Selective Disclosure and Scoring Bias in Common-Value Contests*

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Abstract

Two ex ante identical players compete for a prize of a common, but initially unknown, value in an all-pay auction. A designer decides whether and how to disclose an informative signal of the prize's value to players and sets the scoring rule. When the designer aims to maximize the expected winner's effort, a tilting-and-releveling contest may be optimal: The designer discloses the signal privately to one player while biasing the scoring rule in favor of the other. In contrast, when the designer maximizes expected total effort, a fully symmetric contest—with symmetric information and a neutral scoring rule—is always optimal. We further show that our main result of tilting-and-releveling is robust to four extensions: credibility of the announced disclosure policy, the expected maximum-effort objective, lottery contests, and endogenous information design.

Keywords: All-pay Auction; Contest Design; Information Favoritism; Scoring Bias.

JEL Classification Codes: C72, D44, D82.

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1 Introduction

The enduring conflict between fairness and efficiency/incentives is reconciled in contest-like competitive activities that are commonplace in the modern socioeconomic landscape. These range from college admissions, sporting events, and competitive procurement (Che and Gale, 2003) to internal labor markets inside firms (Lazear and Rosen, 1981; Green and Stokey, 1983; Nalebuff and Stiglitz, 1983; Rosen, 1986). The conventional wisdom has long emphasized the importance of a level playing field as an incentive device to foster competition (see, e.g., Dixit, 1987). This insight provides a rationale for a diverse array of practices that aim to correct initial disparities among competitors. For example, in horse racing, favorite horses are often required to carry extra weight, and governments offer greater support to small and medium-sized enterprises in public procurement.

Substantial scholarly effort has been dedicated to developing various strategies to narrow the gap between asymmetric contestants who may have different levels of innate abilities. The literature typically focuses on discriminatory measures that advantage underdogs or handicap front-runners to create even races. In reality, however, contest-like competitions often take place in more complex environments. Many factors beyond the differences in players' abilities can influence their incentives. This complexity also creates greater opportunities for a designer to manipulate the rules of the competition to advance her goals.

For instance, contestants frequently face uncertainty regarding the contest's nature and the surrounding environment, such as the prize value. When competing for a promotion, employees may not fully understand the nuances of the new role, e.g., the scope of responsibilities, available resources, and implications for their career trajectory. In another context, contractors competing for government procurement may not know the true costs of fulfilling the contract. Contestants' behavior can arguably be influenced by the information available to them. This environment suggests a potential for strategic information disclosure: whether to disclose any available information and, if the designer chooses to do so, to whom.

Suppose the contest designer has discretion in two dimensions: (i) employing discriminatory measures to alter contestants' relative competitiveness and (ii) choosing to disclose or conceal prize value information among players, either symmetrically or selectively. This scenario raises numerous questions. Will a level playing field that equalizes players' relative competitiveness remain optimal? Is an equal distribution of information always optimal? How may discriminatory measures interact with the information disclosure scheme—do they substitute for or complement each other? Prior literature offers limited insights, because it typically focuses on contest design within a single dimension.

Our paper fills this gap and joins the growing literature on contest design with multiple instruments (Halac, Kartik, and Liu, 2017; Ely, Georgiadis, Khorasani, and Rayo, 2023). Focusing on the joint deployment of discriminatory measures and the information disclosure scheme, we demonstrate that the optimum could drastically depart from the conventional wisdom obtained in the context of one-dimensional contest design (Proposition 1). In particular, we find that, when maximizing the expected *winner's* effort, the designer may optimally distort the contest in both dimensions (i.e., the information disclosure and players' relative competitiveness) to create ex post asymmetry even if players are ex ante identical.

Snapshot of the Baseline Model To highlight the contrast with the conventional wisdom, we consider two ex ante identical players who vie for a prize of a common, but initially unknown, value, which can be either high or low. The designer can receive an informative binary signal about the true prize value. Each player's effort is converted into a score, and the higher scorer wins.

The contest rule consists of two elements. First, a disclosure scheme specifies how the signal is disclosed. It is symmetric if the signal is disclosed to or concealed from both contestants, or asymmetric if only one contestant is provided with the signal and thus awarded an information advantage.[1] Second, a multiplier is imposed on each player's effort to generate his score.[2] We normalize the multiplier for player 1 to one and that for player 2 to $\delta > 0$, which is called a *scoring bias*. The bias can be interpreted as a nominal judging rule, as well as measures that elevate or discount players' (perceived) output. For instance, preferred contenders competing for promotion to a higher rung on the corporate ladder may be intentionally nurtured by the incumbent CEO and board members.

Our main focus is on the maximization of the expected *winner*'s effort (see, e.g., Moldovanu and Sela, 2006; Barbieri and Serena, 2024; Fu and Wu, 2022; Wasser and Zhang, 2023). This objective is relevant when the designer benefits primarily from the winning entry rather than from participants' total effort. For instance, in the competition for a corporate leadership role, the human capital gained by the winner is what will drive the value of the company. We also consider the maximization of expected *total* effort, the standard objective in much of the contest literature (see, e.g., Moldovanu and Sela, 2001; Moldovanu, Sela, and Shi, 2007),

[1]Strategic information disclosure and tolerance for informational asymmetry are also observable in public procurement. For instance, in *Assured Performance Systems, Inc.*, B-418233.2 (2020), the Government Accountability Office (GAO) denied a bid protest involving an Army solicitation for administrative support services. The incumbent possessed an informational advantage regarding exact employee anniversary dates, which were critical for accurately estimating labor costs. The contracting officer declined to eliminate this asymmetry entirely, instead providing non-incumbent bidders only with coarse tenure ranges.

[2]Multiplicative bid weighting rule is of both theoretical interest and practical relevance. In the "Buy American Act," the Federal Acquisition Regulation (FAR) directs the government to inflate foreign bids by an evaluation factor ranging from 6% to 50%. The evaluation factor works as an unfavorable multiplier assigned to foreign bids. See FAR Section 25 for more details.

as a benchmark.

Tilting and Releveling: Results and Extensions Absent the uncertainty in prize value, a fair contest (with $\delta = 1$) is optimal regardless of the design objective. However, with an uncertain prize value and the discretion of selective disclosure, the optimum may depart from the conventional wisdom.

When the designer aims to maximize the expected winner's effort, a *tilting-and-releveling* contest could emerge in the optimum. The tilting-and-releveling contest upsets the balance of the contest in both dimensions, which creates an *ex post dual asymmetry* between the ex ante identical players. Specifically, the contest feeds the signal exclusively to one player, while releveling the playing field by biasing the scoring rule in favor of the other. The two instruments (i.e., the disclosure scheme and scoring bias) are *complementary*: Ex post asymmetry never emerges in the optimum if the designer is restricted to distorting the contest in only one dimension (Remark 1). By contrast, a fully symmetric contest extracts all surplus and thus maximizes expected total effort: The designer symmetrically discloses or conceals the signal and uses a neutral scoring bias, $\delta = 1$.

A player's bidding strategy depends on both his expectation of the prize value and the competition he faces. The signal allows its recipient to update his prize expectation, and thereby revise his willingness to bid, i.e., the maximum effort he may exert. Imagine that only one player is awarded the signal. Suppose that a favorable signal is realized, which elevates the recipient's prize expectation. However, he may not step up his effort, since the other player maintains his prior and his bidding strategy is independent of the realized signal. A biased scoring rule that favors the uninformed player can incentivize the informed player: The informed player has to bid more aggressively to win and is willing to do so with a favorable signal. This mechanism could enable an upward shift in the distribution of the expected winner's effort. We identify the condition under which a properly crafted tilting-and-releveling contest prevails in optimum.

Such distortion never improves the expected total effort, which is the sum of the *means* of the contestants' efforts, and thus benefits equally from the contributions of both players. In contrast, the expected winner's effort is the *modified first-order statistic* of the (random) efforts contributed by the two players, and only the winner's input matters.[3]

We extend the baseline model to further explore the fundamentals of our analysis. First, we take into account the designer's ability to credibly commit to her disclosure policy (Section 3.3.1). Namely, she may deviate from the announced disclosure scheme when she finds

[3]In our context, the expected winner's effort is not necessarily the highest effort, except in the case of $\delta = 1$. We thus call the expected winner's effort a *modified* first-order statistic to reflect the nuance.

it profitable, and we examine contest design with a credibility constraint. Second, we let the designer maximize the expected maximum effort, which departs from the expected winner's effort under a biased scoring rule (Section 3.3.2). Third, we consider the lottery winner-selection mechanism and show that the tilting-and-releveling logic remains robust (Section 3.3.3). In Section 4, we generalize the model to allow for a general value distribution with multiple value states. We explore information design (along with design of the scoring biases) in this setting that endogenizes the designer's information structure and the form of her signal. Our main insights remain qualitatively robust.

Related Literature Our model is a variant of the family of all-pay auctions with interdependent valuations, which include Krishna and Morgan (1997); Lizzeri and Persico (2000); Siegel (2014); Rentschler and Turocy (2016); Lu and Parreiras (2017); and Chi, Murto, and Välimäki (2019). This study is primarily linked to two strands of the literature on contest design: (i) optimal biases (as the identity-dependent differential treatment of players) and (ii) information disclosure. To the best of our knowledge, we are the first to allow the designer to choose the optimal combination of the two instruments.

The literature on optimal biases has conventionally espoused the merits of a level playing field for incentive provision, e.g., Epstein, Mealem, and Nitzan (2011); Franke, Kanzow, Leininger, and Schwartz (2013, 2014); Franke, Leininger, and Wasser (2018). A handful of recent studies—e.g., Drugov and Ryvkin (2017); Fu and Wu (2020); Barbieri and Serena (2022); Wasser and Zhang (2023); Echenique and Li (2025)—identify the contexts in which optimal biases further upset the balance of the playing field.

The literature has increasingly recognized information disclosure as a valuable addition to the toolkit for contest design. For example, Yildirim (2005); Aoyagi (2010); Ederer (2010); Goltsman and Mukherjee (2011); Halac, Kartik, and Liu (2017); Lemus and Marshall (2021); and Ely, Georgiadis, Khorasani, and Rayo (2023) examine information feedback in dynamic contests. Halac et al. (2017) and Ely et al. (2023) consider the combination of feedback scheme and prize allocation rule and focus on symmetric information disclosure. Further, the prize is allocated by outcome and cannot depend on a player's identity. In contrast, we consider a static setting and focus on the interaction between the scoring rule and disclosure scheme; we allow for selective disclosure and identity-dependent preferential treatment.

Our paper is closely related to studies of disclosing information on contestants' types, including Wärneryd (2012); Lu, Ma, and Wang (2018), Serena (2022); Zhang and Zhou (2016); Chen and Chen (2024); Melo-Ponce (2021); and Antsygina and Teteryatnikova (2023). These studies focus exclusively on disclosure schemes and portray strategic information disclosure as a device that balances competition, which aligns with the conventional wisdom of leveling the playing field. In contrast, we show that a designer may prefer information asymmetry when she controls both the disclosure scheme and scoring rule.

In the context of private-value auctions, Bergemann and Pesendorfer (2007) consider a joint design problem that allows the seller to control bidders' learning accuracy and subsequent allocation rule. They demonstrate the optimality of creating informational asymmetry together with an asymmetric follow-up design. Chen, Lu, and Bai (2025) also study the joint design of disclosure and favoritism in all-pay auctions. They allow type-dependent bid weights and focus on total effort maximization in private-value auctions. In contrast, we emphasize the complementary role of information disclosure and (type-independent) scoring rule in maximizing winner's effort in a common-value setting.

The rest of the paper proceeds as follows. Section 2 sets up the baseline model. Section 3 characterizes the optimal contest and presents further discussions and extensions. Section 4 deals with the case of general value distribution and endogenous information structure. Section 5 concludes. Analytical details and proofs are collected in the Appendices.

2 Baseline Model

Two risk-neutral players, indexed by $i \in \{1, 2\}$, compete for a prize of a common value $v \in \{v_H, v_L\}$, with $v_H > v_L > 0$.[4] The high value v_H is realized with a probability $\mu := \Pr(v = v_H) \in (0, 1)$, with the low value v_L to be realized with the complementary probability. Players are initially uninformed about v, but its distribution is common knowledge. They simultaneously exert effort $x_i \geq 0$ to win the prize. We assume the players are ex ante symmetric and have the same marginal and average cost of effort, normalized to one.[5]

Winner-selection Mechanism and Scoring Bias The contest designer imposes a scoring bias $\delta_i > 0$ on each player i's effort entry x_i, which generates his score $\delta_i x_i$. We normalize δ_1 to 1 and set $\delta_2 = \delta > 0$. We call the scoring rule with $\delta = 1$ the neutral scoring rule, which awards favoritism to neither player. The scoring rule is biased when δ deviates from 1, which favors player 2 if $\delta > 1$ and player 1 if $\delta < 1$.

A player wins if his score exceeds that of the opponent. The winner is picked randomly in the event of a tie. For given effort entries $\boldsymbol{x} := (x_1, x_2) \in \mathbb{R}^2_+$, player 1's winning probability

[4] We relax the assumption of binary value states and allow for an arbitrary discrete value distribution in Section 4.

[5] The working-paper version (Deng, Fang, Fu, and Wu, 2023) considers an extension with asymmetric players and shows that the paper's main results are robust to this extension.

is

$$p_1(x_1, x_2) = \begin{cases} 1, & \text{if } x_1 > \delta x_2, \\ \frac{1}{2}, & \text{if } x_1 = \delta x_2, \\ 0, & \text{if } x_1 < \delta x_2, \end{cases}$$

and player 2 wins with the complementary probability.

Disclosure Schemes The designer conducts an investigation and obtains a verifiable noisy signal $s \in \{H, L\}$ regarding the prize value v. We label the signals such that

$$\Pr\left(s = H \,\middle|\, v = v_H\right) \geq \Pr\left(s = H \,\middle|\, v = v_L\right).$$

Further, we assume that the signal is not completely uninformative, so the above inequality holds strictly. For the baseline model, we take the information structure of the signal $\Pr(s|v)$ as given.[6]

The designer precommits to her disclosure scheme—i.e., how the signal will be disclosed. The disclosure scheme can formally be described by $\gamma \in \{CC, CD, DC, DD\}$, where C and D indicate "concealment" and "disclosure," respectively. With a symmetric disclosure scheme $\gamma = CC(DD)$, the realized signal s is conveyed to neither (both) of the players. With $\gamma = CD$, the designer conceals the signal from player 1 while disclosing it to player 2; $\gamma = DC$ is similarly defined.

Contest Design Prior to the contest, the designer chooses a contest scheme (γ, δ) to maximize either (i) the expected total effort, denoted by $\mathrm{TE}(\gamma, \delta)$, or (ii) the expected winner's effort, denoted by $\mathrm{WE}(\gamma, \delta)$.[7]

We introduce the following notations to pave the way for subsequent discussion. Let $\bar{v} := \mu v_H + (1 - \mu) v_L$ denote the ex ante expected prize value. We denote the probability that the signal $s = H$ is realized by $\widehat{\mu} := \mu \Pr(s = H \,|\, v = v_H) + (1 - \mu) \Pr(s = H \,|\, v = v_L)$. Upon receiving a signal $s \in \{H, L\}$, a player's expected prize value, denoted by $\widehat{v}_s$, is updated according to the Bayes rule:

$$\widehat{v}_s := \frac{\mu \Pr(s \,|\, v = v_H) v_H + (1 - \mu) \Pr(s \,|\, v = v_L) v_L}{\mu \Pr(s \,|\, v = v_H) + (1 - \mu) \Pr(s \,|\, v = v_L)}.$$

[6] We endogenize the information structure using a Bayesian persuasion approach à la Kamenica and Gentzkow (2011) in Section 4.

[7] By maximizing the expected winner's effort, we assume the designer is committed to adopting the winning product in the context of R&D contests. If the designer lacks commitment power, she will be tempted to adopt the best product regardless of whether the contestant submitting the best product wins the contest prize. For these contests, the designer's objective is to maximize expected maximum effort. We will consider this alternative design objective in Section 3.3.2.

