieved by an optimal bias rule $\hat{\boldsymbol{\delta}}^* = \hat{\boldsymbol{q}}^*$; each mth-ranked's equilibrium winning probability is given by (8). The contest eliminates $N - \hat{\kappa}$ contestants by placing zero weights on the effort entries of these bottom-ranked contestants after the first-stage competition, and awards the prize to one of the $\hat{\kappa}$ survivors.

The contest design problem is complicated, however, when r exceeds $1/\Gamma_N(\hat{\boldsymbol{q}}^*)$. In this case, $\hat{\boldsymbol{q}}^*$ is infeasible, as it cannot be induced in a symmetric pure-strategy equilibrium due to violation of the first-stage participation constraint. As a result, we select the optimum from a restricted set of candidate bias rules. Lemma 2 (ii) establishes the maximum total effort and expected winner's total effort that can be achieved in a symmetric pure-strategy equilibrium. The optimum fully dissipates the rent, which implies that total effort achieves the maximum despite the restriction. The case of maximizing the expected winner's total effort, however, involves additional complications. We further discuss the issue in Online Appendix B, and show that the main prediction would not lose its bite when the restriction is relaxed.

4.3 Transparency vs. Opacity with Endogenous Biases

We are now ready to identify the optimum.

Proposition 2 ***(Optimal Disclosure Scheme with Endogenous Biases)*** *Suppose that Assumption 2 is satisfied, and fix $N \geq 3$. Let the organizer optimize the contest over the set of bias rules that induce a symmetric pure-strategy equilibrium. Then opacity always generates a larger amount of total effort in the contest and a smaller amount of the expected winner's total effort than transparency.*

Proposition 2 states that the organizer prefers opacity (transparency) when she maximizes the total effort (the expected winner's total effort) in the contest if she can flexibly bias the competition in favor of or against contestants. As in the baseline model, opacity dominates transparency in maximizing total effort.

The flexibility to set the bias rule allows the contest to further boost effort supply regardless of the prevailing disclosure scheme. However, it does not overturn the comparison when the organizer maximizes total effort in the contest. As stated above, under transparency, the maximization of total effort is plagued by the fundamental trade-off between early and late efforts. Such a tension limits the contest organizer's ability to elicit total effort, while biases do not serve to reconcile it. Consider, for instance, a bias rule that favors the frontrunner: It dilutes competition in the finale, while amplifying the value of advancement and encouraging early input. In contrast, this tension, as argued in the beginning of Section 4, is absent under opacity.

The ability to set the bias rule tends to favor transparency in the comparison when the expected winner's total effort is a concern for the contest organizer. As previously mentioned, opacity dilutes individual incentive in the second stage. The advantage of transparency is further strengthened when the organizer is allowed to set the bias rule. As is well known in the literature, individual effort in a Tullock contest increases when the contest involves a smaller number of contestants.[24] The organizer can effectively restrict the size of the finale by placing zero weights on the effort entries of early losers. As shown by Lemma 1(ii), the optimum involves the minimum number of finalists.

Proposition 2 yields useful implications for contest design in practice. Consider, for instance, bids for the Olympic games. The performance and impact of the event depend only on the host city's effort. Thus an elimination decision, according to our theory, should be announced immediately to avoid diluting the actual survivors' incentives. Similarly, consider an architectural design competition. Only the quality of the winning design accrues to the benefit of the organizer, which also tends to favor a more transparent organizing format. In contrast, consider an R&D contest that intends to rally efforts to promote innovation in a certain scientific or technological area, e.g., artificial intelligence or a vaccine for communicable diseases. A more opaque contest is likely to be preferable, because it yields broader impact for a research field.

5 Extensions

In this section, we present four extensions to our baseline model that test the robustness of our results. Section 5.1 analyzes two variations to our model. Section 5.2 endows the organizer with additional instruments that allow her to further manipulate the structural elements of the contest.

5.1 Alternative Contest Models

In this part, we first relax the assumption of linear effort costs and consider convex cost functions. We then allow the levels of discriminatory powers in the winner-selection mechanisms to differ across the two stages, which enables us to further discern the roles played by the parameter r.

[24] Ryvkin and Drugov (2020) demonstrate that this result may not hold more generally, e.g., when alternative contest technologies are assumed.

5.1.1 Convex Cost Functions

The previous literature points out that the optimal contest design often depends on the shape of contestants' effort cost curves in dynamic contests/tournaments (e.g., Aoyagi, 2010 and Moldovanu and Sela, 2006). In this part, we relax the assumption of linear effort cost functions and investigate the impact of the curvature of cost functions on the optimal disclosure scheme. Specifically, we assume that one's effort e incurs a cost $e^{1+\tau}$, with $\tau \geq 0$. The model degenerates to the baseline setting when $\tau = 0$. The following is obtained.

Proposition 3 ***(Optimal Contest with Convex Cost Functions)*** *Suppose that Assumption 1 is satisfied and consider an N-M two-stage elimination contest with convex cost functions $e^{1+\tau}$ and $\tau \geq 0$. There exists a unique symmetric pure-strategy equilibrium under both transparency and opacity. Moreover, the following statements hold:*

(i) The contest generates higher total effort under opacity than it does under transparency for all $\tau \geq 0$.

(ii) The expected winner's total effort in the contest under transparency is higher than that under opacity if τ is sufficiently large.

Our predictions are qualitatively robust in the alternative setting. Denote by $\big(e_1(\tau), e_2(\tau)\big)$ the equilibrium effort profile under transparency, and by $\big(\hat{e}_1(\tau), \hat{e}_2(\tau)\big)$ that under opacity. The observation of $Me_2(0) = N\hat{e}_2(0)$ in the baseline model no longer holds in the current context. The fundamental trade-offs of $e_2(\tau) > \hat{e}_2(\tau)$ and $e_1(\tau) < \hat{e}_1(\tau)$, however, remain in place despite the convex cost function. Again, we demonstrate that opacity outperforms transparency in incentivizing total effort.

The comparison in terms of the expected winner's total effort is more complicated. Recall that with a linear cost function, the outcome depends on the property of the contest—i.e., the set of parameters (N, M, r). Proposition 3 states that the comparison tends to favor transparency when the effort cost function is sufficiently convex. When τ increases, both the gap between $e_2(\tau)$ and $\hat{e}_2(\tau)$ and that between $e_1(\tau)$ and $\hat{e}_1(\tau)$ would diminish asymptomatically. The former, however, converges to zero at a slower rate than the latter, which leads to our prediction.

5.1.2 Two-stage Contests with Different Discriminatory Powers

We have assumed that the winner-selection mechanisms are equally precise across the two stages, in the sense that the contest success functions have the same discriminatory power term r across both stages. In reality, however, competitive environments often differ in their dynamics. Consider, for instance, that in a succession race, candidates can be assigned different tasks at different stages, and the technical nature of a task can affect the precision of performance evaluation. Alternatively, the performance of finalists is arguably monitored and observed more closely than in early stages. It is natural to assume that contest success functions differ in terms of their discriminatory powers along the ladder.

Assume that the discriminatory power for the competition in stage $t \in \{1, 2\}$ is $r_t > 0$. We can derive the equilibrium effort profile under each disclosure scheme.[25] Our results in Proposition 1 are largely robust to this variation in modeling.

Proposition 4 ***(Optimal Contest with Different Discriminatory Powers)*** *Suppose that* $\max\{r_1, r_2\} \leq \overline{r}(N, M)$ *and consider an* N*-*M *two-stage elimination contest. There exists a unique symmetric pure-strategy equilibrium under both transparency and opacity. Moreover, the following statements hold:*

(i) The total effort of the contest under opacity is strictly higher than that under opacity for all (r_1, r_2) *such that* $\max\{r_1, r_2\} \leq \overline{r}(N, M)$*.*

(ii) Transparency outperforms opacity in generating the expected winner's total effort if and only if $r_1 < r^\dagger \equiv \frac{1}{f(N,M)}$*.*

5.2 Alternative Contest Design

In this part, we consider two extensions that expand the organizer's design space. First, we allow the organizer to split the prize purse among several prizes. Second, we allow the organizer to manipulate the contest architecture, i.e., two-stage contest vis-à-vis static contest and the possibility of a longer series of elimination.[26]

5.2.1 Prize Allocation

In the baseline setting, we assume that a single prize is awarded to the grand winner in the finale. We now allow the organizer to freely divide her prize prize into several smaller prizes that reward finalists. To put this formally, consider a two-stage sequential elimination contest (N, M, r) and suppose that a total of M prizes are to be given away based on contestants' ranks in the finale. The prizes are ordered in a decreasing series $\mathcal{V}_1 \geq \ldots \geq \mathcal{V}_M \geq 0$, with $\sum_{m=1}^{M} \mathcal{V}_m = \mathcal{V} \equiv 1$. The model boils down to a winner-take-all competition when $\mathcal{V}_2 = 0$.

The optimization is nontrivial. Consider the case of transparency. Multiple prizes soften the competition in the finale, which reduces e_2. It nevertheless enlarges the payoff one would expect from participating in stage-2 competition, which encourages contestants to step up

[25] See Equations (30) and (31) in Appendix B.

[26] We thank two anonymous referees for suggesting these extensions.

their efforts in the preliminary to strive for advancement, i.e., enlarging e_1. The overall effect requires a closer look.

Proposition 5 ***(Optimality of Winner-take-all Contests)*** *Suppose that Assumption 1 is satisfied. In the N-M two-stage elimination contest, both total effort and the expected winner's total effort are maximized in a winner-take-all contest—i.e., $(\mathcal{V}_1, \ldots, \mathcal{V}_M) = (1, 0, \ldots, 0)$—under either disclosure scheme, transparency or opacity.*

Proposition 5 states that a winner-take-all contest prevails regardless of the design objective—total effort or the expected winner's total effort—and irrespective of the disclosure scheme, transparency or opacity. This result thus endorses Proposition 1, since the comparison assumes a winner-take-all prize structure.

5.2.2 Contest Architecture

We have assumed a dynamic structure for the contest. It is unclear a priori whether a sequential elimination process is necessary. In this part, we first examine whether a one-shot contest would outperform a sequential one. We then proceed to allow the organizer to set up the contest beyond a two-stage structure—i.e., by adding additional stages into the series.

Dynamic vs. Static Contests Note that an N-player static contest is strategically equivalent to an N-M two-stage contest with $M = 1$ or N and thus can be denoted by $(N, 1, r)$ or (N, N, r) without introducing additional notations. The following result can be obtained:

Proposition 6 ***(Dynamic versus Static Contests)*** *Suppose that Assumption 1 is satisfied. Then the following statements hold:*

(i) Under transparency, a dynamic contest always generates a higher total effort and a higher expected winner's total effort than a static contest, i.e., $TE^T(N, M, r) > TE^T(N, 1, r)$ and $WE^T(N, M, r) > WE^T(N, 1, r)$, for all $M \in \{2, \ldots, N-1\}$.

(ii) Under opacity, a dynamic contest always generates a higher total effort and a higher expected winner's total effort than a static contest, i.e., $TE^O(N, M, r) > TE^O(N, 1, r)$ and $WE^O(N, M, r) > WE^O(N, 1, r)$, for all $M \in \{2, \ldots, N-1\}$.

Proposition 6 shows that a two-stage contest always outperforms a static one, irrespective of the design objective or the disclosure scheme. As a result, Proposition 1 remains intact even if the organizer is allowed to squeeze the competition into a single stage.

Multiple Stages Proposition 6 inspires us to further explore the design of contest architecture. It is natural to conjecture that more stages would further improve the performance of the contest if the organizer is able to choose the number of stages and/or the number of surviving contestants in each stage. The optimal disclosure scheme deserves to be reexamined when the contest architecture can also be chosen by the organizer.

In what follows, we consider a four-player example. A general analysis is technically challenging and beyond the scope of this paper, which we leave for future research.[27] Recall that we denote a two-stage contest by (N, M, r). With four players and a given discriminatory power r, a sequential-elimination contest can be constructed in one of three possible forms: $(4, 3, 2, r)$, $(4, 3, r)$, or $(4, 2, r)$. Each sequence indicates the number of surviving contestants in every stage. The sequence $(4, 3, 2, r)$, for example, represents a three-stage contest, with one contestant being eliminated in each stage. The other two restore the two-stage structure, as in our baseline model.