It is straightforward to verify that $\widehat{v}_H > \widehat{v}_L$. By the law of iterated expectations, $\bar{v} = \widehat{\mu}\widehat{v}_H + (1-\widehat{\mu})\widehat{v}_L$.

3 Main Results, Discussions, and Extensions

In this section, we first characterize the equilibrium of the all-pay auction under an arbitrary disclosure scheme and scoring bias. The result allows us to derive the optimal contest. We then discuss our results and present extensions to our baseline model.

3.1 Equilibrium Characterization

An all-pay auction with complete information or a discrete signal structure, in general, does not possess pure-strategy equilibria (see, e.g., Hillman and Riley, 1989; Baye, Kovenock, and De Vries, 1996; Siegel, 2009, 2010, 2014). Siegel (2014) provides a technique for constructing the unique mixed-strategy equilibrium of an all-pay auction under a neutral scoring rule, i.e., $\delta = 1$. We apply his technique in our context to characterize the equilibrium in the interim bidding stage under each (γ, δ) with an arbitrary scoring bias $\delta > 0$.

For a player i who observes signal $s \in \{H, L\}$, let $B_{is}(x; \gamma, \delta)$ denote the equilibrium CDF of his effort. If player i does not observe the signal, we write $B_i(x; \gamma, \delta)$. All CDFs below are right-continuous; hence an atom at zero is represented by $B_{is}(0; \gamma, \delta) > 0$.

Lemma 1 ***(Equilibrium Characterization)*** *The all-pay auction induced by an arbitrary contest scheme (γ, δ) has a unique mixed-strategy equilibrium. The equilibrium CDFs are as follows.*

Symmetric Disclosure: $\boldsymbol{\gamma = DD}$. *For each realized signal $s \in \{H, L\}$, if $\delta < 1$,*

$$B_{1s}(x; DD, \delta) = \begin{cases} 0, & x < 0, \\ \dfrac{x}{\delta \widehat{v}_s}, & 0 \le x < \delta \widehat{v}_s, \\ 1, & x \ge \delta \widehat{v}_s, \end{cases} \quad B_{2s}(x; DD, \delta) = \begin{cases} 0, & x < 0, \\ 1 - \delta + \dfrac{\delta x}{\widehat{v}_s}, & 0 \le x < \widehat{v}_s, \\ 1, & x \ge \widehat{v}_s. \end{cases}$$

If $\delta \ge 1$,

$$B_{1s}(x; DD, \delta) = \begin{cases} 0, & x < 0, \\ 1 - \dfrac{1}{\delta} + \dfrac{x}{\delta \widehat{v}_s}, & 0 \le x < \widehat{v}_s, \\ 1, & x \ge \widehat{v}_s, \end{cases} \quad B_{2s}(x; DD, \delta) = \begin{cases} 0, & x < 0, \\ \dfrac{\delta x}{\widehat{v}_s}, & 0 \le x < \dfrac{\widehat{v}_s}{\delta}, \\ 1, & x \ge \dfrac{\widehat{v}_s}{\delta}. \end{cases}$$

The equilibrium under $\gamma = CC$ is obtained from the expressions above by replacing $\widehat{v}_s$ with $\bar{v}$ and dropping the signal subscript s.

Asymmetric Disclosure: $\boldsymbol{\gamma = DC}$. *Player 1 observes the signal and player 2 does not. We write B_2 for player 2's CDF. If $\delta < 1$,*

$$B_{1L}(x; DC, \delta) = \begin{cases} 0, & x < 0, \\ \dfrac{x}{\delta(1-\widehat{\mu})\widehat{v}_L}, & 0 \le x < \delta(1-\widehat{\mu})\widehat{v}_L, \\ 1, & x \ge \delta(1-\widehat{\mu})\widehat{v}_L, \end{cases}$$

$$B_{1H}(x; DC, \delta) = \begin{cases} 0, & x < \delta(1-\widehat{\mu})\widehat{v}_L, \\ \dfrac{x - \delta(1-\widehat{\mu})\widehat{v}_L}{\delta\widehat{\mu}\widehat{v}_H}, & \delta(1-\widehat{\mu})\widehat{v}_L \le x < \delta\bar{v}, \\ 1, & x \ge \delta\bar{v}, \end{cases}$$

and

$$B_2(x; DC, \delta) = \begin{cases} 0, & x < 0, \\ 1 - \delta + \dfrac{\delta x}{\widehat{v}_L}, & 0 \le x < (1-\widehat{\mu})\widehat{v}_L, \\ 1 - \delta\widehat{\mu} + \dfrac{\delta\big[x - (1-\widehat{\mu})\widehat{v}_L\big]}{\widehat{v}_H}, & (1-\widehat{\mu})\widehat{v}_L \le x < \bar{v}, \\ 1, & x \ge \bar{v}. \end{cases}$$

If $1 \le \delta \le 1/\widehat{\mu}$,

$$B_{1L}(x; DC, \delta) = \begin{cases} 0, & x < 0, \\ \dfrac{1}{1-\widehat{\mu}}\left(1 - \dfrac{1}{\delta}\right) + \dfrac{x}{\delta(1-\widehat{\mu})\widehat{v}_L}, & 0 \le x < (1-\widehat{\mu}\delta)\widehat{v}_L, \\ 1, & x \ge (1-\widehat{\mu}\delta)\widehat{v}_L, \end{cases}$$

$$B_{1H}(x; DC, \delta) = \begin{cases} 0, & x < (1-\widehat{\mu}\delta)\widehat{v}_L, \\ \dfrac{x - (1-\widehat{\mu}\delta)\widehat{v}_L}{\delta\widehat{\mu}\widehat{v}_H}, & (1-\widehat{\mu}\delta)\widehat{v}_L \le x < \widehat{v}_L + \delta\widehat{\mu}(\widehat{v}_H - \widehat{v}_L), \\ 1, & x \ge \widehat{v}_L + \delta\widehat{\mu}(\widehat{v}_H - \widehat{v}_L), \end{cases}$$

and

$$B_2(x;DC,\delta)=\begin{cases}0, & x<0,\\ \dfrac{\delta x}{\widehat{v}_L}, & 0\le x<\left(\dfrac{1}{\delta}-\widehat{\mu}\right)\widehat{v}_L,\\ 1-\widehat{\mu}\delta+\dfrac{\delta\left[x-\left(\frac{1}{\delta}-\widehat{\mu}\right)\widehat{v}_L\right]}{\widehat{v}_H}, & \left(\dfrac{1}{\delta}-\widehat{\mu}\right)\widehat{v}_L\le x<\dfrac{\widehat{v}_L}{\delta}+\widehat{\mu}(\widehat{v}_H-\widehat{v}_L),\\ 1, & x\ge\dfrac{\widehat{v}_L}{\delta}+\widehat{\mu}(\widehat{v}_H-\widehat{v}_L).\end{cases}$$

If $\delta>1/\widehat{\mu}$,

$$B_{1L}(x;DC,\delta)=\begin{cases}0, & x<0,\\ 1, & x\ge 0,\end{cases}$$

$$B_{1H}(x;DC,\delta)=\begin{cases}0, & x<0,\\ 1-\dfrac{1}{\delta\widehat{\mu}}+\dfrac{x}{\delta\widehat{\mu}\widehat{v}_H}, & 0\le x<\widehat{v}_H,\\ 1, & x\ge\widehat{v}_H,\end{cases}$$

and

$$B_2(x;DC,\delta)=\begin{cases}0, & x<0,\\ \dfrac{\delta x}{\widehat{v}_H}, & 0\le x<\dfrac{\widehat{v}_H}{\delta},\\ 1, & x\ge\dfrac{\widehat{v}_H}{\delta}.\end{cases}$$

The equilibrium under $\gamma=CD$ is obtained by relabeling the players and replacing δ by $1/\delta$. Equivalently,

$$B_1(x;CD,\delta)=B_2(x;DC,1/\delta),\qquad B_{2s}(x;CD,\delta)=B_{1s}(x;DC,1/\delta).$$

In Section 3.2, we will provide a visual illustration of the equilibrium characterization in Figures 1 and 2 and discuss equilibrium properties that underpin the optimal contest design.

The equilibrium characterization enables the calculation of expected total effort $\mathrm{TE}(\gamma,\delta)$ and the expected winner's effort $\mathrm{WE}(\gamma,\delta)$. Formally,

$$\mathrm{TE}(\gamma,\delta):=\mathbb{E}_{(\gamma,\delta)}[x_1+x_2]\text{ and }\mathrm{WE}(\gamma,\delta):=\mathbb{E}_{(\gamma,\delta)}\left[x_1\mathbb{1}_{\{x_1>\delta x_2\}}+x_2\mathbb{1}_{\{x_1<\delta x_2\}}\right],$$

where the expectation is taken over the realized signal and the equilibrium mixed strategies induced by the contest scheme (γ,δ). We present the results in Table 1, with functions

	$\text{TE}(\gamma,\delta)$	$\text{WE}(\gamma,\delta)$
$\gamma = DD$ or CC, $\delta < 1$	$\delta\bar{v}$	$\frac{\delta\bar{v}(5-\delta)}{6}$
$\gamma = DD$ or CC, $\delta \geq 1$	$\frac{\bar{v}}{\delta}$	$\frac{\bar{v}(5\delta-1)}{6\delta^2}$
$\gamma = DC$, $\delta < 1$	$\delta\left(\widehat{v}_L + \widehat{\mu}^2\widehat{v}_H - \widehat{\mu}^2\widehat{v}_L\right)$	$\frac{\widehat{v}_L}{6}\mathcal{W}_1\left(\widehat{\mu}, \frac{\widehat{v}_H-\widehat{v}_L}{\widehat{v}_L}, \delta\right)$
$\gamma = DC$, $1 \leq \delta \leq \frac{1}{\widehat{\mu}}$	$\frac{\widehat{v}_L}{\delta} + \delta\widehat{\mu}^2(\widehat{v}_H - \widehat{v}_L)$	$\frac{\widehat{v}_L}{6}\mathcal{W}_2\left(\widehat{\mu}, \frac{\widehat{v}_H-\widehat{v}_L}{\widehat{v}_L}, \delta\right)$
$\gamma = DC$, $\delta > \frac{1}{\widehat{\mu}}$	$\frac{\widehat{v}_H}{\delta}$	$\frac{\widehat{v}_H}{6}\mathcal{W}_3\left(\delta\right)$

Table 1: Expected Total Effort and the Expected Winner's Effort in Equilibrium.

$\mathcal{W}_1(\cdot)$, $\mathcal{W}_2(\cdot)$, and $\mathcal{W}_3(\cdot)$ defined as follows:

$$\begin{aligned}
\mathcal{W}_1(u,z,d) &:= -\left(u^3z+1\right)d^2 + \left[u^2z(6-u)+5\right]d,\\
\mathcal{W}_2(u,z,d) &:= \frac{-d^3u^2z\left[u(1+d)-6\right]+5d-1}{d^2},\\
\mathcal{W}_3(d) &:= \frac{5d-1}{d^2}.
\end{aligned}$$

The case with $\gamma = CD$ is omitted, since a contest scheme $(CD, 1/\delta)$ is outcome equivalent to (DC,δ) with symmetric players.

It is noteworthy that for a given scoring bias $\delta > 0$, symmetric disclosure schemes—$\gamma = CC$ and DD—generate the same ex ante equilibrium outcome—i.e., $\text{TE}(CC,\delta) = \text{TE}(DD,\delta)$ and $\text{WE}(CC,\delta) = \text{WE}(DD,\delta)$.

3.2 Optimal Contest

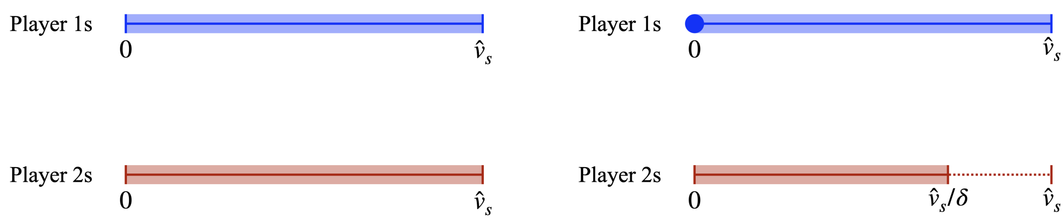
The solutions of equilibrium expected total effort and the expected winner's effort enable analysis of the optimum.

Proposition 1 ***(Optimal Contest)***

(i) If the designer aims to maximize expected total effort, then the optimal contests are $(CC,1)$ and $(DD,1)$.

(ii) If the designer aims to maximize the expected winner's effort, then the optimal contests are as follows. If $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L$, the optimal contests are $\left(CD,\widehat{\mu}\right)$ and $\left(DC,1/\widehat{\mu}\right)$. If $\widehat{\mu}\widehat{v}_H < 4\widehat{v}_L$, the optimal contests are $(CC,1)$ and $(DD,1)$. If $\widehat{\mu}\widehat{v}_H = 4\widehat{v}_L$, the optimal contests are $(CC,1)$, $(DD,1)$, $\left(CD,\widehat{\mu}\right)$, and $\left(DC,1/\widehat{\mu}\right)$.

Proposition 1(i) echoes the conventional wisdom of the contest literature: The total-effort-maximizing contest maintains symmetry. It follows immediately from the fact that,



(a) $\delta = 1$. (b) $\delta > 1$.

Figure 1: Equilibrium Strategies with Symmetric Players: $\gamma = DD$, $s \in \{L, H\}$.

in an unbiased common-value all-pay auction with symmetric information, the expected aggregate effort equals the expected valuation and thus implements the first best for the designer.

However, Proposition 1(ii) shows that to maximize the expected winner's effort, the designer may deliberately create ex post dual asymmetry between players: She *tilts* the playing field by awarding an information advantage to one player, while *releveling* the playing field by biasing the scoring rule in favor of the other. A *tilting-and-releveling* contest, $\big(CD, \widehat{\mu}\big)$ or $\big(DC, 1/\widehat{\mu}\big)$, is optimal when the condition $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L$ is met.

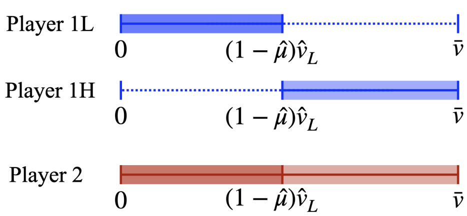
It is useful to first understand the bidding equilibrium under symmetric disclosure vis-à-vis that under asymmetric disclosure, which is summarized in Figure 1.

Equilibrium under Symmetric Disclosure The equilibrium under a symmetric disclosure scheme with discrete signal spaces resembles that in a standard complete-information all-pay auction. A symmetric contest, $(DD, 1)$ or $(CC, 1)$, fully extracts the players' surplus and maximizes expected total effort.[8]

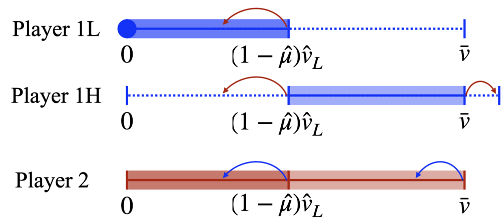
Assume instead a biased scoring rule $\delta > 1$. Player 2 secures a sure win by bidding $\widehat{v}_s/\delta$ under DD (or $\bar{v}/\delta$ under CC), which leaves him with positive surplus. The handicapped player 1 continues to bid up to $\widehat{v}_s$ under DD (or $\bar{v}$ under CC), but he now stays inactive (i.e., exerting zero effort) with a positive probability. The biased scoring rule is obviously suboptimal. We illustrate this rationale in Figure 1 for the case of $\gamma = DD$.

Equilibrium under Asymmetric Disclosure Asymmetric disclosure fundamentally changes the nature of the equilibrium. Assuming $\gamma = DC$ and $\delta = 1$, we illustrate players' equilibrium bidding strategies in Figure 2(a). Player 1 is informed, and his equilibrium bidding strategy is signal-dependent. Player 1, upon receiving signal L, is referred to as player $1L$;

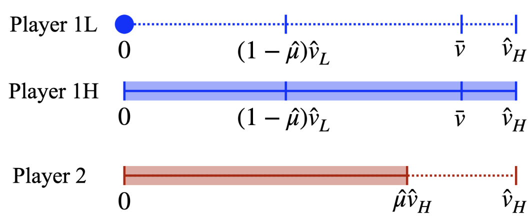
[8] With $\delta = 1$, a player's effort is uniformly distributed over the interval $[0, \widehat{v}_s]$ under DD, where $\widehat{v}_s$ is the updated expected prize value upon receiving a signal $s \in \{H, L\}$. Analogously, one's effort under CC is uniformly distributed over $[0, \bar{v}]$.



(a) $\delta = 1$.



(b) $1 < \delta < 1/\widehat{\mu}$.



(c) $\delta = 1/\widehat{\mu}$.

Figure 2: Equilibrium Strategies with Symmetric Players: $\gamma = DC$.

his efforts are uniformly distributed on $\big[0, (1-\widehat{\mu})\widehat{v}_L\big]$, while those of player $1H$ are distributed on $\big[(1-\widehat{\mu})\widehat{v}_L, \bar{v}\big]$ (see Lemma 1). Player 2's efforts are distributed over the interval $[0, \bar{v}]$.[9]

Player $1L$, due to his lower updated expected prize valuation, is effectively an underdog when competing with the uninformed player 2. The distribution of his efforts includes zero, which implies a zero equilibrium payoff for him. In contrast, player $1H$ has a higher expected prize valuation and becomes a favorite vis-à-vis player 2. The upper support of his efforts remains at $\bar{v}$, although he can bid up to $\widehat{v}_H$. He has no incentive to bid more than $\bar{v}$ because player 2's effort is capped at that level.

The contest $(DC, 1)$ is obviously suboptimal. This naturally prompts the question of how player $1H$ can be further incentivized to bid more than $\bar{v}$, which inspires tilting and releveling.

Tilting and Releveling Raising δ above 1 incentivizes player $1H$. We illustrate this rationale in Figure 2(b). A scoring bias $\delta > 1$ favors player 2 and discourages player $1L$. In contrast, although player $1H$ continues to enjoy the upper hand for δ in the range of $\big[1, 1/\widehat{\mu}\big]$, effort $\bar{v}$ no longer guarantees a sure win. Thus the unfavorable scoring rule compels him to step up his effort: The upper support of his effort increases with δ.

[9]The bidding supports of players $1L$ and $1H$ are adjacent intervals that intersect only at the cutoff $(1-\widehat{\mu})\widehat{v}_L$, a probability-zero point. The distribution of x_2 has different densities for efforts above and below the cutoff. This occurs because in this common-value all-pay auction, the uninformed player 2 takes into account and strategically responds to player 1's type-dependent bidding strategy when placing his bid.

Tilting and releveling "gives up" the low-type informed player 1, but could benefit from his better-incentivized high-type counterpart, since the latter may bid more than $\bar{v}$. This is obviously suboptimal for total-effort maximization, but suggests a potential for elevating the expected winner's effort, in which case only the winner's effort (i.e., the modified first-order statistic) matters. The details are discussed below.

Tilting and Releveling as an Optimal Contest By Proposition 1(ii), with $\gamma = DC$, a bias $\delta = 1/\widehat{\mu}$ could maximize the expected winner's effort. Recall that contests under symmetric disclosure, either DD or CC, generate the same ex ante equilibrium outcomes for a given δ. We compare the tilting-and-releveling contest $\big(DC, 1/\widehat{\mu}\big)$ with a fully symmetric contest $(CC, 1)$ to elucidate the underlying trade-off.

Under $(CC, 1)$, players maintain their prior, so their efforts are uniformly distributed over $[0, \bar{v}]$. Players' equilibrium strategies in the contest $\big(DC, 1/\widehat{\mu}\big)$ are illustrated in Figure 2(c). Imagine first that a low signal $s = L$ is realized. The negative shock, together with the unfavorable scoring rule, forces player $1L$ to give up (i.e., his bidding strategy degenerates to a singleton at zero), which clearly causes a loss compared with the case of $(CC, 1)$. However, player 2 remains uninformed and is immune to the negative shock; he remains active, which provides insurance for the performance of the contest. Then suppose $s = H$. Player 1—because of the upwardly revised prize expectation and the unfavorable scoring rule—may bid more than $\bar{v}$, with the upper support reaching $\widehat{v}_H$. The contest, when maximizing the expected winner's effort, could outperform $(CC, 1)$.

The trade-off between $\big(DC, 1/\widehat{\mu}\big)$ and $\big(CC, 1\big)$ ultimately depends on $\widehat{\mu}$, $\widehat{v}_H$, and $\widehat{v}_L$. First, tilting and releveling could yield a gain when a high signal is realized, which occurs with a probability of $\widehat{\mu}$. Therefore, the former is more likely to prevail with a large $\widehat{\mu}$. Second, the gain of the tilting-and-releveling contest is more significant when the signal prompts substantial upward revision in prize valuation (i.e., from $\bar{v}$ to $\widehat{v}_H$), which requires a larger $\widehat{v}_H$ relative to $\widehat{v}_L$. Summing these leads to the condition $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L$ for the optimality of $\big(DC, 1/\widehat{\mu}\big)$. The scoring bias $\delta = 1/\widehat{\mu}$ relevels the contest under $\gamma = DC$ and enables players $1H$ and 2 to win with an equal probability with $s = H$; it also perfectly eliminates the rent afforded to player 1 by his information advantage.

Complementarity Between Information Disclosure and Scoring Bias The two instruments, information disclosure and scoring bias, play *complementary* roles. That is, the optimum either requires full symmetry or embraces dual asymmetry. Suppose that the designer is allowed to distort the contest in only one dimension, either by setting the disclosure scheme while maintaining a neutral scoring rule or biasing the scoring rule while being constrained by symmetric disclosure. The following ensues.

Remark 1 ***(Unidimensional Contest Design)***

(i) Fix $\delta = 1$. A symmetric disclosure scheme ($\gamma \in \{CC, DD\}$) maximizes both expected total effort and the expected winner's effort simultaneously.

(ii) Fix $\gamma \in \{CC, DD\}$. The neutral scoring bias ($\delta = 1$) maximizes both expected total effort and the expected winner's effort simultaneously.

With $\delta = 1$, an asymmetric disclosure scheme cannot force the high-type player 1 to raise his maximum effort above $\bar{v}$, as Figure 2(a) illustrates. Similarly, with a symmetric disclosure scheme, biasing the scoring rule only allows the favored player to slack off, as Figure 1(b) shows. Asymmetry in only one dimension is always suboptimal.

3.3 Discussions and Extensions

In this part, we consider three extensions that shed further light on the principles of optimal contest design that combines the tools of scoring rule and disclosure policy.

3.3.1 Credibility of Disclosure Policies

The baseline model assumes that the designer can commit to her announced disclosure policy and abstracts away the issue of *credibility* (Akbarpour and Li, 2020; Lin and Liu, 2024): The commitment power can be called into question, since the designer may find it profitable to deviate from her precommitted prescriptions. We now consider the possibility of deviation. To the best of our knowledge, we are the first to examine the issue of credibility in the contest literature.

Fix a disclosure scheme $\gamma \in \{CC, CD, DC, DD\}$ and let $\gamma(i)$ indicate the specific disclosure to a player $i \in \{1, 2\}$ under γ. For example, $\gamma(1) = C$ and $\gamma(2) = D$ for $\gamma = CD$. Consider a disclosure scheme γ announced by the designer and a potential deviation $\gamma' \neq \gamma$. Assume that player i can detect the deviation if and only if $\gamma(i) \neq \gamma'(i)$. For example, the player detects deviation if he unexpectedly learned about the signal from the designer with $\gamma(i) = C$, or was denied access to it with $\gamma(i) = D$. He maintains his belief about the disclosure to his opponent regardless, since he cannot detect deviation in that respect. Suppose that the designer deviates from her disclosure to one player i, i.e., $\gamma(i) \neq \gamma'(i)$. Player i, upon detecting the deviation, would have an incentive to *complain* if he becomes worse off by adopting a bidding strategy that best responds to his opponent's equilibrium strategy under the announced contest scheme (γ, δ); the contest dissolves when a complaint arises. The player *remains silent* otherwise and the contest proceeds.

A credible contest is formally defined as follows.

Definition 1 ***(Credible Contest)*** *A contest (γ, δ) is credible if for every deviation of disclosure policy $\gamma' \neq \gamma$, either (i) at least one player who detects it complains, or (ii) every player who detects it remains silent but such a deviation does not increase the designer's expected payoff.*[10]

In short, a credible contest prevents profitable deviation, which imposes an additional constraint on our joint design problem. The optimal contest that satisfies this requirement is established as follows.

Proposition 2 ***(Optimal Credible Contest)*** *The optimal credible contests are given as follows.*

(i) If the designer aims to maximize expected total effort, then $(\gamma^*_{\mathrm{TE}}, \delta^*_{\mathrm{TE}}) = (DD, 1)$ is optimal. $(\gamma^*_{\mathrm{TE}}, \delta^*_{\mathrm{TE}}) = (CC, 1)$ is also optimal if and only if $\widehat{\mu} \leq 1/2$.

(ii) If the designer aims to maximize the expected winner's effort, then

(a) in the case with $\widehat{\mu} \leq 5/7$ and $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L$, the optimal credible contests are $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = \big(CD, \widehat{\mu}\big)$ and $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = \big(DC, 1/\widehat{\mu}\big)$;

(b) in the case with $\widehat{\mu} > 5/7$ and $\frac{(13\widehat{\mu} - 18\widehat{\mu}^2)\widehat{v}_H}{2(1-\widehat{\mu})} > \widehat{v}_L$, the optimal credible contests are $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = \big(CD, 5 - 6\widehat{\mu}\big)$ and $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = \big(DC, 1/(5 - 6\widehat{\mu})\big)$;

(c) in all other cases, $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = (DD, 1)$ is an optimal credible contest; so is $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = (CC, 1)$ if and only if $\widehat{\mu} \leq 1/3$.

By Proposition 2(i), $(\gamma^*_{\mathrm{TE}}, \delta^*_{\mathrm{TE}}) = (DD, 1)$ remains optimal for total effort maximization. The disclosure policy DD is credible: Any deviation can be detected by at least one player; simple analysis can verify that one will complain if he suffers from a loss of information.

However, the ex ante equivalence of symmetric disclosure schemes CC and DD no longer holds. Suppose that the designer announces CC while disclosing her signal privately to one

[10] Our approach to credible information disclosure is reminiscent of the setup of Lin and Liu (2024) to model credible persuasion, which assumes that a sender's deviation will be detected if it alters the message distribution. Their notion of credibility rules out detectable deviations and requires that a sender not profit from an undetectable deviation. They focus on a sender-receiver problem in which the message is publicly observed, while we consider a game-theoretic environment that allows for private disclosure. Notably, we allow for detectable deviations, provided that they do not reduce the player's payoff—i.e., the player would not file a complaint.

player. The player would bid $\bar{v} := \mu v_H + (1-\mu)v_L$ to secure a win if $s = H$—which benefits the designer—and bid zero if $s = L$. This deviation benefits the designer if the probability of realizing a high signal is large; the privately informed player also benefits, so he would not complain. These render the announced policy non-credible. As a result, by Proposition 2(i), with $\delta = 1$, $\gamma = CC$ is credible if and only if a high signal is less likely, i.e., $\widehat{\mu} \leq 1/2$.

By Proposition 2(ii), to maximize the expected winner's effort, a tilting-and-releveling contest can still prevail. However, the credibility requirement may be a binding constraint. Recall by Proposition 1 that absent credibility concerns, a tilting-and-releveling contest with $(\gamma^*_{\text{WE}}, \delta^*_{\text{WE}}) = \big(CD, \widehat{\mu}\big)$ or $(\gamma^*_{\text{WE}}, \delta^*_{\text{WE}}) = \big(DC, 1/\widehat{\mu}\big)$ is optimal for $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L$. By Proposition 2(ii), its optimality can be preserved for $\widehat{\mu} \leq 5/7$, but dissolves when the probability of a high signal $\widehat{\mu}$ is high.

To satisfy the credibility requirement, the scoring rule for a tilting-and-releveling contest has to favor the uninformed player further: The uninformed player would be excessively privileged if the designer deviates and awards him the signal, in which case the designer would be worse off.[11] This altered tilting-and-releveling contest emerges in the optimum in case (b) of Proposition 2(ii). Alternatively, the designer can simply feed the signal to both players and resort to a neutral scoring rule, in which case a fully symmetric contest $(DD, 1)$ arises, as in case (c) of Proposition 2(ii).

3.3.2 Expected Maximum Effort

With a scoring bias $\delta \neq 1$, the winner of the contest may not be the one who contributes the highest effort. Maximizing the expected winner's effort presumes that the designer benefits only from the winning entry, which is plausible when the designer cannot separate the prize allocation from the adoption of contestants' output, e.g., admissions contests at universities or competitions for promotions within firms. In some contexts, the designer may award the prize based on her preferred rules, yet choose to use a higher-quality entry from another contestant. We now allow the designer to maximize the expected maximum effort of the contest, denoted by $\text{ME}(\gamma, \delta)$. Formally, $\text{ME}(\gamma, \delta) := \mathbb{E}_{(\gamma,\delta)}[\max\{x_1, x_2\}]$.

Proposition 3 ***(Optimal Contest for Expected Maximum Effort)*** *If the designer aims to maximize expected maximum effort, then in the case with $\widehat{\mu}(2-\widehat{\mu})\widehat{v}_H > 4\widehat{v}_L$, the optimal contests are $(\gamma^*_{\text{ME}}, \delta^*_{\text{ME}}) = \big(CD, \widehat{\mu}\big)$ and $(\gamma^*_{\text{ME}}, \delta^*_{\text{ME}}) = \big(DC, 1/\widehat{\mu}\big)$; in the case with $\widehat{\mu}(2-\widehat{\mu})\widehat{v}_H < 4\widehat{v}_L$, the optimal contests are $(\gamma^*_{\text{ME}}, \delta^*_{\text{ME}}) = (CC, 1)$ and $(\gamma^*_{\text{ME}}, \delta^*_{\text{ME}}) = (DD, 1)$; in the case with equality, the optimal contests are $(CC, 1)$, $(DD, 1)$, $\big(CD, \widehat{\mu}\big)$, and $\big(DC, 1/\widehat{\mu}\big)$.*

[11] It is straightforward to verify $1/(5 - 6\widehat{\mu}) > 1/\widehat{\mu}$ for $\widehat{\mu} > 5/7$.

Compared with the objective of expected winner's effort, tilting-and-releveling is more often optimal for the expected maximum effort.[12]

3.3.3 Lottery Contests

The baseline model assumes a deterministic all-pay allocation rule: The player with the higher score wins. We now show that the main logic also survives in a noisy winner-selection environment. Suppose that the winner is selected according to the following lottery contest success function:

$$p_1(x_1, x_2) = \frac{x_1}{x_1 + \delta x_2}, \qquad p_2(x_1, x_2) = \frac{\delta x_2}{x_1 + \delta x_2},$$

whenever $x_1 + \delta x_2 > 0$, with ties at zero broken evenly.

The expected total effort and the expected winner's effort are defined as before, except that the winner's effort is now weighted by the winning probabilities:

$$\mathrm{TE}(\gamma, \delta) := \mathbb{E}_{(\gamma,\delta)}[x_1 + x_2],$$

and

$$\mathrm{WE}(\gamma, \delta) := \mathbb{E}_{(\gamma,\delta)}\left[p_1(x_1, x_2)x_1 + p_2(x_1, x_2)x_2\right].$$

Proposition 4 ***(Optimal Contest under the Lottery Winner-selection Mechanism)*** *Suppose the winner is selected according to the lottery contest success function specified above. The following statements hold.*

(i) If the designer aims to maximize expected total effort, the optimal contests are $(\gamma^*_{\mathrm{TE}}, \delta^*_{\mathrm{TE}}) = (CC, 1)$ *and* $(\gamma^*_{\mathrm{TE}}, \delta^*_{\mathrm{TE}}) = (DD, 1)$.