Table 2 summarizes the performance of the contest under different combinations of disclosure scheme and contest architecture. To identify the optimum for each design objective, we only need to compare the maximum under transparency to its counterpart under opacity.

Transparency	TE	WE
$(4,3,2,r)$	$\frac{13}{864}r(2-r)(12-5r)+\frac{5}{24}r(2-r)+\frac{1}{2}r$	$\frac{13}{3456}r(2-r)(12-5r)+\frac{5}{72}r(2-r)+\frac{1}{4}r$
$(4,3,r)$	$\frac{13}{108}r(3-2r)+\frac{2}{3}r$	$\frac{13}{432}r(3-2r)+\frac{2}{9}r$
$(4,2,r)$	$\frac{7}{24}r(2-r)+\frac{1}{2}r$	$\frac{7}{96}r(2-r)+\frac{1}{4}r$
Opacity	**TE**	**WE**
$(4,3,2,r)$	$\frac{23}{18}r$	$\frac{23}{72}r$
$(4,3,r)$	$\frac{37}{36}r$	$\frac{37}{144}r$
$(4,2,r)$	$\frac{13}{12}r$	$\frac{13}{48}r$

Table 2: Total Effort and Expected Winner's Total Effort under Different Disclosure Schemes and Contest Architectures.

The following result naturally ensues.

Proposition 7 ***(Optimal Disclosure Scheme under Endogenous Contest Architecture)*** *Suppose there are four contestants and $r \leq \frac{18}{23}$. Moreover, the organizer is able to choose the number of stages. Then the following statements hold:*

(i) A three-stage elimination contest $(4, 3, 2, r)$ generates the highest total effort and expected winner's total effort under both transparency and opacity.

[27] It can be verified that the total number of dynamic contest arrangements under the two disclosure schemes with $N \geq 3$ contestants is $2^{N-1} - 2$, which increases exponentially with N.

(ii) Opacity generates a larger amount of total effort in the three-stage elimination contest $(4, 3, 2, r)$ *and a smaller amount of the expected winner's total effort than transparency.*

Proposition 7(i) states that a three-stage contest—with the maximum number of stages in a four-player setting—maximizes both total effort and the expected winner's total effort regardless of the prevailing disclosure scheme. Consistent with Proposition 1(i), Proposition 7(ii) predicts that opacity leads to a larger total effort. Interestingly, Proposition 7(ii) indicates that an additional stage amplifies the advantage of transparency when maximizing the expected winner's total effort.

6 Concluding Remarks

In this paper, we explore the optimal disclosure scheme in elimination contests. We demonstrate that the optimum depends on the organizer's objective and the environmental factors of the contest (Proposition 1). We further allow the organizer to choose the bias rule for the finale based on contestants' early performance. This yields a more distinct comparison, which shows that transparency yields a higher expected winner's total effort, while opacity leads to a gain in total effort (Proposition 2). In addition, we show that an asymmetric finale always emerges in the optimum regardless of the prevailing disclosure scheme.

To the best of our knowledge, we are the first to study sequential-elimination contests under an opaque disclosure scheme. We demonstrate that strategic interaction in the contest differs fundamentally between different disclosure schemes, which allows wide latitude for contest design. In addition to the theoretical contributions, our results provide a lucid playbook for practical contest design in various scenarios.

Our paper focuses on specific instruments for contest design, primarily the choice of disclosure scheme; this differs from a typical mechanism design problem. As in the majority of studies on contest design, our model mirrors the usual exercise of information management prevalent in practice—e.g., control of the amount of information accessible to employees—which differs across firms/organizations. A standard mechanism design approach (e.g., Myerson, 1981) does not apply because of the organizer's limited freedom to manipulate the structure or the winner-selection mechanism of the contest. Our study can thus be viewed as an exercise within a given class of mechanisms, as in Olszewski and Siegel (2020) and many others.[28]

As mentioned in Footnote 10, we assume symmetry among contestants, which allows for a tractable equilibrium analysis in a multi-prize contest environment. This setting enables

[28]See Polishchuk and Tonis (2013); Letina, Liu, and Netzer (2020); and Zhang (2020) for studies that apply the mechanism design approach to optimal contest design.

an analysis that elucidates the incentive effects of the disclosure scheme when the variance among contestants remains relatively mild in terms of their abilities. It limits the scope of this study, however, as it abstracts away the concern about the selection efficiency of a contest. An analysis of selection efficiency requires a multi-prize contest model with heterogeneous players, which imposes a tremendous technical challenge. Nevertheless, it definitely merits future research.

Our paper assumes a complete-information multi-winner nested Tullock contest. An alternative modeling approach is to assume an all-pay auction in which contestants' private valuations or marginal costs are independently and identically distributed (e.g., Moldovanu and Sela, 2001, 2006 and Zhang and Wang, 2009). However, this setting also entails technical complications in a two-stage structure. Finalists are able to infer their opponents' types from the interim outcome (their bids or rankings), which entices contestants to bid strategically in the first stage not only to strive for advancement but also to manipulate competitors' beliefs in their own favor. A comprehensive analysis is definitely worthwhile and should be attempted in future studies.

Finally, in Section 5.2.2 we reexamine the optimal disclosure scheme when the organizer is endowed with the freedom to design the contest architecture and focus on a four-player model. A more general analysis is required to further test the limit of the prediction, which warrants future research.

References

Alcalde, José and Matthias Dahm, "Rent seeking and rent dissipation: A neutrality result," *Journal of Public Economics*, 2010, *94* (1-2), 1–7.

Aoyagi, Masaki, "Information feedback in a dynamic tournament," *Games and Economic Behavior*, 2010, *70* (2), 242–260.

Arbatskaya, Maria and Hugo M. Mialon, "Multi-activity contests," *Economic Theory*, 2010, *43* (1), 23–43.

_ and _ , "Dynamic multi-activity contests," *Scandinavian Journal of Economics*, 2012, *114* (2), 520–538.

Arve, Malin and Olga Chiappinelli, "The role of budget constraints in sequential elimination tournaments," *Scandinavian Journal of Economics*, 2019, *forthcoming.*

Barbieri, Stefano and Marco Serena, "Biasing unbiased dynamic contests," *Working Paper*, 2018.

Baye, Michael R. and Heidrun C. Hoppe, “The strategic equivalence of rent-seeking, innovation, and patent-race games,” *Games and Economic Behavior*, 2003, *44* (2), 217–226.

Brown, Jennifer and Dylan B. Minor, “Selecting the best? Spillover and shadows in elimination tournaments,” *Management Science*, 2014, *60* (12), 3087–3102.

Che, Yeon-Koo and Ian L. Gale, “Optimal design of research contests,” *American Economic Review*, 2003, *93* (3), 646–671.

Chen, Zhijun, “Collusion and biased tournaments,” *European Economic Review*, 2016, *85*, 127–143.

Clark, Derek J. and Christian Riis, “A multi-winner nested rent-seeking contest,” *Public Choice*, 1996, *87* (1–2), 177–184.

_ and _ , “Influence and the discretionary allocation of several prizes,” *European Journal of Political Economy*, 1998, *14* (4), 605–625.

Cohen, Noam, Guy Maor, and Aner Sela, “Two-stage elimination contests with optimal head starts,” *Review of Economic Design*, 2018, *22* (3-4), 177–192.

Conger, Jay A. and Robert M. Fulmer, “Developing your leadership pipeline,” *Harvard Business Review*, 2003, *81* (12), 76–85.

Deng, Shanglyu, Qiang Fu, and Zenan Wu, “Optimally biased Tullock contests,” *Journal of Mathematical Economics*, 2021, *92*, 10–21.

Drugov, Mikhail and Dmitry Ryvkin, “Biased contests for symmetric players,” *Games and Economic Behavior*, 2017, *103*, 116–144.

Ederer, Florian, “Feedback and motivation in dynamic tournaments,” *Journal of Economics & Management Strategy*, 2010, *19* (3), 733–769.

Ewerhart, Christian, “Mixed equilibria in Tullock contests,” *Economic Theory*, 2015, *60* (1), 59–71.

_ , “Contests with small noise and the robustness of the all-pay auction,” *Games and Economic Behavior*, 2017, *105*, 195–211.

Franke, Jörg, “Affirmative action in contest games,” *European Journal of Political Economy*, 2012, *28* (1), 105–118.

— , Christian Kanzow, Wolfgang Leininger, and Alexandra Schwartz, "Effort maximization in asymmetric contest games with heterogeneous contestants," *Economic Theory*, 2013, *52* (2), 589–630.

— , — , — , and — , "Lottery versus all-pay auction contests: A revenue dominance theorem," *Games and Economic Behavior*, 2014, *83*, 116–126.

Fu, Qiang, "A theory of affirmative action in college admissions," *Economic Inquiry*, 2006, *44* (3), 420–428.

— and Jingfeng Lu, "The optimal multi-stage contest," *Economic Theory*, 2012a, *51* (2), 351–382.

— and — , "Micro foundations of multi-prize lottery contests: A perspective of noisy performance ranking," *Social Choice and Welfare*, 2012b, *38* (3), 497–517.

— and Zenan Wu, "On the optimal design of biased contests," *Theoretical Economics*, 2020, *15* (4), 1435–1470.

— , — , and Yuxuan Zhu, "On equilibrium existence in generalized multi-prize lottery contests," *Working Paper*, 2020.

Gershkov, Alex and Motty Perry, "Tournaments with midterm reviews," *Games and Economic Behavior*, 2009, *66* (1), 162–190.

Goltsman, Maria and Arijit Mukherjee, "Interim performance feedback in multistage tournaments: The optimality of partial disclosure," *Journal of Labor Economics*, 2011, *29* (2), 229–265.

Gradstein, Mark and Kai A. Konrad, "Orchestrating rent seeking contests," *Economic Journal*, 1999, *109* (458), 536–545.

Gürtler, Oliver and Christine Harbring, "Feedback in tournaments under commitment problems: Experimental evidence," *Journal of Economics & Management Strategy*, 2010, *19* (3), 771–810.

Kirkegaard, René, "Favoritism in asymmetric contests: Head starts and handicaps," *Games and Economic Behavior*, 2012, *76* (1), 226–248.

Konrad, Kai A., "Investment in the absence of property rights: The role of incumbency advantages," *European Economic Review*, 2002, *46* (8), 1521–1537.

Lagerlöf, Johan N., "Hybrid all-pay and winner-pay contests," *American Economic Journal: Microeconomics*, 2020, *forthcoming.*

Letina, Igor, Shuo Liu, and Nick Netzer, "Optimal contest design: A general approach," *Working Paper*, 2020.

Levitt, Steven D., "Optimal incentive schemes when only the agents' "best" output matters to the principal," *RAND Journal of Economics*, 1995, *26* (4), 744–760.

Lizzeri, Alessandro, Margaret A. Meyer, and Nicola Persico, "The incentive effects of interim performance evaluations," *Working Paper*, 2002.

Lu, Jingfeng and Zhewei Wang, "Axiomatizing multi-prize nested lottery contests: A complete and strict ranking perspective," *Journal of Economic Behavior & Organization*, 2015, *116*, 127–141.

_ , Bo Shen, and Zhewei Wang, "Performance bundling across multiple competitions," *Working Paper*, 2019.

Melkonyan, Tigran, "Hybrid contests," *Journal of Public Economic Theory*, 2013, *15* (6), 968–992.

Meyer, Margaret A., "Learning from coarse information: Biased contests and career profiles," *Review of Economic Studies*, 1991, *58* (1), 15–41.

_ , "Biased contests and moral hazard: Implications for career profiles," *Annales d'Économie et de Statistique*, 1992, *25/26*, 165–187.

Moldovanu, Benny and Aner Sela, "The optimal allocation of prizes in contests," *American Economic Review*, 2001, *91* (3), 542–558.

_ and _ , "Contest architecture," *Journal of Economic Theory*, 2006, *126* (1), 70–96.

Mulcahy, Anne, "Xerox's former CEO on why succession shouldn't be a horse race," *Harvard Business Review*, 2010, *88* (10).

Myerson, Roger B., "Optimal auction design," *Mathematics of Operations Research*, 1981, *6* (1), 58–73.

Nti, Kofi O., "Maximum efforts in contests with asymmetric valuations," *European Journal of Political Economy*, 2004, *20* (4), 1059–1066.

Olszewski, Wojciech and Ron Siegel, "Performance-maximizing large contests," *Theoretical Economics*, 2020, *15* (1), 57–88.

Polishchuk, Leonid and Alexander Tonis, "Endogenous contest success functions: A mechanism design approach," *Economic Theory*, 2013, *52* (1), 271–297.

Rosen, Sherwin, "Prizes and incentives in elimination tournaments," *American Economic Review*, 1986, *76* (4), 701–715.

Ryvkin, Dmitry and Mikhail Drugov, "The shape of luck and competition in winner-take-all tournaments," *Theoretical Economics*, 2020, *15* (4), 1587–1626.

Serena, Marco, "Quality contests," *European Journal of Political Economy*, 2017, *46*, 15–25.

Welch, Jack and John A. Byrne, *Jack: Straight from the Gut*, Business Plus, 2003.

Zhang, Jun and Ruqu Wang, "The role of information revelation in elimination contests," *Economic Journal*, 2009, *119* (536), 613–641.

Zhang, Mengxi, "Contest with incomplete information: When to turn up the heat, and how?," *Working Paper*, 2020.

Appendix A: Equilibrium Analysis in Baseline Model

We first consider the case under transparency.

Lemma 3 ***(Equilibrium under Transparency)*** *Suppose that Assumption 1 is satisfied and consider an N-M two-stage elimination contest under transparency. There exists a unique symmetric subgame perfect equilibrium in pure strategies. In the equilibrium, a contestant chooses stage-*1 *effort*

$$e_1 = \frac{r\left[M-(M-1)r\right]}{NM^2} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right), \tag{10}$$

*and stage-*2 *effort*

$$e_2 = \frac{(M-1)r}{M^2}, \tag{11}$$

provided that he has survived the elimination.

Proof. Recall that a representative contestant's probability of obtaining the mth rank in the first stage when he exerts effort e_1' and all others exert effort e_1 is

$$P_m(e_1', e_1) \equiv \frac{(N-1)!}{(N-m)!} \times \left(\prod_{j=1}^{m-1}\frac{(e_1)^r}{(N-j)(e_1)^r+(e_1')^r}\right) \times \frac{(e_1')^r}{(N-m)(e_1)^r+(e_1')^r}.$$

It is straightforward to verify that $P_m(e_1, e_1) = 1/N$ and

$$\frac{\partial \log P_m(e_1', e_1)}{\partial e_1'} = \frac{r}{e_1'} - \sum_{j=1}^{m}\frac{r(e_1')^{r-1}}{(N-j)(e_1)^r+(e_1')^r}.$$

Therefore, in a symmetric equilibrium with $e_1' = e_1$, we have that

$$\left.\frac{\partial P_m(e_1', e_1)}{\partial e_1'}\right|_{e_1'=e_1} = P_m(e_1, e_1) \times \left.\frac{\partial \log P_m(e_1', e_1)}{\partial e_1'}\right|_{e_1'=e_1} = \frac{r}{Ne_1} \times \left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right). \tag{12}$$

The term $\left.\frac{\partial P_m(e_1', e_1)}{\partial e_1'}\right|_{e_1'=e_1}$ measures the marginal impact of a contestant's effort on his probability of obtaining the mth rank when he places a bid in the same amount as the others. It is straightforward to verify that $\left.\frac{\partial P_m(e_1', e_1)}{\partial e_1'}\right|_{e_1'=e_1}$ strictly decreases with m. That is, additional effort affords him a higher probability of obtaining a higher rank, and, equivalently, renders him less likely to fall behind.

Next, we solve for the equilibrium effort profile (e_1, e_2) by backward induction.

Stage 2 Fixing his opponents' symmetric effort e_2, let a contestant exert effort e_2' for the following optimization problem:

$$\max_{\{e_2'\}} \frac{(e_2')^r}{(e_2')^r + (M-1)(e_2)^r} - e_2'.$$

By standard technique, we can solve for the symmetric equilibrium, in which each finalist bids

$$e_2 = \frac{(M-1)r}{M^2}.$$

By participating in the second-stage competition, a contestant obtains an expected equilibrium payoff

$$V \equiv \frac{1}{M} - e_2 = \frac{M-(M-1)r}{M^2}, \tag{13}$$

which represents the continuation value for a contestant of surviving the elimination process in the first-stage competition.

Stage 1 Contestants compete to advance, and advancing provides an expected value V as characterized in Equation (13). Fixing his opponents' symmetric effort supply e_1, let a contestant choose e_1' for the following optimization problem:

$$\max_{\{e_1'\}} \left[\sum_{m=1}^{M} P_m(e_1', e_1)\right] \times V - e_1'.$$

Imposing symmetry condition of $e_1' = e_1$, we obtain the first-order condition

$$\frac{r}{Ne_1}\sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right)V = 1 \Rightarrow e_1 = \frac{r\left[M-(M-1)r\right]}{NM^2} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right),$$

which uniquely determines the symmetric pure-strategy equilibrium whenever it exists.

It can be verified that $V > 0$ and $e_1 + \frac{M}{N}e_2 \leq \frac{1}{N}$ under Assumption 1. Therefore, a representative contestant's participation constraints for both stages are satisfied and the derived pure strategy indeed constitutes a symmetric pure-strategy equilibrium. This completes the proof. ■

We then consider the case of opacity.

Lemma 4 ***(Equilibrium under Opacity)*** *Suppose that Assumption 1 is satisfied and consider an N-M two-stage elimination contest under opacity. Then a symmetric pure-*

strategy equilibrium exists, and in it, each contestant has a bidding strategy $(\hat{e}_1, \hat{e}_2)$ *with*

$$\hat{e}_1 = \frac{r}{NM} \times \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g}\right), \tag{14}$$

and

$$\hat{e}_2 = \frac{(M-1)r}{NM}. \tag{15}$$

Proof. The solution can be obtained by simple algebra. To establish it as an equilibrium, it suffices to show that a representative contestant's participation constraint is satisfied, i.e., $\hat{e}_1 + \hat{e}_2 \leq \frac{1}{N}$. Carrying out the algebra, this inequality is equivalent to $r \leq \frac{M}{(M-1)+(N-M)f(N,M)}$. This concludes the proof. ■

Appendix B: Proofs

Proof of Proposition 1

Proof. It follows from Table 1 that the two-stage elimination contest under transparency yields a total equilibrium effort

$$TE^T(N,M,r) = Ne_1 + Me_2 = \frac{r\left[M-(M-1)r\right]}{M^2} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \frac{(M-1)r}{M}, \tag{16}$$

and an expected winner's total effort

$$WE^T(N,M,r) = e_1 + e_2 = \frac{r\left[M-(M-1)r\right]}{NM^2} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \frac{(M-1)r}{M^2}. \tag{17}$$

Similarly, the contest under opacity generates a total effort

$$TE^O(N,M,r) = N\hat{e}_1 + N\hat{e}_2 = \frac{r}{M}\sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \frac{(M-1)r}{M}, \tag{18}$$

and an expected winner's total effort

$$WE^O(N,M,r) = \hat{e}_1 + \hat{e}_2 = \frac{r}{NM}\sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \frac{(M-1)r}{NM}. \tag{19}$$

The comparison of total effort and the expected winner's total effort between transparency and opacity is straightforward for $N = 3$, and it remains to analyze the case where $N \geq 4$. It follows from Equations (16) and (18) that

$$\begin{aligned}
& TE^O(N,M,r) - TE^T(N,M,r) \\
= \; & \left[\frac{r}{M} - \frac{r\left[M-(M-1)r\right]}{M^2}\right] \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) \\
= \; & \frac{(M-1)r^2}{M^2} \times \left[M - \sum_{g=0}^{M-1}\frac{M-g}{N-g}\right] \\
= \; & \frac{(N-M)(M-1)r^2}{M^2} \times \sum_{g=0}^{M-1}\frac{1}{N-g} > 0.
\end{aligned}$$

Therefore, opacity always generates more total effort than does transparency. Next, it follows from Equations (17) and (19) that

$$\begin{aligned}
&WE^O(N,M,r) - WE^T(N,M,r)\\
= &\left[\frac{r}{NM} - \frac{r\left[M-(M-1)r\right]}{NM^2}\right] \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \left[\frac{(M-1)r}{NM} - \frac{(M-1)r}{M^2}\right]\\
= &\frac{(N-M)(M-1)r^2}{NM^2} \times \sum_{g=0}^{M-1}\frac{1}{N-g} - \frac{(N-M)(M-1)r}{NM^2}\\
= &\frac{(N-M)(M-1)r^2}{NM^2} \times \left[\sum_{g=0}^{M-1}\frac{1}{N-g} - \frac{1}{r}\right]\\
\equiv &\frac{(N-M)(M-1)r^2}{NM^2} \times \left[f(N,M) - \frac{1}{r}\right].
\end{aligned}$$

Therefore, $WE^O(N,M,r) \gtreqless WE^T(N,M,r)$ is equivalent to $r \gtreqless \frac{1}{f(N,M)} \equiv r^\dagger$. Next, we show that there exists a cutoff $M^\dagger$ such that $\overline{r}(N,M) \leq r^\dagger$ for $M \leq M^\dagger$ and $\overline{r}(N,M) > r^\dagger$ for $M > M^\dagger$. It can be verified that $\overline{r}(N,M) \leq r^\dagger$ is equivalent to

$$(2M-N)f(N,M) \leq M-1.$$

Clearly, the above inequality holds for $M \leq \lfloor \frac{N}{2} \rfloor$. For $M \geq \lfloor \frac{N}{2} \rfloor + 1$, the above inequality is equivalent to

$$f(N,M) \leq \frac{M-1}{2M-N}.$$

Note that the left-hand side strictly increases with M while the right-hand side strictly decreases with M. Moreover, we have that

$$f(N,N-1) = \frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N} \geq \frac{1}{2}+\frac{1}{3}+\frac{1}{4} = \frac{13}{12} > 1 = \frac{N-2}{2(N-1)-N},$$

where the first inequality follows from the postulated $N \geq 4$. Therefore, there exists a cutoff $M^\dagger$ such that $\overline{r}(N,M) \leq r^\dagger$ for $M \leq M^\dagger$ and $\overline{r}(N,M) > r^\dagger$ for $M > M^\dagger$. This completes the proof. ■

Proof of Remark 1

Proof. Under transparency, it follows instantly from Equation (11) that $\frac{de_2}{dr} = \frac{M-1}{M^2} > 0$.

Moreover, Equation (10) implies that

$$\frac{de_1}{dr} = \frac{M - 2(M-1)r}{NM^2} \times \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g} \right).$$

Therefore, $\frac{de_1}{dr} > 0$ is equivalent to $r < \frac{M}{2(M-1)}$.

Under opacity, it follows from Equations (14) and (15) that

$$\frac{d\hat{e}_1}{dr} = \frac{1}{NM} \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g} \right) > 0,$$

and

$$\frac{d\hat{e}_2}{dr} = \frac{M-1}{NM} > 0.$$

This concludes the proof. ■

Proof of Lemma 1

Proof. It is useful to state several intermediary results. The following lemma constructs a potential bias rule that would induce an arbitrary set of equilibrium winning probabilities $\boldsymbol{q} \equiv (q_1, \ldots, q_N) \in \Delta^{N-1}$ under transparency.