(ii) If the designer aims to maximize the expected winner's effort, then there exists a threshold function $\Phi : [0, 1] \to [0, 1]$*, which is continuous and strictly increasing on* $(0, 1/2)$*, with*

$$\Phi(0) = 0, \qquad \Phi(\widehat{\mu}) = 1 \quad \text{for all } \widehat{\mu} \geq 1/2,$$

such that the following holds. If $\widehat{v}_L/\widehat{v}_H < \Phi(\widehat{\mu})$*, then the optimal contest scheme features tilting-and-releveling: The designer discloses the signal to one player and biases the contest in favor of the other. If* $\widehat{v}_L/\widehat{v}_H > \Phi(\widehat{\mu})$*, then the optimal contests are* $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = (CC, 1)$ *and* $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = (DD, 1)$*. At equality, both the symmetric contests and the corresponding tilting-and-releveling contests are optimal.*

[12] Indeed, the threshold for expected winner's effort is $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L$, whereas the threshold for expected maximum effort is $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L/(2 - \widehat{\mu})$. We thank an anonymous reviewer for pointing this out.

The proposition confirms that the tilting-and-releveling logic is not an artifact of deterministic winner selection. Under the lottery rule, a scoring bias still relevels the contest by favoring the uninformed player, which induces the informed player to exert more effort after a favorable signal. At the same time, the uninformed player remains active and provides insurance after an unfavorable signal.

Notably, we have that $\Phi(\widehat{\mu}) > \widehat{\mu}/4$ for all $\widehat{\mu} \in (0,1)$.[13] Thus, relative to the all-pay auction, the lottery winner-selection mechanism expands the parameter region in which tilting-and-releveling is optimal for maximizing expected winner's effort.

We conclude this extension by clarifying the role of the scoring-bias specification. In the deterministic all-pay auction, an additive bias (i.e., a headstart) would be powerful: If the designer can use an affine rule (i.e., player 2's score is given by $\delta x_2 + \alpha$), then setting $\delta = 0$ and choosing α equal (or close) to the expected prize value makes α a deterministic cutoff. Player 1 must exert effort α to win, so the designer can implement the first-best winner's effort.

Implementing such contest rules requires a precise deterministic winner-selection mechanism: once a contestant crosses the cutoff, her winning probability jumps discontinuously from zero to one. Such a cutoff rule is less plausible in environments where scores are noisy measures of performance, or where a higher score merely increases one's probability of winning rather than guaranteeing victory. A lottery contest provides a more natural account of such environments, and we show that a headstart is irrelevant in this setting.

Remark 2 ***(Affine Scoring Bias in Lottery Contests)*** *The conclusion of Proposition 4 is unchanged if the designer is allowed to use an affine scoring bias, i.e., if player 2's score is given by $\delta x_2 + \alpha$, where $\delta > 0$ and $\alpha \geq 0$. In particular, the optimal contest sets zero headstart.*

4 General Value Distribution and Endogenous Information Structure for Disclosure

In this section, we extend the baseline all-pay auction model to allow for multiple value states and let the designer flexibly design the information structure of her investigation. Specifically, suppose that the common value for the prize v is distributed on the set $\{v_1, v_2, \ldots, v_K\}$ with $K \geq 2$ and $\mu_k := \Pr(v = v_k) > 0$ for $k \in \{1, 2, \ldots, K\}$. Without loss of generality, assume that $0 < v_1 < v_2 < \cdots < v_K$. Again, we denote by $\bar{v}$ the ex ante expected prize value. The designer has full control over the amount of information to be revealed

[13]See the proof of Proposition 4 in Appendix A.7 for details.

and the form of the signal disclosed to players. This corresponds to the concept of Bayesian persuasion (Kamenica and Gentzkow, 2011). An information structure comprises a signal space $\mathcal{S}$ and a collection of likelihood distributions $\pi(\cdot|v)$ over $\mathcal{S}$.[14] The designer sets $\mathcal{S}$ and $\big(\gamma, \delta, \pi(\cdot|v)\big)$.[15]

A fully symmetric contest, $(CC, 1)$ or $(DD, 1)$, fully dissipates the rent, thereby maximizing total effort. We thus focus on the maximization of the expected winner's effort. Fixing $\gamma \in \{CC, DD\}$, as in the baseline case, it is straightforward to verify that $\delta = 1$ maximizes the expected winner's effort, which equals $2\bar{v}/3$. For the case of $\gamma = DC$, the following lemma establishes the optimality of binary signal spaces, which simplifies the joint design problem.

Lemma 2 ***(Optimality of Binary Signals)*** *Fix $\gamma = DC$ and $\delta > 0$. An information structure with binary signals—i.e., $\mathcal{S} = \{H, L\}$—maximizes the expected winner's effort.*

In principle, the optimal information structure may require more than two signals. Lemma 2 states that two signals suffice within our context (see Appendix B for more details). Specifically, we show in the proof that holding $\gamma = DC$ and $\delta > 0$ fixed, for any information structure with a signal space that contains three or more elements, we can construct an alternative information structure with one fewer signal to attain a weakly higher expected winner's effort. This allows us to restrict attention to binary signal spaces, which significantly reduces the dimensionality of the information design problem and enables a closed-form characterization of the optimum.

Given a binary signal space $\mathcal{S} = \{H, L\}$, denote by v_s^π the expected prize value conditional on s, i.e., $\mathbb{E}(v|s)$. Without loss of generality, assume that a realization of $s = H$ gives rise to a higher expected prize value, i.e., $v_H^\pi \geq v_L^\pi$. In addition, define $\mu^\pi := \Pr(s = H)$.

In our context, designing the information structure $\pi(\cdot|v)$ with $\mathcal{S} = \{H, L\}$ is equivalent to choosing a distribution of posterior expectations, $(v_H^\pi, v_L^\pi, \mu^\pi)$, subject to the constraint that the distribution can be induced by a binary signal structure. Denote the cumulative distribution functions of v and v^π by $F(x)$ and $G(x)$, respectively. It is well known in the literature that we can find a binary signal structure that generates $(v_H^\pi, v_L^\pi, \mu^\pi)$ if and only if the following conditions are satisfied (see, e.g., Gentzkow and Kamenica, 2016; Kolotilin,

[14]For example, the baseline information structure described in Section 2 involves a binary signal space $\mathcal{S} = \{H, L\}$ and an arbitrary conditional likelihood distribution $\Pr(s|v)$ satisfying the monotone-labeling condition.

[15]It is noteworthy that this remains a limited information design exercise. Addressing a fully general information design problem in our context is technically demanding, because potential correlation between signals would significantly complicate the analysis of common-value all-pay auctions.

2018):[16]

$$\int_0^t F(x)dx \geq \int_0^t G(x)dx \text{ for } v_1 \leq t \leq v_K, \text{ and} \tag{1}$$

$$\mu^\pi v_H^\pi + (1-\mu^\pi)v_L^\pi = \bar{v}. \tag{2}$$

The following proposition fully characterizes the optimal contest.

Proposition 5 ***(Optimal Contest with Multiple Value States and an Endogenous Information Structure)*** *Consider the joint design of scoring bias $\delta > 0$, disclosure scheme γ, and information structure $\pi(\cdot|v)$. If the designer aims to maximize the expected winner's effort, the following holds.*

(i) In the case with $\bar{v}/v_1 > 4$, the optimal contest consists of $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = \big(DC, 1/\mu^\pi\big)$, or equivalently, $\big(CD, \mu^\pi\big)$, with

$$\mu^\pi = \min\left\{\sum_{\ell=k^*}^{K} \mu_\ell, \mu^*(k^*)\right\},$$
$$v_H^\pi = \frac{[\mu^\pi - \Pr(v > v_{k^*})]v_{k^*} + \Pr(v > v_{k^*})\mathbb{E}[v|v > v_{k^*}]}{\mu^\pi}, \text{ and}$$
$$v_L^\pi = \frac{[1-\mu^\pi - \Pr(v \leq v_{k^*})]v_{k^*} + \Pr(v \leq v_{k^*})\mathbb{E}[v|v \leq v_{k^*}]}{1-\mu^\pi},$$

where $\mu^*(k) := 3 - \frac{\sum_{\ell=1}^{k}\mu_\ell}{2} - \frac{\sum_{\ell=k+1}^{K}\mu_\ell v_\ell}{2v_k}$, and $k^* := \min\Big\{k : \mu^*(k) \geq \sum_{\ell=k+1}^{K}\mu_\ell\Big\}$. This distribution of posterior expectations is achieved by a signal structure with $\pi(H|v) = 0$ for $v < v_{k^*}$, $\pi(H|v) = \frac{\mu^\pi - \Pr(v > v_{k^*})}{\mu_{k^*}}$ for $v = v_{k^*}$, and $\pi(H|v) = 1$ for $v > v_{k^*}$.

(ii) In the case with $\bar{v}/v_1 \leq 4$, both $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = (CC, 1)$ and $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = (DD, 1)$, with an arbitrary information structure $\pi(\cdot|v)$, are optimal.

The implications of our baseline model remain intact. The designer may, again, tilt and relevel to maximize the expected winner's effort, provided that the condition $\bar{v}/v_1 > 4$ is met. The optimal information structure takes a simple form: There exists a threshold value state v_{k^*} such that the designer sends a low signal if $v < v_{k^*}$ and a high signal if $v > v_{k^*}$. For $v = v_{k^*}$, the designer may randomize between the two signals.

[16] The first condition says that the prior distribution of the prize value is a mean-preserving spread of the distribution of posterior expectations. The second condition imposes the requirement of Bayes plausibility: the expected posterior expectation must equal the prior mean.

5 Concluding Remarks

This paper studies the optimal design of a contest in which two players compete for a common-valued prize. The designer chooses a combination of two instruments: an information disclosure scheme and a scoring bias. Fully symmetric contests (symmetrically disclosed or concealed information and a neutral scoring rule) maximize expected total effort, which embraces the conventional wisdom of leveling the playing field. However, when maximizing the expected winner's effort, the contest may feature dual asymmetry that distorts the contest in both dimensions: The designer discloses the signal privately to one player, while a favorable scoring rule compensates the other. Such tilting-and-releveling contests could prevail even if the players are ex ante identical.

Our paper is one of the first in the contest literature to examine the optimal combination of multiple design instruments. We demonstrate the complementarity between the instruments, in that the optimum requires either ex post full symmetry or dual asymmetry. Our results generate novel implications for contest design and shed fresh light on the debate regarding the relationship between (a)symmetry and the performance of a contest.

For future research, it would be promising to revisit our research question within the context of all-pay auction models in more general settings, such as those that involve nonlinear cost functions or multiple players. Also, the information design exercise could be expanded by allowing for correlated signals. Although these extensions are technically challenging, they clearly merit further exploration.

Declarations

The authors have no competing interests to declare that are relevant to the content of this article.

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Appendix A Proofs

A.1 Proof of Lemma 1

Proof. The argument is a direct application of the equilibrium construction in Siegel (2014), adapted to the biased scoring rule. Let $y_1 = x_1$, $y_2 = \delta x_2$. The allocation rule depends only on the score bids (y_1, y_2): Player 1 wins if $y_1 > y_2$, player 2 wins if $y_2 > y_1$, and ties are broken evenly. In score space, player 1's cost is y_1, while player 2's cost is y_2/δ. Multiplying player 2's payoff by the positive constant δ does not affect his best responses. Hence the original biased contest is strategically equivalent to a neutral-scoring all-pay auction in score bids in which player 1's value is the relevant expected prize value and player 2's value is δ times that expected prize value. This equivalence applies pointwise to each information set.

For symmetric disclosure, $\gamma = DD$, after each signal realization $s \in \{H, L\}$, the score-space game is a complete-information all-pay auction with values $\widehat{v}_s$ for player 1 and $\delta\widehat{v}_s$ for player 2. The standard complete-information all-pay auction equilibrium therefore gives the two cases reported in the lemma. When $\delta < 1$, player 1 is the stronger player in score space; player 2 puts an atom $1 - \delta$ at zero and otherwise mixes uniformly in score space on $[0, \delta\widehat{v}_s]$. Translating back to efforts gives the displayed CDFs. When $\delta \geq 1$, player 2 is the stronger player in score space; player 1 puts an atom $1 - 1/\delta$ at zero and the corresponding CDFs are again obtained by translating score bids back into efforts. The case $\gamma = CC$ is identical, with $\widehat{v}_s$ replaced by $\bar{v}$.

Consider next $\gamma = DC$. Let F_{1L}, F_{1H}, and F_2 denote the score-bid CDFs induced by the effort CDFs displayed in the lemma; that is, $F_{1s}(y) = B_{1s}(y; DC, \delta)$ and $F_2(y) = B_2(y/\delta; DC, \delta)$. Type $s \in \{L, H\}$ of player 1 obtains the payoff

$$U_{1s}(y) = \widehat{v}_s F_2(y) - y$$

from score bid y, while player 2 obtains

$$U_2(y) = (1 - \widehat{\mu})\delta\widehat{v}_L F_{1L}(y) + \widehat{\mu}\delta\widehat{v}_H F_{1H}(y) - y.$$

The CDFs in the lemma are constructed so that these payoffs are constant on the relevant supports. On any interval over which type s of player 1 mixes, player 1's indifference condition requires

$$F_2'(y) = \frac{1}{\widehat{v}_s}.$$

On any interval over which player 2 mixes and only type s of player 1's CDF increases, player

2's indifference condition requires

$$\Pr(s)\delta\widehat{v}_s F'_{1s}(y) = 1,$$

where $\Pr(H) = \widehat{\mu}$ and $\Pr(L) = 1 - \widehat{\mu}$. These equations pin down the continuous densities reported in the lemma. The atoms at zero are chosen so that a type who puts mass at zero is indifferent between zero and any positive score in his support, and the support endpoints are chosen so that the relevant CDFs are right-continuous and reach one.

These indifference equations also rule out profitable deviations. A score above the top of the relevant support wins with probability one and only increases the cost. A score in a gap can be lowered without changing the winning probability. In the asymmetric-disclosure case, type L's payoff is weakly decreasing above his support because the slope of F_2 there is $1/\widehat{v}_H < 1/\widehat{v}_L$, while type H's payoff is weakly increasing below his support and reaches its equilibrium value at the lower endpoint of that support. The same indifference and no-deviation logic applies to player 2. Thus the displayed CDFs constitute an equilibrium. The case $\gamma = CD$ follows by relabeling the players and replacing δ by $1/\delta$. Uniqueness follows from the uniqueness result for the corresponding score-space all-pay auction. ■

A.2 Proof of Table 1

Proof. Recall that $\bar{v} = (1 - \widehat{\mu})\widehat{v}_L + \widehat{\mu}\widehat{v}_H$. We calculate expected efforts from the CDFs in Lemma 1. Since ties occur with probability zero except at zero, and zero effort does not affect the expected winner's effort, atoms at zero do not create additional terms in the winner-effort calculations.

First consider a symmetric disclosure scheme. Conditional on a common expected value V, where $V = \widehat{v}_s$ under DD and $V = \bar{v}$ under CC, if $\delta < 1$, both players' expected efforts equal $\delta V/2$. Hence conditional total effort is δV. The conditional expected winner's effort is

$$\int_0^{\delta V} x\left(1 - \delta + \frac{x}{V}\right)\frac{dx}{\delta V} + \int_0^{V} x\frac{x}{V}\frac{\delta\, dx}{V} = \frac{\delta V(5-\delta)}{6}.$$

If $\delta \geq 1$, both players' expected efforts equal $V/(2\delta)$. Hence conditional total effort is V/δ. The conditional expected winner's effort is

$$\int_0^{V} x\frac{x}{V}\frac{dx}{\delta V} + \int_0^{V/\delta} x\left(1 - \frac{1}{\delta} + \frac{x}{V}\right)\frac{\delta\, dx}{V} = \frac{V(5\delta - 1)}{6\delta^2}.$$

Taking expectations over s under DD, and using $\mathbb{E}[\widehat{v}_s] = \bar{v}$, gives the first two rows of Table 1. The case CC is the same because $V = \bar{v}$.