Lemma 5 *An equilibrium winning probability distribution $\boldsymbol{q} \equiv (q_1, \ldots, q_N)$ can be induced by a bias rule*

$$\delta_m = \begin{cases} \frac{1}{1-q_m} (q_m)^{\frac{1-r}{r}}, & \text{if } q_m > 0, \\ 0, & \text{if } q_m = 0. \end{cases}$$

Proof. In stage 2, the mth-ranked contestant chooses effort e_2^m for the maximization problem:

$$\max_{\{e_2^m\}} q_m(\boldsymbol{e_2}, \boldsymbol{\delta}) - e_2^m.$$

With simple algebraic transformation, the first-order condition with respect to e_2^m for an interior solution can be written as

$$\frac{r}{e_2^m} \times q_m(\boldsymbol{e_2}, \boldsymbol{\delta}) \times \left[1 - q_m(\boldsymbol{e_2}, \boldsymbol{\delta})\right] = 1, \text{ for } m \in \{1, \ldots, N\},$$

which allows us to rewrite the stage-2 equilibrium effort as follows:

$$e_2^m = q_m(\boldsymbol{e_2}, \boldsymbol{\delta}) \times \left[1 - q_m(\boldsymbol{e_2}, \boldsymbol{\delta})\right] r, \text{ for } m \in \{1, \ldots, N\}. \tag{20}$$

Note that Equation (20) also holds for a corner solution with $e_2^m = 0$, because $q_m = 0$ in this instance.

Clearly, for $q_m = 0$, the contest organizer can simply exclude the contestant by setting $\delta_m = 0$. For $q_i > 0$ and $q_j > 0$, it follows from Equations (4) and (20) that

$$\left(\frac{\delta_i}{\delta_j}\right)^r = \left(\frac{q_i}{q_j}\right)^{1-r} \times \left(\frac{1-q_i}{1-q_j}\right)^{-r} \Rightarrow \frac{\delta_i}{\delta_j} = \frac{\frac{1}{1-q_i}\left(q_i\right)^{\frac{1-r}{r}}}{\frac{1}{1-q_j}\left(q_j\right)^{\frac{1-r}{r}}}.$$

Therefore, one potential bias rule $\boldsymbol{\delta} \equiv (\delta_1, \ldots, \delta_N)$ that induces the winning probabilities $\boldsymbol{q} \equiv (q_1, \ldots, q_N)$ is

$$\delta_m = \begin{cases} \frac{1}{1-q_m}\left(q_m\right)^{\frac{1-r}{r}}, & \text{if } q_m > 0, \\ 0, & \text{if } q_m = 0. \end{cases}$$

This completes the proof. ■

The next lemma characterizes the optimal equilibrium winning probability distribution $\boldsymbol{q}^* \equiv (q_1^*, \ldots, q_N^*)$ that maximizes total effort under transparency.

Lemma 6 *Fix $N \geq 3$ and $r \in (0, 1]$. Suppose that the contest organizer maximizes the total effort in the contest. Then*

$$q_m^* = \begin{cases} \frac{1}{2} + \frac{1}{2}\frac{\alpha_m - \mu_\kappa}{1-\alpha_m r}, & \textit{if } m \in \{1, \ldots, \kappa\}, \\ 0, & \textit{if } m \in \mathcal{N} \setminus \{1, \ldots, \kappa\}, \end{cases} \tag{21}$$

where μ_κ is defined by (5) *and* (6)*.*

Proof. By Equation (20), the continuation value for obtaining the mth rank, denoted by V_m, can then be written as

$$V_m = q_m(\boldsymbol{e_2}, \boldsymbol{\delta}) - e_2^m = \left[1 - (1 - q_m)r\right] \times q_m. \tag{22}$$

Fixing $\boldsymbol{V} \equiv (V_1, \ldots, V_N)$ and other contestants' effort e_1, a representative contestant in the first stage chooses e_1' for the maximization problem:

$$\max_{\{e_1'\}} \left[\sum_{m=1}^{N} P_m(e_1', e_1) \times V_m\right] - e_1'.$$

The first-order condition, together with the symmetry condition $e_1' = e_1$ and Equation (12), gives

$$e_1 = \frac{r}{N} \times \sum_{m=1}^{N} \left[\left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g} \right) \times V_m \right] \equiv \frac{r}{N} \times \sum_{m=1}^{N} \left[\alpha_m V_m \right], \tag{23}$$

where $\alpha_m < 1$ is defined in (7). Combining (20), (22), and (23), we can express the total effort in the contest, which we denote by TE^{RT}, as a function of $\boldsymbol{q} \equiv (q_1, \ldots, q_N)$:

$$\begin{aligned} TE^{RT}(\boldsymbol{q}) &\equiv Ne_1 + \sum_{m=1}^{N} e_2^m \\ &= r \times \left[\sum_{m=1}^{N} \left\{ \alpha_m \left[(1-r)q_m + r(q_m)^2 \right] \right\} + \sum_{m=1}^{N} \left[q_m(1-q_m) \right] \right]. \end{aligned} \tag{24}$$

By Lemma 5, an arbitrary set of equilibrium winning probabilities $\boldsymbol{q} \equiv (q_1, \ldots, q_N)$ can be induced by some contest rule $\boldsymbol{\delta} \equiv (\delta_1, \ldots, \delta_N)$. Therefore, the contest organizer's optimization problem can be reformulated as the following: She chooses the equilibrium winning probability distribution $\boldsymbol{q}$ to maximize $TE^{RT}(\boldsymbol{q})$ in expression (24), subject to the plausibility constraint

$$\sum_{m=1}^{N} q_m = 1, \tag{25}$$

and the non-negativity constraints

$$q_m \geq 0, \text{ for } m \in \{1, \ldots, N\}. \tag{26}$$

Further, the usual participation constraint must be satisfied. The following lemma can be established:

Lemma 7 *Suppose that $i, j \in \mathcal{N}$ and $i < j$. Then $q_i^* \geq q_j^*$.*

Proof. Suppose to the contrary that $q_i^* < q_j^*$. Consider an alternative vector of the equilibrium winning probabilities, denoted by $\tilde{\boldsymbol{q}}^* \equiv (\tilde{q}_1^*, \ldots, \tilde{q}_N^*)$, as follows:

$$\tilde{q}_m^* = \begin{cases} q_i^*, & \text{if } m = j, \\ q_j^*, & \text{if } m = i, \\ q_m^*, & \text{if } m \in \mathcal{N} \setminus \{i, j\}. \end{cases}$$

Next, we show that $\tilde{\boldsymbol{q}}^*$ generates higher total effort than does $\boldsymbol{q}^*$. It follows from Equation (24) that

$$\frac{TE^{RT}(\boldsymbol{q}^*) - TE^{RT}(\tilde{\boldsymbol{q}}^*)}{r} = \sum_{m=1}^{N} \left\{ \alpha_m \left[(1-r) q_m^* + r (q_m^*)^2 \right] \right\} - \sum_{m=1}^{N} \left\{ \alpha_m \left[(1-r) \tilde{q}_m^* + r (\tilde{q}_m^*)^2 \right] \right\}$$
$$= (\alpha_i - \alpha_j) \times \left[(1-r) + r \left(q_i^* + q_j^* \right) \right] \times \left(q_i^* - q_j^* \right) < 0,$$

where the strict inequality follows from $\alpha_i > \alpha_j$, $(q_i^*, q_j^*) \geq (0,0)$ and the postulated $q_i^* < q_j^*$. Therefore, we have that $TE^{RT}(\boldsymbol{q}^*) < TE^{RT}(\tilde{\boldsymbol{q}}^*)$, which is a contradiction to the definition of $\boldsymbol{q}^*$. This completes the proof. ■

We are now ready to characterize the effort-maximizing equilibrium winning probability distribution $\boldsymbol{q}^*$. Consider the following sequence of auxiliary problems $(\mathcal{P}_n)$: for each $n = 2, \ldots, N$, the contest organizer maximizes $TE^{RT}(\boldsymbol{q})$ in (24) by ignoring the non-negativity constraint $q_m \geq 0$ for $m \in \{1, \ldots, n\}$ and setting $q_m = 0$ for $m \in \mathcal{N} \setminus \{1, \ldots, n\}$. By Lemma 7, the optimal solution to the original maximization problem must be one of the solutions to the $N-1$ auxiliary maximization problems.

It can further be verified that the solution to the auxiliary problem $(\mathcal{P}_n)$, which we denote by $\check{\boldsymbol{q}}^n \equiv (\check{q}_1^n, \ldots, \check{q}_N^n)$, is

$$\check{q}_m^n = \begin{cases} \frac{1}{2} + \frac{1}{2} \frac{\alpha_m - \mu_n}{1 - \alpha_m r}, & \text{if } m \in \{1, \ldots, n\}, \\ 0, & \text{if } m \in \mathcal{N} \setminus \{1, \ldots, n\}, \end{cases}$$

where μ_n is defined in (5) in the main text. It can also be verified that $\check{q}_1^n > \ldots > \check{q}_n^n$. If $\check{q}_n^n > 0$, then $\check{\boldsymbol{q}}^n \equiv (\check{q}_1^n, \ldots, \check{q}_N^n)$ is a potential maximizer to the original optimization problem.

Define

$$\kappa := \max \left\{ n = 2, \ldots, N \,|\, \check{q}_n^n > 0 \right\} \equiv \max \left\{ n = 2, \ldots, N \,|\, 1 + (1-r)\alpha_n > \mu_n \right\}.$$

Note that $\check{q}_2^2 > 0$ and thus the set $\left\{ n = 2, \ldots, N \,|\, \check{q}_n^n > 0 \right\}$ is finite and nonempty. Therefore, κ is well-defined and unique. For the auxiliary problem that maximizes $TE^{RT}(\boldsymbol{q})$, the maximal value of the objective function increases with n because more equality constraints—i.e., $q_m = 0$ for $m \in \mathcal{N} \setminus \{1, \ldots, n\}$—are dropped when n increases. This implies that the solution to the auxiliary problem $(\mathcal{P}_\kappa)$ is also the solution to the original maximization problem of the contest organizer. This completes the proof. ■

Denote the equilibrium stage-2 winning probabilities that maximize the expected winner's total effort under transparency by $\boldsymbol{q}^{**} \equiv (q_1^{**}, \ldots, q_N^{**})$. We further obtain the following lemma.

Lemma 8 *Fix $N \geq 3$ and $r \in (0,1]$. Suppose that the organizer aims to maximize the expected winner's total effort. Then the optimum requires $q_1^{**} = \frac{1}{2} \times \left[1 + \frac{1}{N(N-1)-[2(N+\frac{1}{N})-5]r}\right]$, $q_2^{**} = 1 - q_1^{**}$, and $q_m^{**} = 0$ for $m \geq 3$.*

Proof. Combining (20), (22), and (23), the expected winner's total effort under transparency, denoted by WE^{RT}, can be expressed with respect to $\boldsymbol{q} \equiv (q_1, \ldots, q_N)$ as follows:

$$\begin{aligned} WE^{RT}(\boldsymbol{q}) &\equiv e_1 + \sum_{m=1}^{N} [q_m e_2^m] \\ &= r \times \left[\frac{1}{N} \times \sum_{m=1}^{N} \left\{ \alpha_m \left[(1-r) q_m + r (q_m)^2 \right] \right\} + \sum_{m=1}^{N} \left[(q_m)^2 (1 - q_m) \right] \right]. \end{aligned} \tag{27}$$

The contest organizer chooses the equilibrium winning probabilities $\boldsymbol{q}$ to maximize $WE^{RT}(\boldsymbol{q})$, subject to the plausibility constraint (25) and the non-negativity constraints (26).

Lemma 9 *Suppose that $i, j \in \mathcal{N}$ and $i < j$. Then $q_i^{**} \geq q_j^{**}$.*

Proof. The proof is similar to that of Lemma 7, and is omitted for brevity. ■

Lemma 10 *$q_1^{**} > 0$, $q_2^{**} > 0$ and $q_m^{**} = 0$ for $m \geq 3$.*

Proof. It is evident that at least two contestants are active in the second stage and it suffices to rule out the possibility that $(q_1^{**}, q_2^{**}, q_3^{**}) > (0,0,0)$ by Lemma 9. Suppose that $(q_1^{**}, q_2^{**}, q_3^{**}) > (0,0,0)$. Fix $a_{ij} := q_i^{**} + q_j^{**} \in (0,1]$. Consider the following function $\phi_{ij}(x)$ with domain $x \in [0, a_{ij}]$.

$$\begin{aligned} \phi_{ij}(x) := &\frac{\alpha_i \left[(1-r)x + rx^2 \right] + \alpha_j \left[(1-r)(a_{ij} - x) + r(a_{ij} - x)^2 \right]}{N} \\ &+ \left[x^2 (1-x) + (a_{ij} - x)^2 (1 - a_{ij} + x) \right], \end{aligned}$$

where $i, j \in \{1,2,3\}$ and $i \neq j$. It is straightforward to verify that $\phi_{ij}(x)$ is quadratic in x. For $(q_1^{**}, q_2^{**}, q_3^{**}) > (0,0,0)$, we must have that $\arg\max_{x \in [0, a_{ij}]} \phi_{ij}(x) \in (0, a_{ij})$ for $i, j \in \{1,2,3\}$ and $i \neq j$, which requires $\phi_{ij}(x)$ to be strictly concave in x. It can be verified that $\phi_{ij}''(x) < 0$ is equivalent to

$$3 \left(q_i^{**} + q_j^{**} \right) \equiv 3a_{ij} > 2 + \frac{r}{N} \left(\alpha_i + \alpha_j \right).$$

Therefore, we must have that

$$q_1^{**} + q_2^{**} + q_3^{**} \equiv \frac{a_{12} + a_{13} + a_{23}}{2} > 1 + \frac{r}{6N} \sum_{i=1}^{3} \alpha_i = 1 + \frac{r}{6N} \left(3 - \frac{3}{N} - \frac{2}{N-1} - \frac{1}{N-2} \right) \geq 1,$$

where the last inequality follows from the postulated $N \geq 3$. Note that the above inequality contradicts $q_1^{**} + q_2^{**} + q_3^{**} \leq \sum_{m=1}^{N} q_m^{**} = 1$. This completes the proof. ■

We can now prove Lemma 8. By Lemma 10 and Equation (27), the contest organizer's optimization problem can be simplified as follows: She chooses $q_1 \in [0, 1]$ to maximize

$$\max_{\{q_1\}} \left\{ \frac{1}{N}\left(1 - \frac{1}{N}\right)\left[(1-r)q_1 + r(q_1)^2\right] \right.$$
$$\left. + \frac{1}{N}\left(1 - \frac{1}{N} - \frac{1}{N-1}\right)\left[(1-r)(1-q_1) + r(1-q_1)^2\right] + (1-q_1)q_1 \right\} r.$$

Note that the above expression is quadratic in q_1 and it is straightforward to verify that

$$q_1^{**} = \frac{1}{2} \times \frac{N + \left[\frac{1}{N-1} - \left(2 - \frac{2}{N} - \frac{1}{N-1}\right) r\right]}{N - \left(2 - \frac{2}{N} - \frac{1}{N-1}\right) r} = \frac{1}{2} \times \left[1 + \frac{1}{N(N-1) - \left[2\left(N + \frac{1}{N}\right) - 5\right] r}\right].$$

This completes the proof. ■

Part (i) of Lemma 1 follows immediately from Lemmata 5 and 6. Part (ii) follows immediately from Lemmata 5 and 8. This concludes the proof. ■

Proof of Lemma 2

Proof. The analysis is largely similar to that under transparency. We focus on the set of bias rules under which a symmetric pure-strategy equilibrium exists. Consider an arbitrary contestant's strategic problem by choosing his effort $(\hat{e}_1', \hat{e}_2')$ when other contestants have the same effort pair $(\hat{e}_1, \hat{e}_2)$. With slight abuse of notation, denote by $\hat{q}_m(\hat{e}_2', \hat{e}_2; \boldsymbol{\delta})$ a contestant's probability of winning the prize with a bias rule $\boldsymbol{\delta} \equiv (\delta_1, \ldots, \delta_N)$ in place, given that he is ranked in mth place after the first-stage competition. Recall that $P_m(\hat{e}_1', \hat{e}_1)$ is the probability of a contestant's being ranked in mth place. A representative contestant solves the following maximization problem:

$$\max_{\{\hat{e}_1', \hat{e}_2'\}} \sum_{m=1}^{N} \left[P_m(\hat{e}_1', \hat{e}_1) \times \hat{q}_m(\hat{e}_2', \hat{e}_2; \boldsymbol{\delta})\right] - \hat{e}_1' - \hat{e}_2',$$

where $P_m(\hat{e}_1', \hat{e}_1) \times \hat{q}_m(\hat{e}_2', \hat{e}_2; \boldsymbol{\delta})$ is his probability of being ranked in mth place while winning the prize in the end. The sum $\sum_{m=1}^{N} \left[P_m(\hat{e}_1', \hat{e}_1) \times \hat{q}_m(\hat{e}_2', \hat{e}_2; \boldsymbol{\delta})\right]$ is thus his overall probability of winning the contest. Anticipating a symmetric equilibrium, the organizer chooses $\hat{\boldsymbol{q}} \equiv$

$(\hat{q}_1, \ldots, \hat{q}_N)$ to maximize the total effort, denoted by $TE^{RO}(\hat{\boldsymbol{q}})$, in the contest

$$TE^{RO}(\hat{\boldsymbol{q}}) \equiv N\hat{e}_1 + N\hat{e}_2 = r \times \left\{ \sum_{m=1}^{N} [\alpha_m \hat{q}_m] + \sum_{m=1}^{N} \left[\hat{q}_m(1-\hat{q}_m)\right] \right\} =: r\Gamma_N(\hat{\boldsymbol{q}}), \tag{28}$$

or the expected winner's total effort, denoted by $WE^{RO}(\hat{\boldsymbol{q}})$,

$$WE^{RO}(\hat{\boldsymbol{q}}) \equiv \hat{e}_1 + \hat{e}_2 = \frac{r}{N}\Gamma_N(\hat{\boldsymbol{q}}), \tag{29}$$

subject to constraints (25) and (26), as well as to the usual participation constraint.

To proceed, we consider the relaxed optimization problem that maximizes $\Gamma_N(\cdot)$ subject to constraints (25) and (26), i.e., without considering the participation constraint. Simple algebra can verify that the relaxed optimization problem has the following maximizer:

$$\hat{q}_m^* = \begin{cases} \frac{1}{\hat{\kappa}} + \frac{\alpha_m}{2} - \frac{\sum_{s=1}^{\hat{\kappa}} \alpha_s}{2\hat{\kappa}}, & \text{if } m \in \{1, \ldots, \hat{\kappa}\}, \\ 0, & \text{if } m \in \mathcal{N} \setminus \{1, \ldots, \hat{\kappa}\}, \end{cases}$$

where $\hat{\kappa}$ is defined as

$$\hat{\kappa} := \max\left\{ n = 2, \ldots, N \,\middle|\, \frac{1}{n} + \frac{\alpha_n}{2} - \frac{\sum_{s=1}^{n} \alpha_s}{2n} > 0 \right\}.$$

We consider the following two cases:

Case I: $r \leq \frac{1}{\Gamma_N(\hat{\boldsymbol{q}}^*)}$. Clearly, a contestant's participation constraint is satisfied for the contest bias rule $\hat{\boldsymbol{q}}^*$, and thus $\hat{\boldsymbol{q}}^*$ is the optimal equilibrium winning probability distribution. Moreover, it can be verified that $\hat{\boldsymbol{q}}^*$ can be induced by a bias rule $\hat{\boldsymbol{\delta}}^* = \hat{\boldsymbol{q}}^*$.

Case II: $r > \frac{1}{\Gamma_N(\hat{\boldsymbol{q}}^*)}$. First, note that the total effort in any symmetric pure-strategy equilibrium does not exceed the value of the prize $\mathcal{V} = 1$; and thus the expected winner's total effort in any symmetric pure-strategy equilibrium does not exceed $\frac{1}{N}$. Therefore, if there exists a contest rule that yields a symmetric pure-strategy equilibrium that fully dissipates the rent, it must be optimal to the contest organizer.

Second, note that $\Gamma_N(\cdot)$ is continuous in all arguments. Moreover, we have that

$$\Gamma_N(1, 0, \ldots, 0) = \alpha_1 = 1 - \frac{1}{N} < 1 \leq \frac{1}{r}, \text{ and } \Gamma_N(\hat{\boldsymbol{q}}^*) > \frac{1}{r}.$$

Therefore, there must exist an equilibrium winning probability distribution $\grave{\boldsymbol{q}} = (\grave{q}_1, \ldots, \grave{q}_N)$

such that

$$\Gamma_N(\dot{\boldsymbol{q}}) = \frac{1}{r}.$$

Clearly, the total effort in the contest under the constructed bias rule $\dot{\boldsymbol{q}}$ is 1 and the rent is fully dissipated. Moreover, the contest rule where $\delta_m = \dot{q}_m$ for all $m \in \{1, \ldots, N\}$ leads to the desirable equilibrium winning probability distribution $\dot{\boldsymbol{q}}$. This completes the proof. ■

Proof of Proposition 2

Proof. We first prove the result for total effort. The comparison between opacity and transparency is obvious for $r > \frac{1}{\Gamma_N(\hat{\boldsymbol{q}}^*)}$, and it remains to consider the case where $r \leq \frac{1}{\Gamma_N(\hat{\boldsymbol{q}}^*)}$. Recall that the optimal winning probability distribution under transparency is $\boldsymbol{q}^* \equiv (q_1^*, \ldots, q_N^*)$ as defined in (21). It follows from Equations (24) and (28) that

$$\begin{aligned} TE^{RO}(\boldsymbol{q}^*) - TE^{RT}(\boldsymbol{q}^*) &= r \times \left[\sum_{m=1}^{N} [\alpha_m q_m^*] + \sum_{m=1}^{N} \left[q_m^*(1 - q_m^*)\right]\right] \\ &\quad - r \times \left[\sum_{m=1}^{N} \alpha_m \left[(1-r)q_m^* + r(q_m^*)^2\right] + \sum_{m=1}^{N} \left[q_m^*(1 - q_m^*)\right]\right] \\ &= r^2 \times \left[\sum_{m=1}^{N} \alpha_m q_m^*(1 - q_m^*)\right], \end{aligned}$$

and hence it remains to show that $\sum_{m=1}^{N} \alpha_m q_m^*(1 - q_m^*) > 0$. Note that

$$\sum_{m=1}^{N} \alpha_m = \sum_{m=1}^{N} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g}\right) = N - \sum_{g=0}^{N-1} \frac{N-g}{N-g} = 0.$$

Together with the fact that $\alpha_1 > \ldots > \alpha_N$, there exists a cutoff $3 \leq \overline{m} \leq N-1$ such that $\alpha_m \geq 0$ for $m < \overline{m}$ and $\alpha_m < 0$ otherwise. Let $\overline{q}$ be the corresponding q that maximizes $q_m^*(1 - q_m^*)$ for $m \geq \overline{m}$. It follows from Lemma 7 that $q_m^* \geq \overline{q}$ for $m < \overline{m}$. Therefore, we have that

$$\begin{aligned}\sum_{m=1}^{N}\left[\alpha_m q_m^*(1-q_m^*)\right] &= \sum_{m=1}^{\overline{m}-1}\left[\alpha_m q_m^*(1-q_m^*)\right] + \sum_{m=\overline{m}}^{N}\left[\alpha_m q_m^*(1-q_m^*)\right] \\ &\geq \sum_{m=1}^{\overline{m}-1}\left[\alpha_m q_m^*(1-q_m^*)\right] + \sum_{m=\overline{m}}^{N}\left[\alpha_m \overline{q}(1-\overline{q})\right] \\ &= \sum_{m=1}^{\overline{m}-1}\left[\alpha_m q_m^*(1-q_m^*)\right] - \overline{q}(1-\overline{q})\sum_{m=1}^{\overline{m}-1}\alpha_m \\ &= \sum_{m=1}^{\overline{m}-1}\left[\alpha_m\left(q_m^*-\overline{q}\right)\left(1-q_m^*-\overline{q}\right)\right] \geq \alpha_1\left(q_1^*-\overline{q}\right)\left(1-q_1^*-\overline{q}\right) > 0,\end{aligned}$$

where the first inequality follows from $\alpha_m < 0$ and $\overline{q}(1-\overline{q}) \geq q_m^*(1-q_m^*)$ for $m \geq \overline{m}$; the second equality follows from $\sum_{m=1}^{N}\alpha_m = 0$; the second inequality follows from $\alpha_m \geq 0$, $q_m^* \geq \overline{q}$ and $1-q_m^*-\overline{q} \geq 0$ for $2 \leq m \leq \overline{m}-1$; and the last strict inequality follows from the fact that $\alpha_1 = 1-\frac{1}{N} > 0$, $q_1^* > q_2^* \geq \overline{q}$ and $q_1^*+\overline{q} \leq 1-q_2^* < 1$. Therefore, opacity always generates higher total effort than transparency.