Now consider $\gamma = DC$. From the CDFs in Lemma 1, direct integration gives

$$\mathrm{TE}(DC,\delta) = \begin{cases} \delta\left[\widehat{v}_L + \widehat{\mu}^2(\widehat{v}_H - \widehat{v}_L)\right], & \delta < 1, \\ \dfrac{\widehat{v}_L}{\delta} + \delta\widehat{\mu}^2(\widehat{v}_H - \widehat{v}_L), & 1 \le \delta \le \dfrac{1}{\widehat{\mu}}, \\ \dfrac{\widehat{v}_H}{\delta}, & \delta > \dfrac{1}{\widehat{\mu}}. \end{cases}$$

This gives the total-effort entries in the last three rows of Table 1.

It remains to compute the expected winner's effort under $\gamma = DC$. For $\delta < 1$, conditional on $s = L$, the winner's effort is

$$WE_L = \int_0^{\delta(1-\widehat{\mu})\widehat{v}_L} x\left(1 - \delta + \frac{x}{\widehat{v}_L}\right)\frac{dx}{\delta(1-\widehat{\mu})\widehat{v}_L} + \int_0^{(1-\widehat{\mu})\widehat{v}_L} x\frac{x}{(1-\widehat{\mu})\widehat{v}_L}\frac{\delta\,dx}{\widehat{v}_L} + \int_{(1-\widehat{\mu})\widehat{v}_L}^{\bar{v}} x\frac{\delta\,dx}{\widehat{v}_H}.$$

Conditional on $s = H$, the winner's effort is

$$WE_H = \int_{\delta(1-\widehat{\mu})\widehat{v}_L}^{\delta\bar{v}} x\left(1 - \delta\widehat{\mu} + \frac{x - \delta(1-\widehat{\mu})\widehat{v}_L}{\widehat{v}_H}\right)\frac{dx}{\delta\widehat{\mu}\widehat{v}_H} + \int_{(1-\widehat{\mu})\widehat{v}_L}^{\bar{v}} x\frac{x - (1-\widehat{\mu})\widehat{v}_L}{\widehat{\mu}\widehat{v}_H}\frac{\delta\,dx}{\widehat{v}_H}.$$

Therefore

$$(1-\widehat{\mu})WE_L + \widehat{\mu}WE_H = \frac{\widehat{v}_L}{6}\mathcal{W}_1\left(\widehat{\mu}, \frac{\widehat{v}_H - \widehat{v}_L}{\widehat{v}_L}, \delta\right).$$

For $1 \le \delta \le 1/\widehat{\mu}$, conditional on $s = L$, the winner's effort is

$$WE_L = \int_0^{(1-\widehat{\mu}\delta)\widehat{v}_L} x\frac{x}{\widehat{v}_L}\frac{dx}{\delta(1-\widehat{\mu})\widehat{v}_L} + \int_0^{(1-\widehat{\mu}\delta)\widehat{v}_L/\delta} x\left(\frac{1-1/\delta}{1-\widehat{\mu}} + \frac{x}{(1-\widehat{\mu})\widehat{v}_L}\right)\frac{\delta\,dx}{\widehat{v}_L} + \int_{(1-\widehat{\mu}\delta)\widehat{v}_L/\delta}^{\widehat{v}_L/\delta+\widehat{\mu}(\widehat{v}_H-\widehat{v}_L)} x\frac{\delta\,dx}{\widehat{v}_H}.$$

Conditional on $s = H$, the winner's effort is

$$WE_H = \int_{(1-\widehat{\mu}\delta)\widehat{v}_L}^{\widehat{v}_L+\delta\widehat{\mu}(\widehat{v}_H-\widehat{v}_L)} x\left(1 - \widehat{\mu}\delta + \frac{x - (1-\widehat{\mu}\delta)\widehat{v}_L}{\widehat{v}_H}\right)\frac{dx}{\delta\widehat{\mu}\widehat{v}_H} + \int_{(1-\widehat{\mu}\delta)\widehat{v}_L/\delta}^{\widehat{v}_L/\delta+\widehat{\mu}(\widehat{v}_H-\widehat{v}_L)} x\frac{x - (1-\widehat{\mu}\delta)\widehat{v}_L/\delta}{\widehat{\mu}\widehat{v}_H}\frac{\delta\,dx}{\widehat{v}_H}.$$

Hence

$$(1-\widehat{\mu})WE_L + \widehat{\mu}WE_H = \frac{\widehat{v}_L}{6}\mathcal{W}_2\left(\widehat{\mu}, \frac{\widehat{v}_H - \widehat{v}_L}{\widehat{v}_L}, \delta\right).$$

Finally, for $\delta > 1/\widehat{\mu}$, type $1L$ exerts zero effort for sure. Using the last case in Lemma 1, the expected winner's effort is

$$
\begin{aligned}
\mathrm{WE}(DC,\delta) =& (1-\widehat{\mu})\int_0^{\widehat{v}_H/\delta} x\frac{\delta\, dx}{\widehat{v}_H} \\
&+\widehat{\mu}\left[\int_0^{\widehat{v}_H} x\frac{x}{\widehat{v}_H}\frac{dx}{\delta\widehat{\mu}\widehat{v}_H}+\int_0^{\widehat{v}_H/\delta} x\left(1-\frac{1}{\delta\widehat{\mu}}+\frac{x}{\widehat{\mu}\widehat{v}_H}\right)\frac{\delta\, dx}{\widehat{v}_H}\right] \\
=& \frac{\widehat{v}_H(5\delta-1)}{6\delta^2}=\frac{\widehat{v}_H}{6}\mathcal{W}_3(\delta).
\end{aligned}
$$

This completes the derivation of all entries in Table 1. ■

A.3 Proof of Proposition 1

Proof. We first prove part (i). From Table 1,

$$
\mathrm{TE}(CC,\delta)=\mathrm{TE}(DD,\delta)=\begin{cases}\delta\bar{v}, & \delta<1,\\ \bar{v}/\delta, & \delta\geq 1,\end{cases}
$$

which is maximized at $\delta=1$, with maximum $\bar{v}$. Under $\gamma=DC$, the three cases in Table 1 imply

$$
\mathrm{TE}(DC,\delta)\leq\bar{v}.
$$

Indeed, if $\delta<1$, then

$$
\mathrm{TE}(DC,\delta)=\delta\left[\widehat{v}_L+\widehat{\mu}^2(\widehat{v}_H-\widehat{v}_L)\right]\leq\widehat{v}_L+\widehat{\mu}(\widehat{v}_H-\widehat{v}_L)=\bar{v}.
$$

If $1\leq\delta\leq 1/\widehat{\mu}$, then

$$
\mathrm{TE}(DC,\delta)=\frac{\widehat{v}_L}{\delta}+\delta\widehat{\mu}^2(\widehat{v}_H-\widehat{v}_L)\leq\widehat{v}_L+\widehat{\mu}(\widehat{v}_H-\widehat{v}_L)=\bar{v}.
$$

If $\delta>1/\widehat{\mu}$, then

$$
\mathrm{TE}(DC,\delta)=\frac{\widehat{v}_H}{\delta}<\widehat{\mu}\widehat{v}_H\leq\bar{v}.
$$

The case $\gamma=CD$ is symmetric. Therefore expected total effort is maximized by $(CC,1)$ and $(DD,1)$.

We next prove part (ii). The following observation follows by direct maximization of the three expressions for $\mathrm{WE}(DC,\delta)$ in Table 1.

Lemma A1 *For $\gamma=DC$, the expected winner's effort is maximized at either $\delta=1$ or $\delta=1/\widehat{\mu}$.*

Proof. For $\delta < 1$, the expression for $\mathrm{WE}(DC, \delta)$ is increasing in δ on $(0, 1)$. For $\delta > 1/\widehat{\mu}$, the expression $\widehat{v}_H(5\delta - 1)/(6\delta^2)$ is decreasing in δ. On the intermediate interval $1 \leq \delta \leq 1/\widehat{\mu}$, differentiating the middle expression in Table 1 shows that no interior point yields a value exceeding both endpoints. Hence the maximum over all $\delta > 0$ is attained at either $\delta = 1$ or $\delta = 1/\widehat{\mu}$. ■

Under symmetric disclosure,

$$\mathrm{WE}(CC, 1) = \mathrm{WE}(DD, 1) = \frac{2}{3}\bar{v}.$$

At $\delta = 1$ under asymmetric disclosure,

$$\mathrm{WE}(DC, 1) = \frac{1}{6}\left[4\widehat{v}_L + \widehat{\mu}^2(6 - 2\widehat{\mu})(\widehat{v}_H - \widehat{v}_L)\right],$$

and thus

$$\mathrm{WE}(CC, 1) - \mathrm{WE}(DC, 1) = \frac{\widehat{\mu}(1 - \widehat{\mu})(2 - \widehat{\mu})(\widehat{v}_H - \widehat{v}_L)}{3} > 0.$$

Therefore $(DC, 1)$ is dominated by the fully symmetric contest. At the releveling bias $\delta = 1/\widehat{\mu}$,

$$\mathrm{WE}\left(DC, \frac{1}{\widehat{\mu}}\right) = \frac{\widehat{\mu}\widehat{v}_H(5 - \widehat{\mu})}{6}.$$

Comparing this with $\mathrm{WE}(CC, 1) = 2\bar{v}/3$, and using $\bar{v} = (1 - \widehat{\mu})\widehat{v}_L + \widehat{\mu}\widehat{v}_H$, yields

$$\mathrm{WE}\left(DC, \frac{1}{\widehat{\mu}}\right) - \mathrm{WE}(CC, 1) = \frac{(1 - \widehat{\mu})\left(\widehat{\mu}\widehat{v}_H - 4\widehat{v}_L\right)}{6}.$$

Hence the tilting-and-releveling contest $(DC, 1/\widehat{\mu})$ is optimal if and only if $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L$; otherwise the fully symmetric contest is optimal. By relabeling the players, $(CD, \widehat{\mu})$ is outcome equivalent to $(DC, 1/\widehat{\mu})$. This proves part (ii). ■

A.4 Proof of Remark 1

Proof. First fix $\delta = 1$. Under symmetric disclosure, Table 1 gives

$$\mathrm{TE}(CC, 1) = \mathrm{TE}(DD, 1) = \bar{v}, \qquad \mathrm{WE}(CC, 1) = \mathrm{WE}(DD, 1) = \frac{2}{3}\bar{v}.$$

Under DC,

$$\mathrm{TE}(DC, 1) = \widehat{v}_L + \widehat{\mu}^2(\widehat{v}_H - \widehat{v}_L),$$

so

$$\mathrm{TE}(CC, 1) - \mathrm{TE}(DC, 1) = \widehat{\mu}(1 - \widehat{\mu})(\widehat{v}_H - \widehat{v}_L) > 0.$$

Moreover, as computed in the proof of Proposition 1,

$$\mathrm{WE}(CC,1) - \mathrm{WE}(DC,1) = \frac{\widehat{\mu}(1-\widehat{\mu})(2-\widehat{\mu})(\widehat{v}_H - \widehat{v}_L)}{3} > 0.$$

The case CD is symmetric. Hence, when the scoring rule is neutral, a symmetric disclosure scheme maximizes both objectives.

Next fix $\gamma \in \{CC, DD\}$. From Table 1,

$$\mathrm{TE}(\gamma,\delta) = \begin{cases} \delta\bar{v}, & \delta < 1, \\ \bar{v}/\delta, & \delta \geq 1, \end{cases}$$

which is maximized at $\delta = 1$. The same table gives

$$\mathrm{WE}(\gamma,\delta) = \begin{cases} \delta\bar{v}(5-\delta)/6, & \delta < 1, \\ \bar{v}(5\delta-1)/(6\delta^2), & \delta \geq 1. \end{cases}$$

The first expression is increasing on $(0,1)$, and the second is decreasing on $[1,\infty)$. Thus $\delta = 1$ maximizes expected winner's effort under either symmetric disclosure scheme. This proves both parts. ■

A.5 Proof of Proposition 2

Proof. To proceed, we derive how the designer's deviation in disclosure policy changes a players' bidding strategy. Due to symmetry, it is without loss to focus on player 2. Further, when $\gamma(2) = D$, either player 2 cannot detect and react to the designer's deviation, or he would complain for sure due to loss of information, rendering the deviation infeasible. Therefore, it suffices to consider the case in which $\gamma(2) = C$.

Let $\bar{b}_{1s}(\gamma,\delta)$ denote player 1's highest equilibrium bid under (γ,δ) and the signal realization s for $\gamma \in \{DC, DD\}$. Similarly, let $\bar{b}_1(\gamma,\delta)$ denote player 1's highest equilibrium bid under (γ,δ) for $\gamma \in \{CC, CD\}$. The following result ensues.

Lemma A2 ***(Player's Bidding Strategy upon Detecting Designer's Deviation)*** *Fix an announced policy (γ,δ) and the designer's deviation $\gamma' \neq \gamma$. Player 2's bidding strategy is described as follows.*

(i) If $\gamma = CC$ and $\gamma' \in \{CD, DD\}$, player 2 bids 0 when receiving a low signal and bids $\bar{b}_1(CC,\delta)/\delta$ when receiving a high signal.

(ii) If $\gamma = CC$ and $\gamma' = DC$, player 2 follows his equilibrium bidding strategy under (γ,δ) since he is not aware of the deviation.

(iii) If $\gamma = DC$ and $\gamma' = DD$, player 2 bids $\bar{b}_{1s}(DC, \delta)/\delta$ when receiving signal $s \in \{L, H\}$ and wins the contest with certainty.

Proof. It is straightforward to verify that the strategies described in the lemma are player 2's best responses to player 1's equilibrium bidding strategy under (γ, δ) following the designer's deviations. ■

Part (i) follows immediately from Lemma A2 and it remains to prove part (ii).

For part (ii), first note that the contest $(\gamma, \delta) = (DD, 1)$ is credible. Therefore, in the case in which $(DD, 1)$ or $(CC, 1)$ is optimal absent credibility concern—i.e., when $\widehat{\mu}\widehat{v}_H \leq 4\widehat{v}_L$—$(DD, 1)$ is still optimal in the presence of credibility concern. Further, note that $\text{WE}(CC, 1) = \text{WE}(DD, 1)$. As long as $(\gamma, \delta) = (DD, 1)$ emerges as an optimal credible contest, $(CC, 1)$ is also optimal, provided that it is credible. By Lemma A2, we can verify that $(CC, 1)$ is credible if and only if $\widehat{\mu} \leq 1/3$.

It remains to consider the case in which $\widehat{\mu}\widehat{v}_H > 4\widehat{v}_L$, or equivalently, $\text{WE}(DC, 1/\widehat{\mu}) > \text{WE}(DD, 1)$. By Lemma A2, fixing δ, the designer's deviation from DC to DD generates the following expected winner's effort:

$$\text{WE}(DC \to DD, \delta) := \begin{cases} (1-\widehat{\mu})\widehat{v}_L + \widehat{\mu}^2\widehat{v}_H, & \text{if } \delta < 1, \\ \frac{1-\widehat{\mu}\delta}{\delta}\widehat{v}_L + \widehat{\mu}^2\widehat{v}_H, & \text{if } 1 \leq \delta \leq \frac{1}{\widehat{\mu}}, \\ \frac{\widehat{\mu}\widehat{v}_H}{\delta}, & \text{if } \delta > \frac{1}{\widehat{\mu}}. \end{cases}$$

Fixing $\delta = 1/\widehat{\mu}$, simple algebra would verify that $\text{WE}(DC \to DD, \delta) \leq \text{WE}(DC, \delta)$ if and only if $\widehat{\mu} \leq 5/7$. Therefore, the optimal credible contest is $(DC, 1/\widehat{\mu})$ or $(CD, \widehat{\mu})$ if $\widehat{\mu} \leq 5/7$.

We now turn to the case of $\widehat{\mu} > 5/7$. Carrying out the algebra, we can show that $\text{WE}(DC \to DD, \delta) > \text{WE}(DC, \delta)$ for all $\delta \leq 1/\widehat{\mu}$. Put differently, any contest (DC, δ) with $\delta \leq 1/\widehat{\mu}$ is not credible. For $\delta > 1/\widehat{\mu}$, it holds that

$$\text{WE}(DC, \delta) - \text{WE}(DC \to DD, \delta) = \frac{(5-6\widehat{\mu})\delta - 1}{6\delta^2}\widehat{v}_H.$$

Suppose $\widehat{\mu} > 5/6$. It can be verified that $\text{WE}(DC, \delta) - \text{WE}(DC \to DD, \delta) < 0$ and thus (DC, δ) is not credible for all $\delta > 1/\widehat{\mu}$, from which we can conclude that $(DD, 1)$ is the optimal credible contest.

Next, suppose $5/7 < \widehat{\mu} \leq 5/6$. It follows from the above equation that (DC, δ) is credible if and only if $\delta \geq 1/(5 - 6\widehat{\mu})$. Recall from the proof of Lemma A1 that $\text{WE}(DC, \delta)$ decreases with δ for $\delta > 1/\widehat{\mu}$. Further, $1/(5 - 6\widehat{\mu}) > 1/\widehat{\mu}$ for $\widehat{\mu} > 5/7$. Therefore, $\delta = 1/(5 - 6\widehat{\mu})$ maximizes the expected winner's effort among all credible contests with $\gamma = DC$. Moreover, simple algebra would verify that $\mathrm{WE}(DC, 1/(5-6\widehat{\mu})) > \mathrm{WE}(DD, 1)$ is equivalent to

$$\frac{13\widehat{\mu} - 18\widehat{\mu}^2}{2(1-\widehat{\mu})} > \frac{\widehat{v}_L}{\widehat{v}_H}.$$

Therefore, both $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = \big(CD, 5-6\widehat{\mu}\big)$ and $(\gamma^*_{\mathrm{WE}}, \delta^*_{\mathrm{WE}}) = \big(DC, 1/(5-6\widehat{\mu})\big)$ are an optimal credible contest scheme if the above inequality holds and $(DD, 1)$ is optimal otherwise. ■

A.6 Proof of Proposition 3

Proof. For any pair of equilibrium effort CDFs, the expected maximum effort can be computed as

$$\mathbb{E}[\max\{x_1, x_2\}] = \int_0^\infty \big[1 - \Pr(x_1 \le x)\Pr(x_2 \le x)\big]\, dx.$$

We first consider symmetric disclosure. Using the equilibrium CDFs in Lemma 1, direct integration gives

$$\mathrm{ME}(CC, \delta) = \mathrm{ME}(DD, \delta) = \begin{cases} \bar{v}\left(\delta - \dfrac{\delta^2}{2} + \dfrac{\delta^3}{6}\right), & 0 < \delta \le 1, \\[2ex] \bar{v}\left(\dfrac{1}{\delta} - \dfrac{1}{2\delta^2} + \dfrac{1}{6\delta^3}\right), & \delta \ge 1. \end{cases}$$

It is straightforward to verify that symmetric disclosure is maximized at $\delta = 1$, and

$$\mathrm{ME}(CC, 1) = \mathrm{ME}(DD, 1) = \frac{2}{3}\bar{v}.$$

It remains to analyze asymmetric disclosure. By symmetry,

$$\mathrm{ME}(CD, \delta) = \mathrm{ME}(DC, 1/\delta),$$

so it suffices to consider DC.

First suppose $0 < \delta < 1$. Under DC, player 1's unconditional effort CDF is

$$(1-\widehat{\mu})B_{1L}(x; DC, \delta) + \widehat{\mu}B_{1H}(x; DC, \delta),$$

and player 2's effort CDF is $B_2(x; DC, \delta)$. From the CDFs in Lemma 1, for every $x \in [0, \bar{v}]$,

$$(1-\widehat{\mu})B_{1L}(x; DC, \delta) + \widehat{\mu}B_{1H}(x; DC, \delta) \ge \frac{x}{\bar{v}},$$

and

$$B_2(x; DC, \delta) \geq \frac{x}{\bar{v}}.$$

Therefore,

$$\begin{aligned}
\mathrm{ME}(DC, \delta) &= \int_0^{\bar{v}} \left[1 - \big((1-\widehat{\mu})B_{1L}(x; DC, \delta) + \widehat{\mu}B_{1H}(x; DC, \delta)\big)\, B_2(x; DC, \delta)\right] dx \\
&\leq \int_0^{\bar{v}} \left[1 - \left(\frac{x}{\bar{v}}\right)^2\right] dx = \frac{2}{3}\bar{v}.
\end{aligned}$$

Thus no asymmetric-disclosure contest with $0 < \delta < 1$ can outperform $(CC, 1)$ or $(DD, 1)$.

Next suppose

$$1 \leq \delta \leq \frac{1}{\widehat{\mu}}.$$

Again using the CDFs in Lemma 1, player 1's unconditional effort CDF satisfies

$$(1-\widehat{\mu})B_{1L}(x; DC, \delta) + \widehat{\mu}B_{1H}(x; DC, \delta) \geq 1 - \frac{1}{\delta} + \frac{x}{\delta\left[\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)\right]}$$

for every

$$x \in \left[0, \widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)\right].$$

Similarly, player 2's effort CDF satisfies

$$B_2(x; DC, \delta) \geq \frac{\delta x}{\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)}$$

for every

$$x \in \left[0, \frac{\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)}{\delta}\right],$$

and $B_2(x; DC, \delta) = 1$ above this interval. Hence

$$\begin{aligned}
\mathrm{ME}(DC, \delta) \leq &\int_0^{\frac{\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)}{\delta}} \left[1 - \left(1 - \frac{1}{\delta} + \frac{x}{\delta\left[\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)\right]}\right) \frac{\delta x}{\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)}\right] dx \\
&+ \int_{\frac{\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)}{\delta}}^{\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)} \left[1 - \left(1 - \frac{1}{\delta} + \frac{x}{\delta\left[\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)\right]}\right)\right] dx \\
&= \left[\widehat{v}_L + \widehat{\mu}\delta(\widehat{v}_H - \widehat{v}_L)\right] \left(\frac{1}{\delta} - \frac{1}{2\delta^2} + \frac{1}{6\delta^3}\right).
\end{aligned}$$

We now show that this upper bound is never larger than both endpoint values

$$\frac{2}{3}\bar{v} \quad \text{and} \quad \widehat{v}_H\left(\widehat{\mu}-\frac{\widehat{\mu}^2}{2}+\frac{\widehat{\mu}^3}{6}\right).$$

The first endpoint is the symmetric-disclosure value, and the second will be shown below to be the value of $\mathrm{ME}(DC, 1/\widehat{\mu})$.

For $1 \leq \delta \leq 1/\widehat{\mu}$, straightforward algebra gives

$$\begin{aligned}
&6\left\{[\widehat{v}_L+\widehat{\mu}\delta(\widehat{v}_H-\widehat{v}_L)]\left(\frac{1}{\delta}-\frac{1}{2\delta^2}+\frac{1}{6\delta^3}\right)-\frac{2}{3}\bar{v}\right\}\\
&=\left(\frac{1}{\delta}-1\right)\left[\widehat{v}_L\left(\frac{1}{\delta^2}-\frac{2}{\delta}+4\right)-\widehat{\mu}(\widehat{v}_H-\widehat{v}_L)\left(2-\frac{1}{\delta}\right)\right].
\end{aligned}$$

Since $1/\delta - 1 \leq 0$, the upper bound being larger than $\frac{2}{3}\bar{v}$ requires

$$\frac{\widehat{v}_H-\widehat{v}_L}{\widehat{v}_L} > \frac{\frac{1}{\delta^2}-\frac{2}{\delta}+4}{\widehat{\mu}\left(2-\frac{1}{\delta}\right)}.$$

Similarly,

$$\begin{aligned}
&6\left\{[\widehat{v}_L+\widehat{\mu}\delta(\widehat{v}_H-\widehat{v}_L)]\left(\frac{1}{\delta}-\frac{1}{2\delta^2}+\frac{1}{6\delta^3}\right)-\widehat{v}_H\left(\widehat{\mu}-\frac{\widehat{\mu}^2}{2}+\frac{\widehat{\mu}^3}{6}\right)\right\}\\
&=\left(\frac{1}{\delta}-\widehat{\mu}\right)\left[\widehat{\mu}\widehat{v}_H\left(\widehat{\mu}+\frac{1}{\delta}-3\right)+\widehat{v}_L\left(\frac{1}{\delta^2}-\frac{3}{\delta}+6\right)\right].
\end{aligned}$$

Since $1/\delta - \widehat{\mu} \geq 0$, the upper bound being larger than

$$\widehat{v}_H\left(\widehat{\mu}-\frac{\widehat{\mu}^2}{2}+\frac{\widehat{\mu}^3}{6}\right)$$

requires

$$\frac{\widehat{v}_H-\widehat{v}_L}{\widehat{v}_L} < \frac{\widehat{\mu}\left(\widehat{\mu}+\frac{1}{\delta}-3\right)+\frac{1}{\delta^2}-\frac{3}{\delta}+6}{\widehat{\mu}\left(3-\widehat{\mu}-\frac{1}{\delta}\right)}.$$

But

$$\begin{aligned}
&\frac{\frac{1}{\delta^2}-\frac{2}{\delta}+4}{\widehat{\mu}\left(2-\frac{1}{\delta}\right)}-\frac{\widehat{\mu}\left(\widehat{\mu}+\frac{1}{\delta}-3\right)+\frac{1}{\delta^2}-\frac{3}{\delta}+6}{\widehat{\mu}\left(3-\widehat{\mu}-\frac{1}{\delta}\right)} \\
=&\frac{(1-\widehat{\mu})\left(2\widehat{\mu}+\frac{2}{\delta}-\frac{\widehat{\mu}}{\delta}\right)}{\widehat{\mu}\left(2-\frac{1}{\delta}\right)\left(3-\widehat{\mu}-\frac{1}{\delta}\right)}>0.
\end{aligned}$$

Therefore the two required inequalities cannot hold simultaneously. Hence, for every $1 \leq \delta \leq 1/\widehat{\mu}$,

$$\mathrm{ME}(DC,\delta) \leq \max\left\{\frac{2}{3}\bar{v},\ \widehat{v}_H\left(\widehat{\mu}-\frac{\widehat{\mu}^2}{2}+\frac{\widehat{\mu}^3}{6}\right)\right\}.$$

Finally, suppose

$$\delta \geq \frac{1}{\widehat{\mu}}.$$

In this region, type $1L$ exerts zero effort for sure. From the CDFs in Lemma 1, player 1's unconditional effort distribution has an atom $1-1/\delta$ at zero and density $1/(\delta\widehat{v}_H)$ on $(0,\widehat{v}_H]$, while player 2 is uniformly distributed on $[0,\widehat{v}_H/\delta]$. Therefore,

$$\mathrm{ME}(DC,\delta)=\widehat{v}_H\left(\frac{1}{\delta}-\frac{1}{2\delta^2}+\frac{1}{6\delta^3}\right).$$

This expression is strictly decreasing for $\delta>1$, so this region is maximized at $\delta=1/\widehat{\mu}$. Thus

$$\mathrm{ME}(DC,1/\widehat{\mu})=\widehat{v}_H\left(\widehat{\mu}-\frac{\widehat{\mu}^2}{2}+\frac{\widehat{\mu}^3}{6}\right).$$

Combining the three cases, $\mathrm{ME}(DC,\delta)$ is maximized either by $(DC,1/\widehat{\mu})$ or by the symmetric benchmark $(CC,1)$ and $(DD,1)$. By player symmetry, the same conclusion applies to CD, with $(CD,\widehat{\mu})$ outcome-equivalent to $(DC,1/\widehat{\mu})$.

It remains to compare the two candidate values. We have

$$\mathrm{ME}(CC,1)=\mathrm{ME}(DD,1)=\frac{2}{3}\bar{v}=\frac{2}{3}\left[\widehat{\mu}\widehat{v}_H+(1-\widehat{\mu})\widehat{v}_L\right],$$

and

$$\mathrm{ME}(DC,1/\widehat{\mu})=\mathrm{ME}(CD,\widehat{\mu})=\widehat{v}_H\left(\widehat{\mu}-\frac{\widehat{\mu}^2}{2}+\frac{\widehat{\mu}^3}{6}\right).$$

Therefore,

$$
\begin{aligned}
&\mathrm{ME}(DC, 1/\widehat{\mu}) - \mathrm{ME}(CC, 1) \\
=&\widehat{v}_H\left(\widehat{\mu} - \frac{\widehat{\mu}^2}{2} + \frac{\widehat{\mu}^3}{6}\right) - \frac{2}{3}\left[\widehat{\mu}\widehat{v}_H + (1-\widehat{\mu})\widehat{v}_L\right] \\
=&\frac{1-\widehat{\mu}}{6}\left[\widehat{\mu}(2-\widehat{\mu})\widehat{v}_H - 4\widehat{v}_L\right].
\end{aligned}
$$

Hence the symmetric contest is optimal when

$$\widehat{\mu}(2-\widehat{\mu})\widehat{v}_H < 4\widehat{v}_L,$$

the tilting-and-releveling contest is optimal when

$$\widehat{\mu}(2-\widehat{\mu})\widehat{v}_H > 4\widehat{v}_L,$$

and both are optimal at equality. This completes the proof. ■

A.7 Proof of Proposition 4

Proof. We first consider symmetric disclosure. Fix a common expected prize value $V > 0$, where $V = \bar{v}$ under CC and $V = \widehat{v}_s$ after signal $s \in \{H, L\}$ under DD. Under the Tullock winner-selection mechanism, player 1 solves

$$\max_{x_1 \geq 0} V \frac{x_1}{x_1 + \delta x_2} - x_1,$$

and player 2 solves

$$\max_{x_2 \geq 0} V \frac{\delta x_2}{x_1 + \delta x_2} - x_2.$$

The first-order conditions of an interior equilibrium are

$$\frac{V\delta x_2}{(x_1 + \delta x_2)^2} = 1, \qquad \frac{V\delta x_1}{(x_1 + \delta x_2)^2} = 1.$$

Hence $x_1 = x_2$, and therefore

$$x_1 = x_2 = \frac{V\delta}{(1+\delta)^2}.$$

Thus

$$\mathrm{TE} = \frac{2V\delta}{(1+\delta)^2}, \qquad \mathrm{WE} = \frac{V\delta}{(1+\delta)^2}.$$

Both expressions are maximized at $\delta = 1$. Taking expectations over the signal under DD, and using

$$\bar{v} = \widehat{\mu}\widehat{v}_H + (1 - \widehat{\mu})\widehat{v}_L,$$

we obtain

$$\text{TE}(CC, 1) = \text{TE}(DD, 1) = \frac{\bar{v}}{2}, \qquad \text{WE}(CC, 1) = \text{WE}(DD, 1) = \frac{\bar{v}}{4}.$$

We next consider asymmetric disclosure. It suffices to analyze the symmetric disclosure schemes and the asymmetric disclosure scheme DC. The case CD follows by relabeling the players and replacing δ by $1/\delta$. Let $\gamma = DC$, so player 1 observes the signal and player 2 does not.

Our approach is to consider what equilibrium effort profiles can be induced by varying δ. For that purpose, we write

$$\delta x_2 = \widehat{v}_H \alpha^2,$$

and analyze what values of $\alpha \in [0, 1]$ is attainable in equilibrium, and then optimize over the feasible set.

Given δx_2, type $s \in \{H, L\}$ of player 1 solves

$$\max_{x_1 \geq 0} \widehat{v}_s \frac{x_1}{x_1 + \delta x_2} - x_1.$$

Hence type H's best response is $x_{1H} = \widehat{v}_H \alpha(1 - \alpha)$, and type L's best response is $x_{1L} = \widehat{v}_H \alpha \left(\sqrt{\widehat{v}_L / \widehat{v}_H} - \alpha \right)$ when $0 \leq \alpha \leq \sqrt{\widehat{v}_L / \widehat{v}_H}$, while type L exerts zero effort when $\alpha \geq \sqrt{\widehat{v}_L / \widehat{v}_H}$.

Player 2's first-order condition is

$$\delta \left[\widehat{\mu} \frac{\widehat{v}_H x_{1H}}{(x_{1H} + \delta x_2)^2} + (1 - \widehat{\mu}) \frac{\widehat{v}_L x_{1L}}{(x_{1L} + \delta x_2)^2} \right] = 1,$$

where the second term is omitted when type L is inactive. For every active type s, player 1's first-order condition implies

$$\frac{\widehat{v}_s \delta x_2}{(x_{1s} + \delta x_2)^2} = 1.$$

Substituting this identity into player 2's first-order condition gives $x_2 = \widehat{\mu} x_{1H} + (1 - \widehat{\mu}) x_{1L}$ when both informed types are active, and $x_2 = \widehat{\mu} x_{1H}$ when only type H is active.