Next, we show that transparency generates a greater expected winner's total effort than does opacity. We consider the following two cases.

Case I: $N = 3$. By Lemma 8, the optimal equilibrium winning probability distribution under transparency is $(q_1^*, q_2^*, q_3^*) = \left(\frac{21-5r}{36-10r}, \frac{15-5r}{36-10r}, 0\right)$. Similarly, the optimal equilibrium winning probability distribution under opacity is $(\hat{q}_1^*, \hat{q}_2^*, \hat{q}_3^*) = \left(\frac{5}{8}, \frac{3}{8}, 0\right)$ from the proof of Lemma 2. Carrying out the algebra, we see that the difference between the maximum expected winner's total effort under transparency and that under opacity is

$$WE^{RT}\left(q_1^*, q_2^*, q_3^*\right) - WE^{RO}\left(\hat{q}_1^*, \hat{q}_2^*, \hat{q}_3^*\right) = \frac{r(6-5r)(69-20r)}{288(18-5r)},$$

which is strictly positive for all $r \in (0, 1]$.

Case II: $N \geq 4$. From a contestant's participation constraint, we can obtain that $WE^{RO}(\boldsymbol{q}) \leq \frac{1}{N}$. Together with Equation (29), we have that

$$\begin{aligned} WE^{RO}(\hat{\boldsymbol{q}}) &\leq \min\left\{\frac{1}{N}, \frac{r}{N} \times \left\{\sum_{m=1}^{N}\left[\alpha_m \hat{q}_m\right] + \sum_{m=1}^{N}\left[\hat{q}_m(1-\hat{q}_m)\right]\right\}\right\} \\ &\leq \frac{r}{N} \times \left\{\sum_{m=1}^{N}\left[\max\left\{\alpha_m, \alpha_2\right\}\hat{q}_m\right] + \sum_{m=1}^{N}\left[\hat{q}_m(1-\hat{q}_m)\right]\right\} \\ &= \frac{r}{N} \times \left\{\alpha_1\hat{q}_1 + \alpha_2\sum_{m=2}^{N}\hat{q}_m + \sum_{m=1}^{N}\left[\hat{q}_m(1-\hat{q}_m)\right]\right\} =: WE^{\dagger}_{max}(\hat{\boldsymbol{q}}), \end{aligned}$$

where the second inequality follows from $\alpha_1 > \ldots > \alpha_N$ and $\hat{q}_m \geq 0$ for all $m \in \{1, \ldots, N\}$. Therefore, we have that

$$WE^{RO}(\hat{\boldsymbol{q}}^*) \leq \max\left\{WE^{\dagger}_{max}(\hat{\boldsymbol{q}}) \,\middle|\, \sum_{m=1}^{N}\hat{q}_m = 1, \hat{q}_m \geq 0, m \in \{1, \ldots, N\}\right\}.$$

It can be verified that the solution to the maximization problem on the right-hand side of the above inequality, denoted by $\boldsymbol{q}^{\dagger} \equiv (q_1^{\dagger}, \ldots, q_N^{\dagger})$, is

$$q_1^{\dagger} = \frac{1}{N} + \frac{N-1}{N} \times \frac{\alpha_1 - \alpha_2}{2} = \frac{3}{2N}, \text{ and } q_2^{\dagger} = \ldots = q_N^{\dagger} = \frac{1-q_1^{\dagger}}{N-1} = \frac{2N-3}{2N(N-1)}.$$

Therefore, the expected winner's total effort under opacity can be bounded above by

$$WE^{RO}(\hat{\boldsymbol{q}}^*) \leq WE^{\dagger}_{max}(q_1^{\dagger}, \ldots, q_N^{\dagger}) = \frac{8N^2 - 20N + 13}{4N^2(N-1)}r.$$

Next, it follows from Equation (27) that the maximum expected winner's total effort under transparency can be bounded below by

$$\begin{aligned} WE^{RT}(\boldsymbol{q}^*) \geq WE^{RT}\left(\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0\right) &= r\left[\frac{\alpha_2}{N} + \frac{1}{2N(N-1)} + \frac{N - (\alpha_1+\alpha_2)r}{4N}\right] \\ &\geq r\left[\frac{\alpha_2}{N} + \frac{1}{2N(N-1)} + \frac{N - (\alpha_1+\alpha_2)}{4N}\right] \\ &= \frac{N^3 + N^2 - 5N + 2}{4N^2(N-1)}r, \end{aligned}$$

where the second inequality follows from $r \leq 1$. Therefore, it suffices to show that

$$\frac{N^3 + N^2 - 5N + 2}{4N^2(N-1)} r > \frac{8N^2 - 20N + 13}{4N^2(N-1)} r,$$

which is equivalent to

$$N^3 - 7N^2 + 15N - 11 > 0.$$

It is straightforward to verify that the above strict inequality holds for $N \geq 4$. This completes the proof. ■

Proof of Proposition 3

Proof. It can be verified that the equilibrium effort profile under transparency, denoted by $\big(e_1(\tau), e_2(\tau)\big)$, is

$$e_1(\tau) = \left\{ \frac{1}{1+\tau} \times \frac{r}{N} \times \left[\frac{1}{M} - \frac{1}{1+\tau} \times \frac{(M-1)r}{M^2} \right] \times \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g} \right) \right\}^{\frac{1}{1+\tau}},$$

and

$$e_2(\tau) = \left[\frac{1}{1+\tau} \times \frac{(M-1)r}{M^2} \right]^{\frac{1}{1+\tau}}.$$

Similarly, the equilibrium effort profile under opacity, denoted by $\big(\hat{e}_1(\tau), \hat{e}_2(\tau)\big)$, is given by

$$\hat{e}_1(\tau) = \left[\frac{1}{1+\tau} \times \frac{r}{NM} \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g} \right) \right]^{\frac{1}{1+\tau}},$$

and

$$\hat{e}_2(\tau) = \left[\frac{1}{1+\tau} \times \frac{(M-1)r}{NM} \right]^{\frac{1}{1+\tau}}.$$

Carrying out the algebra, we have that

$$\begin{aligned} N\left[e_1(\tau)\right]^{1+\tau} + M\left[e_2(\tau)\right]^{1+\tau} &\leq N\left[\hat{e}_1(\tau)\right]^{1+\tau} + N\left[\hat{e}_2(\tau)\right]^{1+\tau} \\ &= \frac{1}{1+\tau} \times \left[\frac{r}{M} \times \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g} \right) + \frac{(M-1)r}{M} \right] \\ &\leq \frac{r}{M} \times \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g} \right) + \frac{(M-1)r}{M} \leq 1, \end{aligned}$$

where the second inequality follows from $\tau \geq 0$, and the third inequality follows from Assumption 1. Therefore, the participation constraints are satisfied under both transparency and opacity, and thus the existence and the uniqueness of pure-strategy equilibria are guaranteed for both disclosure schemes.

Part (i) of the proposition follows instantly from the facts that $Me_2(\tau) < N\hat{e}_2(\tau)$ and $e_1(\tau) < \hat{e}_1(\tau)$; and it remains to prove part (ii), which is equivalent to showing that

$$\begin{aligned}\mathcal{G}(\tau) :=& \left[\frac{r}{NM}\sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right)\right]^{\frac{1}{1+\tau}} \\ & - \left\{\frac{r}{N}\times\left[\frac{1}{M}-\frac{1}{1+\tau}\times\frac{(M-1)r}{M^2}\right]\times\sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right)\right\}^{\frac{1}{1+\tau}} \\ & < \left[\frac{(M-1)r}{M^2}\right]^{\frac{1}{1+\tau}} - \left[\frac{(M-1)r}{NM}\right]^{\frac{1}{1+\tau}} =: \mathcal{H}(\tau), \text{ for sufficiently large } \tau.\end{aligned}$$

Note that $\mathcal{G}(\tau) > 0$ and $\mathcal{H}(\tau) > 0$ for all $\tau \geq 0$; and $\lim_{\tau\to\infty}\mathcal{G}(\tau) = \lim_{\tau\to\infty}\mathcal{H}(\tau) = 0$. Moreover, it can be verified that

$$\mathcal{H}'(\tau) = -\frac{1}{(1+\tau)^2}\times\left\{\log\left[\frac{(M-1)r}{M^2}\right]\times\left[\frac{(M-1)r}{M^2}\right]^{\frac{1}{1+\tau}} - \log\left[\frac{(M-1)r}{NM}\right]\times\left[\frac{(M-1)r}{NM}\right]^{\frac{1}{1+\tau}}\right\},$$

and

$$\begin{aligned}\mathcal{G}'(\tau) =& -\frac{1}{(1+\tau)^2}\times\left(\frac{\Delta}{M}\right)^{\frac{1}{1+\tau}}\times\log\left(\frac{\Delta}{M}\right) \\ & -\frac{1}{(1+\tau)^2}\times\left\{\Delta\times\left[\frac{1}{M}-\frac{1}{1+\tau}\times\frac{(M-1)r}{M^2}\right]\right\}^{\frac{1}{1+\tau}}\times\log\left\{\Delta\times\left[\frac{1}{M}-\frac{1}{1+\tau}\times\frac{(M-1)r}{M^2}\right]\right\} \\ & +\frac{1}{(1+\tau)^3}\times\left\{\Delta\times\left[\frac{1}{M}-\frac{1}{1+\tau}\times\frac{(M-1)r}{M^2}\right]\right\}^{\frac{1}{1+\tau}}\times\frac{\frac{(M-1)r}{M^2}}{\frac{1}{M}-\frac{1}{1+\tau}\times\frac{(M-1)r}{M^2}},\end{aligned}$$

where $\Delta := \frac{r}{N}\sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right)$. Applying L'Hospital's Rule, we obtain

$$\lim_{\tau\to\infty}\frac{\mathcal{G}(\tau)}{\mathcal{H}(\tau)} = \lim_{\tau\to\infty}\frac{\mathcal{G}'(\tau)}{\mathcal{H}'(\tau)} = \frac{\log\left(\frac{\Delta}{M}\right)-\log\left(\frac{\Delta}{M}\right)}{\log\left[\frac{(M-1)r}{M^2}\right]-\log\left[\frac{(M-1)r}{NM}\right]} = 0.$$

Therefore, $\frac{\mathcal{G}(\tau)}{\mathcal{H}(\tau)} < 1$, or equivalently, $\mathcal{G}(\tau) < \mathcal{H}(\tau)$, when τ is sufficiently large. This completes the proof. ■

Proof of Proposition 4

Proof. Simple algebra would verify that the equilibrium effort profile under transparency, denoted by $\big(e_1(r_1, r_2), e_2(r_1, r_2)\big)$, is

$$e_1(r_1, r_2) = \frac{r_1 \left[M - (M-1)r_2\right]}{NM^2} \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g}\right), \text{ and } e_2(r_1, r_2) = \frac{(M-1)r_2}{M^2}. \quad (30)$$

From the above expression, as r_2 increases, stage-1 effort decreases and stage-2 effort increases. In contrast, r_1 has no impact on stage-2 effort. Similarly, the equilibrium effort profile under opacity, denoted by $\big(\hat{e}_1(r_1, r_2), \hat{e}_2(r_1, r_2)\big)$, can be derived as the following:

$$\hat{e}_1(r_1, r_2) = \frac{r_1}{NM} \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g}\right), \text{ and } \hat{e}_2(r_1, r_2) = \frac{(M-1)r_2}{NM}. \quad (31)$$

Proposition 4 follows immediately from comparing the resulting total effort and the expected winner's total effort under transparency and opacity, and is omitted for brevity. ■