First suppose $0 \leq \alpha \leq \sqrt{\widehat{v}_L / \widehat{v}_H}$. Then both informed types are active, and $x_2 = \widehat{v}_H \alpha \left[\widehat{\mu} + (1 - \widehat{\mu}) \sqrt{\widehat{v}_L / \widehat{v}_H} - \alpha \right]$. Therefore this α is implemented by $\delta = \alpha / [\widehat{\mu} + (1 - \widehat{\mu}) \sqrt{\widehat{v}_L / \widehat{v}_H} - \alpha]$. Total effort is $\text{TE} = 2\widehat{v}_H \alpha \left[\widehat{\mu} + (1 - \widehat{\mu}) \sqrt{\widehat{v}_L / \widehat{v}_H} - \alpha \right]$. Hence $\text{TE} \leq$

$(\widehat{v}_H/2)\left[\widehat{\mu}+(1-\widehat{\mu})\sqrt{\widehat{v}_L/\widehat{v}_H}\right]^2$. Moreover, $\left[\widehat{\mu}+(1-\widehat{\mu})\sqrt{\widehat{v}_L/\widehat{v}_H}\right]^2 \leq \widehat{\mu}+(1-\widehat{\mu})\widehat{v}_L/\widehat{v}_H$. Thus $\mathrm{TE} \leq \bar{v}/2$.

In this region, the expected winner's effort equals

$$\begin{aligned}\mathrm{WE} = \widehat{v}_H \Bigg[& \alpha\left(\widehat{\mu}+(1-\widehat{\mu})\sqrt{\frac{\widehat{v}_L}{\widehat{v}_H}}\right) \\ & + \alpha^2\left(\left[\widehat{\mu}+(1-\widehat{\mu})\sqrt{\frac{\widehat{v}_L}{\widehat{v}_H}}\right]\left[\widehat{\mu}+(1-\widehat{\mu})\sqrt{\frac{\widehat{v}_H}{\widehat{v}_L}}\right]-2\right)\Bigg].\end{aligned}$$

Indeed, conditional on $s=H$, player 1 wins with probability $1-\alpha$ and player 2 wins with probability α. Conditional on $s=L$, player 1 wins with probability $1-\alpha\sqrt{\widehat{v}_H/\widehat{v}_L}$, and player 2 wins with probability $\alpha\sqrt{\widehat{v}_H/\widehat{v}_L}$. Substituting x_{1H}, x_{1L}, and x_2 into the definition of expected winner's effort gives the displayed expression.

Now suppose $\sqrt{\widehat{v}_L/\widehat{v}_H} \leq \alpha \leq 1$. Then only type H is active. Hence

$$x_{1L}=0, \qquad x_{1H}=\widehat{v}_H\alpha(1-\alpha), \qquad x_2=\widehat{\mu}\widehat{v}_H\alpha(1-\alpha).$$

This α is implemented by

$$\delta=\frac{\alpha}{\widehat{\mu}(1-\alpha)}.$$

Total effort is

$$\mathrm{TE}=2\widehat{\mu}\widehat{v}_H\alpha(1-\alpha) \leq \frac{\widehat{\mu}\widehat{v}_H}{2} \leq \frac{\bar{v}}{2}.$$

The expected winner's effort is

$$\mathrm{WE}=\widehat{\mu}\widehat{v}_H\alpha(1-\alpha)\left[2-\widehat{\mu}-(1-\widehat{\mu})\alpha\right].$$

To see this, conditional on $s=H$, player 1 wins with probability $1-\alpha$ and player 2 wins with probability α. Conditional on $s=L$, player 1 exerts zero effort and player 2 wins for sure. Therefore

$$\begin{aligned}\mathrm{WE} &= \widehat{\mu}\left[(1-\alpha)x_{1H}+\alpha x_2\right]+(1-\widehat{\mu})x_2 \\ &= \widehat{\mu}\widehat{v}_H\alpha(1-\alpha)\left[2-\widehat{\mu}-(1-\widehat{\mu})\alpha\right].\end{aligned}$$

Combining the two regions, every asymmetric-disclosure equilibrium under DC is represented by some $\alpha \in [0,1]$, and every such α is implemented by the corresponding value of δ displayed above. Since the case CD is the mirror image of DC, the total-effort bounds above imply

$$\mathrm{TE}(DC,\delta) \le \frac{\bar{v}}{2}, \qquad \mathrm{TE}(CD,\delta) \le \frac{\bar{v}}{2}.$$

Because the symmetric contests $(CC, 1)$ and $(DD, 1)$ attain $\bar{v}/2$, part (i) follows.

It remains to prove part (ii). Since all expected-effort expressions are homogeneous of degree one in $\widehat{v}_H$, the comparison depends on the posterior values only through the ratio $\widehat{v}_L/\widehat{v}_H$. Define Φ as follows. Set

$$\Phi(0) = 0, \qquad \Phi(\widehat{\mu}) = 1 \quad \text{for all} \quad \widehat{\mu} \in [1/2, 1],$$

and for $\widehat{\mu} \in (0, 1/2)$, set

$$\Phi(\widehat{\mu}) := \max\left\{ \left(\frac{\widehat{\mu}}{1-\widehat{\mu}}\right)^2, \right. \\ \left. \frac{4\widehat{\mu}\left(\widehat{\mu} - 1 + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)\left(1 + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)}{(1-\widehat{\mu})\left(\widehat{\mu} + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)^3} - \frac{\widehat{\mu}}{1-\widehat{\mu}} \right\}.$$

The first term in the maximum is the crossing generated by the region in which both informed types can remain active. The second term is the crossing generated by the region in which only type H is active. In the latter region, the unconstrained maximizer solves

$$\frac{\partial}{\partial \alpha}\left\{\widehat{\mu}\alpha(1-\alpha)\left[2 - \widehat{\mu} - (1-\widehat{\mu})\alpha\right]\right\} = 0,$$

which gives

$$\alpha = \frac{\widehat{\mu} - 1 + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}}{\widehat{\mu} + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}}.$$

Substituting this value into the high-only expression for WE, and comparing it with the symmetric benchmark

$$\frac{\bar{v}}{4} = \frac{\widehat{v}_H}{4}\left[\widehat{\mu} + (1-\widehat{\mu})\frac{\widehat{v}_L}{\widehat{v}_H}\right],$$

gives the second term in the definition of Φ. Similarly, maximizing the quadratic expression for WE in the region in which both informed types are active and comparing it with the same symmetric benchmark gives the first term. Therefore, for every informative binary signal $\widehat{v}_H > \widehat{v}_L$,

$$\max_{\delta>0} \mathrm{WE}(DC,\delta) \begin{cases} > \dfrac{\bar{v}}{4}, & \text{if } \dfrac{\widehat{v}_L}{\widehat{v}_H} < \Phi(\widehat{\mu}), \\ = \dfrac{\bar{v}}{4}, & \text{if } \dfrac{\widehat{v}_L}{\widehat{v}_H} = \Phi(\widehat{\mu}), \\ < \dfrac{\bar{v}}{4}, & \text{if } \dfrac{\widehat{v}_L}{\widehat{v}_H} > \Phi(\widehat{\mu}). \end{cases}$$

The same comparison applies to CD by symmetry. Since

$$\mathrm{WE}(CC,1) = \mathrm{WE}(DD,1) = \frac{\bar{v}}{4},$$

the threshold comparison proves the optimality statements in part (ii). In the strict asymmetric region, the implementing value of δ under DC satisfies $\delta > 1$, so the scoring rule favors the uninformed player. The mirror contest CD uses the reciprocal bias and favors player 1, who is then the uninformed player.

It remains only to verify the stated properties of Φ. The displayed formula immediately gives

$$\Phi(0) = 0.$$

For $\widehat{\mu} \geq 1/2$,

$$\left(\frac{\widehat{\mu}}{1-\widehat{\mu}}\right)^2 \geq 1,$$

so the threshold is truncated at one:

$$\Phi(\widehat{\mu}) = 1.$$

Thus Φ is continuous on $[1/2, 1]$, and it is also continuous at $\widehat{\mu} = 1/2$ because

$$\left(\frac{\widehat{\mu}}{1-\widehat{\mu}}\right)^2 \to 1 \qquad \text{as} \qquad \widehat{\mu} \uparrow \frac{1}{2}.$$

For $\widehat{\mu} \in (0, 1/2)$, the first term in the maximum is strictly increasing because

$$\frac{d}{d\widehat{\mu}} \left(\frac{\widehat{\mu}}{1-\widehat{\mu}}\right)^2 = \frac{2\widehat{\mu}}{(1-\widehat{\mu})^3} > 0.$$

The derivative of the second term in the maximum equals

$$\frac{8\widehat{\mu}^2 + 8\widehat{\mu}\sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3} - 24\widehat{\mu} - 9\sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3} + 24}{\left(\widehat{\mu} + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)^4 \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}}.$$

The denominator is positive. The numerator can be written as

$$8\widehat{\mu}^2 - 24\widehat{\mu} + 24 - (9 - 8\widehat{\mu})\sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}.$$

For $\widehat{\mu} \in (0, 1/2)$,

$$\sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3} < \sqrt{3},$$

and therefore the numerator is larger than

$$8\widehat{\mu}^2 - 24\widehat{\mu} + 24 - (9 - 8\widehat{\mu})\sqrt{3}.$$

This lower bound is decreasing on $(0, 1/2)$, and its value at $\widehat{\mu} = 1/2$ is

$$14 - 5\sqrt{3} > 0.$$

Hence the second term in the maximum is also strictly increasing on $(0, 1/2)$. The maximum of two strictly increasing continuous functions is strictly increasing and continuous. Moreover, the first term is below one on $(0, 1/2)$, while the second term is also below one because it is strictly increasing and equals

$$\frac{28\sqrt{7} - 67}{27} < 1$$

at $\widehat{\mu} = 1/2$. Hence no truncation occurs on $(0, 1/2)$, and Φ is strictly increasing on $(0, 1/2)$.

Finally, we show that $\Phi(\widehat{\mu}) > \widehat{\mu}/4$ for all $\widehat{\mu} \in (0, 1)$. If $\widehat{\mu} \in [1/2, 1)$, then $\Phi(\widehat{\mu}) = 1 > \widehat{\mu}/4$. It remains to consider $\widehat{\mu} \in (0, 1/2)$. By the definition of Φ, it suffices to show that

$$\frac{4\widehat{\mu}\left(\widehat{\mu} - 1 + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)\left(1 + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)}{(1 - \widehat{\mu})\left(\widehat{\mu} + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)^3} - \frac{\widehat{\mu}}{1 - \widehat{\mu}} > \frac{\widehat{\mu}}{4}.$$

Since $\widehat{\mu} > 0$, this inequality is equivalent to

$$16\left(\widehat{\mu} - 1 + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)\left(1 + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right) > (5 - \widehat{\mu})\left(\widehat{\mu} + \sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\right)^3.$$

The difference between the left-hand side and the right-hand side is

$$(1 - \widehat{\mu})\Big[32 - 45\widehat{\mu} + 25\widehat{\mu}^2 - 4\widehat{\mu}^3 - \left(4\widehat{\mu}^2 - 19\widehat{\mu} + 15\right)\sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3}\Big].$$

For $\widehat{\mu} \in (0, 1/2)$,

$$4\widehat{\mu}^2 - 19\widehat{\mu} + 15 = (1 - \widehat{\mu})(15 - 4\widehat{\mu}) > 0,$$

and

$$\sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3} < 2 - \widehat{\mu},$$

because

$$(2 - \widehat{\mu})^2 - (\widehat{\mu}^2 - 3\widehat{\mu} + 3) = 1 - \widehat{\mu} > 0.$$

Therefore,

$$\begin{aligned}
&32 - 45\widehat{\mu} + 25\widehat{\mu}^2 - 4\widehat{\mu}^3 - \left(4\widehat{\mu}^2 - 19\widehat{\mu} + 15\right)\sqrt{\widehat{\mu}^2 - 3\widehat{\mu} + 3} \\
&> 32 - 45\widehat{\mu} + 25\widehat{\mu}^2 - 4\widehat{\mu}^3 - \left(4\widehat{\mu}^2 - 19\widehat{\mu} + 15\right)(2 - \widehat{\mu}) \\
&= 2 + 8\widehat{\mu} - 2\widehat{\mu}^2 > 0.
\end{aligned}$$

Since $1 - \widehat{\mu} > 0$, the required difference is strictly positive. Hence the second term in the maximum defining $\Phi(\widehat{\mu})$ is strictly larger than $\widehat{\mu}/4$, and therefore $\Phi(\widehat{\mu}) > \widehat{\mu}/4$ for every $\widehat{\mu} \in (0, 1/2)$. Combining this with the case $\widehat{\mu} \in [1/2, 1)$ proves the claim. This completes the proof. ■

A.8 Proof of Remark 2

Proof. It suffices to consider the case in which the affine bias favors player 2; the case in which it favors player 1 is symmetric. Suppose, for example, that under DC, the scores are

$$y_1 = x_1, \qquad y_2 = \delta x_2 + \alpha, \qquad \delta > 0, \quad \alpha \geq 0.$$

The corresponding lottery probabilities are

$$p_1(x_1, x_2) = \frac{y_1}{y_1 + y_2}, \qquad p_2(x_1, x_2) = \frac{y_2}{y_1 + y_2}.$$

A positive intercept α is never useful for the designer. Fixing player 2's score y_2, player 1's best response and the winning probabilities depend only on y_2, not on whether this score is generated by actual effort δx_2 or by the intercept α. Thus, α is a virtual score rather than actual effort. Whenever player 2 is active, his first-order condition is

$$\delta \mathbb{E}\left[\frac{\widehat{v}_s x_{1s}}{(x_{1s} + y_2)^2}\right] = 1,$$

where the expectation is taken over the signal realizations that induce positive effort by player 1. For every active type s, player 1's first-order condition implies

$$\frac{\widehat{v}_s y_2}{(x_{1s} + y_2)^2} = 1.$$

Substituting this identity into player 2's first-order condition gives

$$y_2 = \delta \mathbb{E}[x_{1s}].$$

Hence the same equilibrium score y_2, the same informed player's efforts, and the same winning probabilities are implemented with $\alpha = 0$. The only difference is that player 2's actual effort rises from

$$x_2 = \frac{y_2 - \alpha}{\delta}$$

to

$$x_2 = \frac{y_2}{\delta}.$$

Therefore, setting $\alpha = 0$ weakly raises both expected total effort and expected winner's effort.

If instead player 2 is inactive and $y_2 = \alpha$, then the intercept is a pure headstart. Such a headstart is also dominated. Given a pure head start α, type s's best response is

$$x_{1s} = \left(\sqrt{\widehat{v}_s \alpha} - \alpha\right)_+ .$$

The associated contribution to expected winner's effort is bounded above by

$$\max_{\alpha \geq 0} \frac{x_{1s}^2}{x_{1s} + \alpha} = \frac{4}{27}\widehat{v}_s < \frac{1}{4}\widehat{v}_s,$$

while the associated contribution to total effort is bounded above by

$$\max_{\alpha \geq 0} \left(\sqrt{\widehat{v}_s \alpha} - \alpha\right) = \frac{1}{4}\widehat{v}_s < \frac{1}{2}\widehat{v}_s.$$

Averaging over signal realizations, a pure-head-start design is dominated by the symmetric lottery contest with $\delta = 1$. Therefore, every optimal affine-score lottery contest has $\alpha = 0$. The optimal designs and the threshold Φ in Proposition 4 are consequently unchanged. ∎

A.9 Proof of Lemma 2

Proof. See Appendix B. ∎

A.10 Proof of Proposition 5

Proof. It is useful to prove an intermediate result.