Proof of Proposition 5

Proof. Fix an arbitrary prize allocation profile $\boldsymbol{\mathcal{V}} := (\mathcal{V}_1, \ldots, \mathcal{V}_M)$, with $\mathcal{V}_1 \geq \ldots \geq \mathcal{V}_M \geq 0$ and $\sum_{m=1}^{M} \mathcal{V}_m = 1$. By standard technique, the equilibrium period-2 effort under transparency, which we denote by $e_2(\boldsymbol{\mathcal{V}})$ with slight abuse of notation, can be derived as

$$e_2(\boldsymbol{\mathcal{V}}) = \frac{r}{M} \times \sum_{j=1}^{M} \left[\beta_j \mathcal{V}_j\right],$$

where $\beta_j < 1$ is defined as

$$\beta_j := 1 - \sum_{g=0}^{j-1} \frac{1}{M-g}, \text{ for } j \in \{1, \ldots, M\}.$$

The equilibrium period-1 effort, denoted by $e_1(\boldsymbol{\mathcal{V}})$, is given by

$$e_1(\boldsymbol{\mathcal{V}}) = \frac{r}{N} \times \left[\sum_{m=1}^{M} \alpha_m\right] \times \left[\frac{1}{M} - e_2(\boldsymbol{\mathcal{V}})\right],$$

where $\alpha_m < 1$ is defined in (7). Therefore, the total effort amounts to

$$Ne_1(\boldsymbol{\mathcal{V}}) + Me_2(\boldsymbol{\mathcal{V}}) = \frac{r}{M} \times \left[\sum_{m=1}^{M} \alpha_m\right] + \left[\sum_{m=1}^{M} (1 - \alpha_m r)\right] \times e_2(\boldsymbol{\mathcal{V}}).$$

It is straightforward to verify that $\sum_{m=1}^{M} (1 - \alpha_m r) > 0$ under Assumption 1 and that $e_2(\boldsymbol{\mathcal{V}})$ is maximized at $\boldsymbol{\mathcal{V}} = (1, 0, \dots, 0)$. Therefore, the total effort is maximized by setting $\boldsymbol{\mathcal{V}} = (1, 0, \dots, 0)$. Similarly, we can show that the expected winner's total effort is also maximized by setting $\boldsymbol{\mathcal{V}} = (1, 0, \dots, 0)$.

The analysis under opacity is similar. Again, by standard technique, a player's effort profile in a symmetric equilibrium under opacity, which we denote by $\big(\hat{e}_1(\boldsymbol{\mathcal{V}}), \hat{e}_2(\boldsymbol{\mathcal{V}})\big)$, is

$$\hat{e}_1(\boldsymbol{\mathcal{V}}) = \frac{r}{NM} \sum_{m=1}^{M} \alpha_m,$$

and

$$\hat{e}_2(\boldsymbol{\mathcal{V}}) = \frac{r}{N} \times \sum_{j=1}^{M} \left[\beta_j \mathcal{V}_j\right].$$

It is evident that $\hat{e}_1(\boldsymbol{\mathcal{V}})$ is independent of the period-2 prize structure $\boldsymbol{\mathcal{V}} \equiv (\mathcal{V}_1, \dots, \mathcal{V}_M)$. Moreover, the fact that $\beta_1 > \dots > \beta_M$ implies immediately that $\hat{e}_2(\boldsymbol{\mathcal{V}})$ is maximized by setting $\boldsymbol{\mathcal{V}} = (1, 0, \dots, 0)$. This concludes the proof. ■

Proof of Proposition 6

Proof. The proof is straightforward for $N = 3$, and it remains to analyze the case where $N \geq 4$. It can be verified that $\bar{r}(N, M) \leq 1$ for $N \geq 4$. We first show that $TE^T(N, M, r) > TE^T(N, 1, r)$. Note that

$$\sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g}\right) = (N - M) \left(\sum_{g=0}^{M-1} \frac{1}{N-g}\right) > (N - M) \times \frac{M}{N}. \tag{32}$$

Together with Equation (16), we have that

$$\begin{aligned}
TE^T(N,M,r) =& \frac{r\left[M-(M-1)r\right]}{M^2} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \frac{(M-1)r}{M} \\
\geq& \frac{r}{M^2} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \frac{(M-1)r}{M} \\
>& \frac{r}{M^2} \times (N-M) \times \frac{M}{N} + \frac{(M-1)r}{M} \\
=& \frac{(N-1)r}{N} = TE^T(N,1,r),
\end{aligned}$$

where the first inequality follows from $r \leq \bar{r}(N,M) \leq 1$ and the second inequality follows from (32).

Similarly, by Equation (17), we can obtain that

$$\begin{aligned}
WE^T(N,M,r) =& \frac{r\left[M-(M-1)r\right]}{NM^2} \times \sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \frac{(M-1)r}{M^2} \\
>& \frac{r}{NM^2} \times (N-M) \times \frac{M}{N} + \frac{(M-1)r}{M^2} \\
>& \frac{(N-1)r}{N^2} = WE^T(N,1,r),
\end{aligned}$$

where the first inequality follows from $r \leq \bar{r}(N,M) \leq 1$ and (32), and the second inequality follows from the fact that $(M-1)/M^2 > (N-1)/N^2$ for $N > M \geq 2$.

Further, by Equation (18), we can obtain that

$$\begin{aligned}
TE^O(N,M,r) =& \frac{r}{M}\sum_{m=1}^{M}\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right) + \frac{(M-1)r}{M} \\
>& (N-M) \times \frac{r}{N} + \frac{(M-1)r}{M} \\
>& \frac{(N-1)r}{N} = TE^O(N,1,r),
\end{aligned}$$

where the first inequality follows again from (32) and the second inequality follows from $N > M$. Last, note that $WE^O(N,M,r) = TE^O(N,M,r)/N$ for all $M \in \{1,\ldots,N\}$. Together with the above inequality, we have that $WE^O(N,M,r) > WE^O(N,1,r)$. This concludes the proof. ■

Proof of Proposition 7

Proof. Standard technique yields the following:

(i) Under transparency, in the symmetric pure-strategy equilibrium, a contestant chooses stage-1 effort $e_1 = \frac{13}{3456}r(2-r)(12-5r)$, stage-2 effort $e_2 = \frac{5}{72}r(2-r)$, and stage-3 effort $e_3 = \frac{1}{4}r$, provided that he has survived the elimination.

(ii) Under opacity, in the symmetric pure-strategy equilibrium, a contestant chooses stage-1 effort $\hat{e}_1 = \frac{13}{144}r$, stage-2 effort $\hat{e}_2 = \frac{5}{48}r$, and stage-3 effort $\hat{e}_3 = \frac{1}{8}r$.

Therefore, the total effort under transparency, denoted by $TE^T(4,3,2,r)$ amounts to

$$TE^T(4,3,2,r) = 4e_1 + 3e_2 + 2e_3 = \frac{13}{864}r(2-r)(12-5r) + \frac{5}{24}r(2-r) + \frac{1}{2}r,$$

and the expected winner's total effort under transparency, denoted by $WE^T(4,3,2,r)$, is

$$WE^T(4,3,2,r) = e_1 + e_2 + e_3 = \frac{13}{3456}r(2-r)(12-5r) + \frac{5}{72}r(2-r) + \frac{1}{4}r.$$

Similarly, the total effort under opacity, denoted by $TE^O(4,3,2,r)$, amounts to

$$TE^O(4,3,2,r) = 4\hat{e}_1 + 4\hat{e}_2 + 4\hat{e}_3 = \frac{23}{18}r,$$

and the expected winner's total effort under opacity, denoted by $WE^O(4,3,2,r)$, is

$$WE^O(4,3,2,r) = \hat{e}_1 + \hat{e}_2 + \hat{e}_3 = \frac{23}{72}r.$$

It is straightforward to verify that a representative contestant's participation constraints under transparency and opacity are satisfied for $r \leq \frac{18}{23}$. Moreover, simple algebra would verify that a three-stage elimination contest $(4,3,2,r)$ generates the highest total effort and expected winner's total effort under both transparency and opacity for $r \leq \frac{18}{23}$. This concludes the proof. ■

Disclosure and Favoritism in Sequential Elimination Contests

ONLINE APPENDIX

Qiang Fu* Zenan Wu†

In this online appendix we collect the materials omitted from the main text of the paper.[1] The appendices are ordered according to where they are first referenced in the main text. Online Appendix A allows the contest organizer to randomize between full disclosure and no disclosure, and shows that partial disclosure is suboptimal. Therefore, it is without loss of generality to focus on the comparison between full disclosure and no disclosure in Proposition 1. Online Appendix B relaxes the restriction that the contest organizer must select the bias rule from those that induce a symmetric pure-strategy equilibrium, and demonstrates that Proposition 2 remains largely intact. Online Appendix C presents a three-player example to illustrate contest design with endogenous biases.

A Randomized Disclosure Schemes

We have assumed that the contest organizer chooses between full disclosure and no disclosure in the main text. In this section, we enrich the set of candidate disclosure schemes by allowing for partial disclosure. For the sake of simplicity, we employ and extend the setup in the baseline N-M two-stage model in Section 2.

Specifically, instead of full disclosure and no disclosure, the contest organizer now commits to a disclosure scheme indexed by $\mu \in [0, 1]$, where μ denotes the probability that the interim rankings are disclosed. Clearly, full transparency corresponds to $\mu = 1$ and full opacity corresponds to $\mu = 0$.

*Department of Strategy and Policy, National University of Singapore, 15 Kent Ridge Drive, Singapore, 119245. Email: bizfq@nus.edu.sg

†School of Economics, Peking University, Beijing, China, 100871. Email: zenan@pku.edu.cn

[1]This note is not self-contained; it is the online appendix of the paper "Disclosure and Favoritism in Sequential Elimination Contests."

Denote the optimal disclosure scheme that maximizes the total effort and the expected winner's total effort by μ^* and μ^{**}, respectively. Standard technique leads to the following.

Proposition A1 ***(Suboptimality of Randomized Disclosure Schemes)*** *Consider a two-stage elimination contest. Suppose that Assumption 1 is satisfied and the contest organizer is allowed to randomize between full disclosure and no disclosure. Then $\mu^* \in \{0,1\}$ and $\mu^{**} \in \{0,1\}$.*

Proof. Fixing a disclosure rule $\mu \in [0,1]$, a symmetric pure-strategy equilibrium is characterized by the triple $(e_{1p}, e_{2p}, \hat{e}_{2p})$, where e_{1p} is a representative contestant's stage-1 effort, and e_{2p} (respectively, $\hat{e}_{2p}$) denotes the contestant's stage-2 effort when the interim rankings are disclosed (respectively, concealed). We use subscript p to indicate "partial disclosure."

Fixing $\mu \in [0,1]$, a contestant's stage-2 effort when the interim rankings are publicized is equal to that under transparency, i.e.,

$$e_{2p} = \frac{M-(M-1)r}{M^2}, \tag{A1}$$

and it remains to pin down $(e_{1p}, \hat{e}_{2p})$. Fixing the other contestants' effort profiles $(e_{1p}, \hat{e}_{2p})$, let a contestant choose $(e'_{1p}, \hat{e}'_{2p})$ for the following optimization problem:

$$\max_{\{\hat{e}'_{1p},\hat{e}'_{2p}\}} \mu V \left[\sum_{m=1}^{M} P_m(e'_{1p}, \boldsymbol{e}_{1p}^{-i})\right] + (1-\mu)\left\{\left[\sum_{m=1}^{M} P_m(\hat{e}'_{1p}, \hat{\boldsymbol{e}}_{1p}^{-i})\right] \times \frac{(\hat{e}'_{2p})^r}{(\hat{e}'_{2p})^r + (M-1)(\hat{e}_{2p})^r} - \hat{e}'_{2p}\right\} - e'_{1p},$$

where V is defined in Equation (13). Analogously to the analysis in the proof of Lemmata 3 and 4, $(e_{1p}, \hat{e}_{2p})$ can be solved as follows:

$$\hat{e}_{2p} = \frac{(M-1)r}{NM}, \tag{A2}$$

and

$$e_{1p} = \frac{\left[\mu MV + (1-\mu)\right] r}{NM} \times \sum_{m=1}^{M} \left(1 - \sum_{g=0}^{m-1} \frac{1}{N-g}\right). \tag{A3}$$

It can be verified that contestants' participation constraints are satisfied under Assumption 1, and thus the effort profile specified by (A1)-(A3) constitutes a unique symmetric pure-strategy equilibrium. In addition, the total effort and the expected winner's total effort, which we denote by $TE_p(\mu)$ and $WE_p(\mu)$, respectively, are

$$TE_p(\mu) \equiv Ne_{1p} + \mu M e_{2p} + (1-\mu) N \hat{e}_{2p},$$

and

$$WE_p(\mu) \equiv e_{1p} + \mu e_{2p} + (1-\mu)\hat{e}_{2p}.$$

Note that e_{2p} and $\hat{e}_{2p}$ are independent of μ from Equations (A1) and (A2), and e_{1p} is linear in μ from Equation (A3). Therefore, $TE_p(\mu)$ and $WE_p(\mu)$ are convex in μ, implying that $\mu^* \in \{0,1\}$ and $\mu^{**} \in \{0,1\}$. This completes the proof. ■

By Proposition A1, randomization (i.e., $\mu \in (0,1)$) is always suboptimal regardless of the organizer's objective.

B Alternative Bidding Equilibria in Excessively Discriminatory Contests

The results in Section 4 are obtained under the condition that the organizer optimizes by choosing a bias rule $\hat{\boldsymbol{\delta}}$ from those that induce a symmetric pure-strategy equilibrium under opacity. In this section, we relax this restriction and show that Proposition 2 remains robust.

As seen from Lemma 2, when $r \leq 1/\Gamma_N(\hat{\boldsymbol{q}}^*)$, the constraint is nonbinding as all bias rules induce symmetric pure-strategy equilibria. The restriction, however, does limit the set of potential bias rules for optimization when r exceeds the cutoff. An increase in r encourages more aggressive bidding, which could cause the participation condition to break down and dissolve symmetric pure-strategy equilibrium, as in typical static contests. The comparison between transparency and opacity in Proposition 2 is immune to this result when the organizer aims to maximize total effort. To be more specific, Lemma 2(ii) states that there exists a bias rule that induces full rent dissipation in a symmetric pure-strategy equilibrium, in which case a total effort of 1 results. This indeed reaches the limit of the contest design, and no other mechanism could outperform it. The global optimality of opacity can therefore be established.

The same, however, cannot be said when the organizer's objective is to maximize the expected winner's total effort. With the restriction of a symmetric pure-strategy equilibrium, the expected winner's total effort is bounded above by $1/N$, which falls below the maximum under transparency from Proposition 2. In this case, more aggressive bias rules exist and they break down the symmetric pure-strategy equilibrium by violating the participation constraint. In general, multiple equilibria could arise under large r; for instance, there could exist symmetric mixed-strategy equilibria. Alternatively, there could exist *semi-symmetric equilibria* that resemble those in contests with endogenous entry depicted by Fu, Jiao and Lu (2015): In such equilibria, a subset of contestants play symmetric pure-strategy bidding among themselves, while the rest stay inactive by bidding zero with probability one. There may also exist many other types of asymmetric equilibria that involve various forms of randomization. These equilibria could lead to a greater expected winner's total effort than under the restriction of symmetric equilibrium in pure strategies. To see that, imagine a situation with $r > 1/\Gamma_N(\hat{\boldsymbol{q}}^*)$. In a restricted optimum, the contest ends up with an expected winner's total effort $1/N$. If the organizer instead sets a rule that breaks down this equilibrium and induces a semi-symmetric equilibrium with $N-1$ active contestants, the expected winner's total effort is then bounded by $1/(N-1)$ instead of $1/N$.

It is technically challenging to fully characterize these equilibria in our context. Recall that the dynamic linkage between stages dissolves in the contest under opacity, which leads contestants to behave as if they were choosing multiple actions simultaneously, i.e., $\hat{e}_1$ and $\hat{e}_2$, in a static contest. The literature provides little guidance in solving for asymmetric or mixed-strategy equilibria in imperfectly discriminatory contests that involve multi-dimensional strategies: In such a scenario, each contestant can randomize in either dimension, i.e., either $\hat{e}_1$ or $\hat{e}_2$. This is particularly challenging in our context because (i) the probability of winning in Tullock contests is discontinuous at the origin; and (ii) one's stage-2 outcome ultimately depends on stage-1 outcome, despite the dissolved dynamic linkage due to opacity. Moreover, the multiplicity of equilibria imposes conceptual limitations on contest design: It is difficult to predict the performance of the contest when the particular equilibrium to be played under a given bias rule remains ambiguous.

Despite the limitations, our result does not lose its bite when we allow for the aforementioned semi-symmetric equilibria and consider bias rules that could induce semi-symmetric equilibria. Imagine an equilibrium that involves $N' \in \{3, \ldots, N-1\}$ active contestants under opacity. The contest in this equilibrium is essentially equivalent to an alternative N'-contest in a symmetric pure-strategy equilibrium. As a result, enlarging the set of eligible bias rules to allow for these semi-symmetric equilibria is no different than letting the contest organizer *shortlist* the contestants—i.e., excluding $N - N'$ contestants and inviting the rest to participate in a two-stage contest—while optimizing over the set of bias rules that induce a symmetric pure-strategy equilibrium. We then consider an alternative optimization problem: Under a given disclosure policy, the organizer sets the optimal number of participants, and chooses the optimal bias rule accordingly over the set of candidate rules that induce a symmetric pure-strategy equilibrium.

Lemma A1 $\Gamma_N(\hat{\boldsymbol{q}}^*)$ *strictly increases with* N.

Proof. To highlight the fact that $\hat{\boldsymbol{q}}^*$ depends on N, let us denote the optimal winning probabilities for the case of N contestants by $\hat{\boldsymbol{q}}^*_N := (\hat{q}^*_{1N}, \ldots, \hat{q}^*_{NN})$. To prove the lemma, it suffices to show that $\Gamma_N(\hat{\boldsymbol{q}}^*_N) < \Gamma_{N+1}(\hat{\boldsymbol{q}}^*_{N+1})$. Let $\hat{\boldsymbol{q}}^\star_{N+1} = (\hat{q}^*_{1N}, \ldots, \hat{q}^*_{NN}, 0)$. It follows from

Equation (28) that

$$\Gamma_{N+1}(\hat{\boldsymbol{q}}^{\star}_{N+1}) - \Gamma_N(\hat{\boldsymbol{q}}^{*}_{N}) = \sum_{m=1}^{N}\left\{\left[\left(1-\sum_{g=0}^{m-1}\frac{1}{N+1-g}\right)-\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right)\right]\times \hat{q}^{*}_{mN}\right\}$$
$$= \frac{1}{N+1}\times\sum_{m=1}^{N}\left[\frac{m}{N-m+1}\times\hat{q}^{*}_{mN}\right] > 0.$$

Therefore, we have that $\Gamma_{N+1}(\hat{\boldsymbol{q}}^{*}_{N+1}) \geq \Gamma_{N+1}(\hat{\boldsymbol{q}}^{\star}_{N+1}) > \Gamma_N(\hat{\boldsymbol{q}}^{*}_{N})$. This completes the proof. ■

By Lemma A1, the cutoff $\frac{1}{\Gamma_N(\hat{\boldsymbol{q}}^*)}$ strictly decreases when the number of participants increases. When fewer participants are involved, a symmetric pure-strategy equilibrium is more likely to emerge. In other words, the organizer, when narrowing the pool, ends up with additional freedom in choosing the bias rule. Note that $\Gamma_3(\hat{\boldsymbol{q}}^*) = \frac{11}{12} < 1$, which implies that if the organizer invites only three participants, she can induce a symmetric pure-strategy equilibrium for any contest rule under Assumption 2. The following result can then be obtained.

Proposition A2 *Fix $N \geq 4$ and $r \in (0, 1]$. Suppose that the contest organizer is allowed to shortlist contestants and select $N' \in \{3, \ldots, N\}$ of them for the competition. When the contest organizer is able to set the bias rule for the second-stage competition, she always prefers transparency to opacity if she aims to maximize the expected winner's total effort.*

We do not have to lay out a formal proof, as the logic is straightforward. Suppose that the optimum under opacity requires N participants. Then the optimum under opacity is outperformed by that under transparency by Proposition 2. Suppose otherwise that it requires $N' \in \{3, \ldots, N-1\}$ participants, which demands that the organizer shortlist. The optimum is still outperformed by that under transparency: The organizer, under transparency, can shortlist the same number N' of participants and set the optimal bias rule accordingly, which again generates a greater expected winner's total effort by Proposition 2. We thus restore the optimality of transparency in a broader setting.

C Three-player Example of Optimal Contest Design with Endogenous Biases

As stated in Section 4, we can establish a correspondence between contestants' efforts and winning probabilities in equilibrium. This further allows us to rewrite design objectives, total effort and the expected winner's total effort, as functions of the equilibrium winning probability distribution. Our optimization approach lets the organizer choose equilibrium winning probability distribution to maximize reformulated objective functions. Table A1 summarizes the equilibrium winning probability distribution in the optimal contest under transparency for the case of $N = 3$.

Transparency	q_1^*	q_2^*	q_3^*	$TE^{RT}(\boldsymbol{q}^*)$
$\frac{63-3\sqrt{241}}{50} < r \leq 1$	$\frac{8r^2+9r-72}{3(7r^2-36)}$	$\frac{r^2+45r-90}{6(7r^2-36)}$	$\frac{25r^2-63r+18}{6(7r^2-36)}$	$\frac{r(5r^3+33r^2+171r-621)}{126r^2-648}$
$0 < r \leq \frac{63-3\sqrt{241}}{50}$	$\frac{15-5r}{2(12-5r)}$	$\frac{9-5r}{2(12-5r)}$	0	$\frac{r(25r^2-170r+273)}{288-120r}$
Transparency	q_1^{**}	q_2^{**}	q_3^{**}	$WE^{RT}(\boldsymbol{q}^{**})$
$0 < r \leq 1$	$\frac{21-5r}{36-10r}$	$\frac{15-5r}{36-10r}$	0	$\frac{r(25r^2-230r+513)}{1296-360r}$

Table A1: Optimal Equilibrium Winning Probabilities under Transparency in Three-Player Contests.

By Table A1, when $r > \frac{63-3\sqrt{241}}{50} \approx 0.3285$, the optimal contest involves three active players in the second stage—i.e., $q_1^* > q_2^* > q_3^* > 0$—and the equilibrium winning distribution can be induced by a bias rule $(\delta_1^*, \delta_2^*, \delta_3^*) = \left(\frac{1}{1-q_1^*}(q_1^*)^{\frac{1-r}{r}}, \frac{1}{1-q_2^*}(q_2^*)^{\frac{1-r}{r}}, \frac{1}{1-q_3^*}(q_3^*)^{\frac{1-r}{r}}\right)$. When $r \leq \frac{63-3\sqrt{241}}{50} \approx 0.3285$, the optimal contest involves two active players in the second stage—i.e., $q_1^* > q_2^* > q_3^* = 0$—and the equilibrium winning distribution can be induced by a bias rule $(\delta_1^*, \delta_2^*, \delta_3^*) = \left(\frac{1}{1-q_1^*}(q_1^*)^{\frac{1-r}{r}}, \frac{1}{1-q_2^*}(q_2^*)^{\frac{1-r}{r}}, 0\right)$.

Table A2 summarizes the equilibrium winning probability distribution in the optimal contest under opacity.

Opacity	$\hat{q}_1^*$	$\hat{q}_2^*$	$\hat{q}_3^*$	$TE^{RT}(\hat{\boldsymbol{q}}^*)$	$WE^{RT}(\hat{\boldsymbol{q}}^*)$
$0 < r \leq 1$	$\frac{5}{8}$	$\frac{3}{8}$	0	$\frac{91}{96}r$	$\frac{91}{298}r$

Table A2: Optimal Equilibrium Winning Probabilities under Opacity in Three-Player Contests.

Although the bottom-ranked contestant has zero chance of winning the prize in the optimum, he is uninformed of his status and continues to exert effort in the second stage. The optimal equilibrium winning probability distribution simultaneously maximizes the total effort exerted in the overall contest and the expected winner's total effort, and is independent of the discriminatory power of the contest technology (i.e., r).

References

Fu, Qiang, Qian Jiao, and Jingfeng Lu, "Contests with endogenous entry," *International Journal of Game Theory*, 2015, *44* (2), 387–424.