Lemma A3 *Suppose that $\gamma = DC$. Fix an arbitrary tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies (2) and let the designer set the scoring bias $\delta > 0$. Then the expected winner's effort from the contest is maximized at $\delta = 1$ or $\delta = \frac{1}{\mu^\pi}$.*

Proof. The proof closely follows that of Lemma A1 and is omitted for brevity. ■

Following the same steps in the proof of Proposition 1 and using Lemma A3, we can show that for an arbitrary tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies (2), the expected winner's effort from the contest is maximized by $(\delta, \gamma) = (1, CC)$, $(\delta, \gamma) = (1, DD)$, or $(\delta, \gamma) = \left(\frac{1}{\mu^\pi}, DC\right)$. The first two contest schemes generate an expected winner's effort of $\frac{2}{3}\bar{v}$, while the third one generates an expected winner's effort of $\frac{\mu^\pi v_H^\pi (5-\mu^\pi)}{6}$. The optimization problem under $\gamma = DC$ is

$$\max_{v_L^\pi, v_H^\pi, \mu^\pi} \frac{\mu^\pi v_H^\pi (5-\mu^\pi)}{6} \ s.t. \ (1) \text{ and } (2).$$

It can be verified that for an arbitrary μ^π, suppose that $\sum_{\ell=k+1}^K \mu_\ell < \mu^\pi \leq \sum_{\ell=k}^K \mu_\ell$ for some k, then the largest v_H^π satisfying (1) and (2) is given by

$$\frac{[\mu^\pi - \Pr(v > v_k)]v_k + \Pr(v > v_k)\mathbb{E}[v|v > v_k]}{\mu^\pi}.$$

So the optimization problem becomes

$$\max_{\mu^\pi} W(\mu^\pi) := \frac{\left\{[\mu^\pi - \Pr(v > v_k)]v_k + \Pr(v > v_k)\mathbb{E}[v|v > v_k]\right\}(5-\mu^\pi)}{6},$$

where k satisfies $\sum_{\ell=k+1}^K \mu_\ell < \mu^\pi \leq \sum_{\ell=k}^K \mu_\ell$. For a fixed k, $W(\mu^\pi)$ is quadratic in μ^π and the axis of symmetry is

$$\mu^*(k) := 3 - \frac{\sum_{\ell=1}^k \mu_\ell}{2} - \frac{\sum_{\ell=k+1}^K \mu_\ell v_\ell}{2v_k},$$

which increases with k. Note that $\sum_{\ell=k+1}^K \mu_\ell$ and $\sum_{\ell=k}^K \mu_\ell$ both decrease with k. Let $k^* := \min\left\{k : \mu^*(k) \geq \sum_{\ell=k+1}^K \mu_\ell\right\}$. Then for all $k > k^*$, $\mu^*(k) \geq \mu^*(k^*) \geq \sum_{\ell=k^*+1}^K \mu_\ell \geq \sum_{\ell=k}^K \mu_\ell$. Therefore, $W(\mu^\pi)$ increases with $\mu^\pi \leq \sum_{\ell=k^*+1}^K \mu_\ell$. Meanwhile, for all $k < k^*$, $\mu^*(k) < \sum_{\ell=k+1}^K \mu_\ell$. So $W(\mu^\pi)$ decreases with $\mu^\pi > \sum_{\ell=k^*}^K \mu_\ell$. Finally, it is straightforward to verify that for $\sum_{\ell=k^*+1}^K \mu_\ell < \mu^\pi \leq \sum_{\ell=k^*}^K \mu_\ell$, $W(\mu^\pi)$ increases with $\mu^\pi \leq \min\left\{\sum_{\ell=k^*}^K \mu_\ell, \mu^*(k^*)\right\}$ and decreases otherwise.

To sum up, $W(\mu^\pi)$ is maximized at $\mu^\pi = \min\left\{\sum_{\ell=k^*}^{K} \mu_\ell, \mu^*(k^*)\right\}$. In the case of $\mu^*(1) \geq 1$, which is equivalent to $\bar{v}/v_1 \leq 4$, $W(\mu^\pi)$ is maximized at $\mu^\pi = 1$. The maximized value is $\frac{2}{3}\bar{v}$, so $(\delta, \gamma) = (1, CC)$ and $(\delta, \gamma) = (1, DD)$ are optimal. In the case of $\mu^*(1) < 1$, the maximized value $W(\mu^\pi) > W(1) = \frac{2}{3}\bar{v}$, so $\gamma = DC$ is optimal. Finally, it is straightforward to verify the desired distribution of posterior expectations can be induced by the signal structure described in the proposition. ■

Appendix B General Information Structure

This appendix concerns the case where the designer can freely choose the information structure. We show that binary signals are optimal for maximizing the expected winner's effort for the $\gamma = DC$ case.

Fix $\delta \in (0, \infty)$. We first characterize the equilibrium under an arbitrary information structure $\pi(s|v)$. Suppose that the corresponding distribution of posterior means is given by

$$\big\langle (m_1, \ldots, m_N), (q_1, \ldots, q_N) \big\rangle,$$

where

$$m_n := \mathbb{E}(v \mid s_n) \quad \text{and} \quad q_n := \Pr(s_n) > 0.$$

We order the signals such that

$$v_1 \leq m_1 < \cdots < m_N \leq v_K, \qquad \sum_{n=1}^{N} q_n = 1, \qquad \sum_{n=1}^{N} q_n m_n = \bar{v}.$$

Proposition B1 *Given the posterior distribution $\big\langle (m_1, \ldots, m_N), (q_1, \ldots, q_N) \big\rangle$ and $\gamma = DC$, the equilibrium of the contest is described as follows.*

(i) Suppose that $\delta < 1$. Contestant 1, upon receiving signal s_n, mixes uniformly over the interval $\left[\delta \sum_{k=1}^{n-1} q_k m_k,\, \delta \sum_{k=1}^{n} q_k m_k\right]$. Contestant 2 exerts zero effort with probability $1 - \delta$ and, for each $n \in \{1, \ldots, N\}$, mixes over $\left[\sum_{k=1}^{n-1} q_k m_k,\, \sum_{k=1}^{n} q_k m_k\right]$ with density δ / m_n.

(ii) Suppose that $\delta \geq 1$. Further, suppose that

$$\frac{1}{\sum_{k=n_0}^{N} q_k} \leq \delta < \frac{1}{\sum_{k=n_0+1}^{N} q_k}$$

for some $n_0 \in \{1, \ldots, N\}$, where $1/0 = \infty$ by convention. Set $A_{n_0} := 1 - \delta \sum_{k > n_0} q_k$. Contestant 1, upon receiving a signal from $\{s_1, \ldots, s_{n_0 - 1}\}$, exerts zero effort for sure. Upon receiving signal s_{n_0}, he exerts zero effort with probability $1 - \frac{A_{n_0}}{\delta q_{n_0}}$ and mixes over $[0,\, m_{n_0} A_{n_0}]$ with density $\frac{1}{\delta q_{n_0} m_{n_0}}$. Upon receiving signal $s_n \in \{s_{n_0+1}, \ldots, s_N\}$, he mixes uniformly over $\left[m_{n_0} A_{n_0} + \delta \sum_{k=n_0+1}^{n-1} q_k m_k,\, m_{n_0} A_{n_0} + \delta \sum_{k=n_0+1}^{n} q_k m_k\right]$. Contestant 2 mixes over $\left[0,\, \frac{m_{n_0} A_{n_0}}{\delta}\right]$ with density δ / m_{n_0} and, for each $n \in \{n_0 + 1, \ldots, N\}$, over $\left[\frac{m_{n_0} A_{n_0}}{\delta} + \sum_{k=n_0+1}^{n-1} q_k m_k,\, \frac{m_{n_0} A_{n_0}}{\delta} + \sum_{k=n_0+1}^{n} q_k m_k\right]$ with density δ / m_n.

Proof of Lemma 2. Take any information structure with $N \geq 3$ signals. We show that it is weakly outperformed by another information structure with fewer signals. Since the number of signals is finite, repeated application of the argument yields an optimal information structure with at most two signals.

Case (i): Suppose that

$$\delta \leq \frac{1}{q_{N-1} + q_N}.$$

In this case, contestant 1 is active upon receiving either s_{N-1} or s_N. Let $\tilde{x}$ denote the left endpoint, in contestant 2's effort scale, of the interval associated with signal s_{N-1}. Thus contestant 1 mixes over

$$\left[\delta\tilde{x},\ \delta(\tilde{x} + q_{N-1}m_{N-1})\right]$$

upon receiving s_{N-1}, and over

$$\left[\delta(\tilde{x} + q_{N-1}m_{N-1}),\ \delta(\tilde{x} + q_{N-1}m_{N-1} + q_N m_N)\right]$$

upon receiving s_N.

Let $\widehat{\pi}$ be the information structure obtained by pooling s_{N-1} and s_N. The pooled signal has probability

$$\widehat{q} := q_{N-1} + q_N$$

and posterior mean

$$\widehat{m} := \frac{q_{N-1}m_{N-1} + q_N m_N}{q_{N-1} + q_N}.$$

All other signals are unchanged.

The expected winner's effort under π can be decomposed as

$$\begin{aligned}
\mathrm{WE}(\pi) = \ & \Pr(x_1 \leq \delta\tilde{x},\ x_2 < \tilde{x})\mathbb{E}\left[x_1 \mathbb{1}_{\{x_1 > \delta x_2\}} + x_2 \mathbb{1}_{\{x_1 < \delta x_2\}} \,\middle|\, x_1 \leq \delta\tilde{x},\ x_2 < \tilde{x}\right] \\
& + \Pr(x_1 > \delta\tilde{x},\ x_2 \geq \tilde{x})\mathbb{E}\left[x_1 \mathbb{1}_{\{x_1 > \delta x_2\}} + x_2 \mathbb{1}_{\{x_1 < \delta x_2\}} \,\middle|\, x_1 > \delta\tilde{x},\ x_2 \geq \tilde{x}\right] \\
& + \Pr(x_1 \leq \delta\tilde{x},\ x_2 \geq \tilde{x})\mathbb{E}[x_2 \,|\, x_1 \leq \delta\tilde{x},\ x_2 \geq \tilde{x}] \\
& + \Pr(x_1 > \delta\tilde{x},\ x_2 < \tilde{x})\mathbb{E}[x_1 \,|\, x_1 > \delta\tilde{x},\ x_2 < \tilde{x}]\,.
\end{aligned}$$

The first line is unchanged under $\widehat{\pi}$. In the third line, contestant 2 wins for sure; in the fourth line, contestant 1 wins for sure. By independence of the players' mixed strategies,

$$\mathbb{E}[x_2 \mid x_1 \leq \delta\tilde{x},\ x_2 \geq \tilde{x}] = \mathbb{E}[x_2 \mid x_2 \geq \tilde{x}],$$

and

$$\mathbb{E}[x_1 \mid x_1 > \delta\tilde{x},\ x_2 < \tilde{x}] = \mathbb{E}[x_1 \mid x_1 > \delta\tilde{x}].$$

Direct calculation gives

$$\widehat{\mathbb{E}}[x_2 \mid x_2 \geq \tilde{x}] - \mathbb{E}[x_2 \mid x_2 \geq \tilde{x}] = \frac{q_{N-1}q_N}{2(q_{N-1}+q_N)}(m_N - m_{N-1}) > 0,$$

and

$$\widehat{\mathbb{E}}[x_1 \mid x_1 > \delta\tilde{x}] - \mathbb{E}[x_1 \mid x_1 > \delta\tilde{x}] = \delta\,\frac{q_{N-1}q_N}{2(q_{N-1}+q_N)}(m_N - m_{N-1}) > 0,$$

where hats denote expectations under $\widehat{\pi}$.

For the second line, direct integration over the two top intervals gives

$$\begin{aligned}
&\widehat{\mathbb{E}}\big[x_1 \mathbb{1}_{\{x_1>\delta x_2\}} + x_2 \mathbb{1}_{\{x_1<\delta x_2\}} \,\big|\, x_1 > \delta\tilde{x},\ x_2 \geq \tilde{x}\big] \\
&\quad - \mathbb{E}\big[x_1 \mathbb{1}_{\{x_1>\delta x_2\}} + x_2 \mathbb{1}_{\{x_1<\delta x_2\}} \,\big|\, x_1 > \delta\tilde{x},\ x_2 \geq \tilde{x}\big] \\
&= \frac{(1+\delta)q_{N-1}q_N(2q_{N-1}+q_N)(m_N - m_{N-1})}{6(q_{N-1}+q_N)^2} > 0.
\end{aligned}$$

Moreover, under both π and $\widehat{\pi}$,

$$\Pr(x_1 > \delta\tilde{x}) = q_{N-1} + q_N, \qquad \Pr(x_2 \geq \tilde{x}) = \delta(q_{N-1} + q_N).$$

Therefore,

$$\begin{aligned}
\mathrm{WE}(\widehat{\pi}) - \mathrm{WE}(\pi) &= (1-\widehat{q})\delta\widehat{q}\,\frac{q_{N-1}q_N}{2\widehat{q}}(m_N - m_{N-1}) \\
&\quad + \widehat{q}(1-\delta\widehat{q})\,\delta\frac{q_{N-1}q_N}{2\widehat{q}}(m_N - m_{N-1}) \\
&\quad + \delta\widehat{q}^2\,\frac{(1+\delta)q_{N-1}q_N(2q_{N-1}+q_N)(m_N - m_{N-1})}{6\widehat{q}^2} > 0.
\end{aligned}$$

Thus pooling the two highest signals strictly increases the expected winner's effort in this case.

Case (ii): Suppose that

$$\frac{1}{q_{N-1}+q_N} < \delta < \frac{1}{q_N}.$$

In this case, only contestant 1 with signals s_{N-1} and s_N is active in equilibrium. Using the equilibrium characterization above, direct integration gives

$$\mathrm{WE}(\pi) = \frac{m_{N-1}(5\delta - 1)}{6\delta^2} + \frac{\delta q_N^2\left[6 - q_N(1+\delta)\right](m_N - m_{N-1})}{6}.$$

It is worth pointing out that this expression is independent of q_{N-1}. Moreover,

$$\frac{\partial \mathrm{WE}(\pi)}{\partial q_N} = \frac{\delta q_N(m_N - m_{N-1})\left[4 - q_N(1+\delta)\right]}{2} > 0,$$

where the inequality follows from $q_N < 1/\delta$.

Construct an alternative information structure π' as follows. Let

$$q'_{N-1} := \frac{1}{\delta} - q_N,$$

which satisfies $0 < q'_{N-1} < q_{N-1}$ under the present case condition. When s_{N-1} is drawn under π, let π' send s_{N-1} with probability q'_{N-1}/q_{N-1} and send s_1 with the complementary probability. All other signals are left unchanged. Equivalently,

$$\pi'(s_1 \mid v) = \pi(s_1 \mid v) + \pi(s_{N-1} \mid v)\frac{q_{N-1} - q'_{N-1}}{q_{N-1}},$$

$$\pi'(s_{N-1} \mid v) = \pi(s_{N-1} \mid v)\frac{q'_{N-1}}{q_{N-1}},$$

and

$$\pi'(s_k \mid v) = \pi(s_k \mid v) \quad \text{for } k \neq 1, N-1.$$

This operation reduces the probability of signal s_{N-1} but leaves m_{N-1}, m_N, and q_N unchanged. It may change the posterior mean associated with s_1, but that signal remains inactive in the present case. Hence

$$\mathrm{WE}(\pi') = \mathrm{WE}(\pi).$$

By construction,

$$\delta = \frac{1}{q'_{N-1} + q_N}.$$

Thus π' falls into Case (i), and pooling its two highest signals weakly increases the expected winner's effort and reduces the number of signal realizations.

Case (iii): Suppose that

$$\delta \geq \frac{1}{q_N}.$$

In this case, only contestant 1 with signal s_N is active in equilibrium. The information structure π is outcome-equivalent to the binary-signal information structure with posterior means

$$\bar{m}_1 = \frac{\sum_{k=1}^{N-1} q_k m_k}{\sum_{k=1}^{N-1} q_k}, \qquad \bar{m}_2 = m_N,$$

and signal probabilities

$$\bar{q}_1 = \sum_{k=1}^{N-1} q_k, \qquad \bar{q}_2 = q_N.$$

The expected winner's effort is unchanged by this replacement.

Combining the three cases, any information structure with $N \geq 3$ signals is weakly outperformed by an information structure with fewer signals. Iterating this argument yields an optimal information structure with at most two signals. ■