A THEORY OF AFFIRMATIVE ACTION IN COLLEGE ADMISSIONS

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To address the issue of when minority and nonminority candidates compete for admissions to a college, we show that an academic quality–oriented college maximizes the test score of its incoming class by adopting an admissions rule that favors the minority. Such a "handicapping" rule increases competition and induces candidates to invest more in educational attainment. These results reconcile the often-assumed conflicts between diversity and academic quality. However, we also show that the non-minority responds to the affirmative action admissions more aggressively, which tends to widen the racial test score gap. (*JEL* H0, J7)

I. INTRODUCTION

Race-conscious preferential admissions have been widely practiced by selective colleges and universities to enhance minority representation in higher education. For example, the College of Arts and Sciences at the University of Michigan automatically added 20 points (out of a possible 150 points) to a minority applicant's score in its rating system. Harvard University has an "unofficial lift"scheme, which also targets minority applicants. However, controversy has surrounded affirmative action ever since its inception. For instance, California, Texas, and Florida have already terminated the use of race-conscious admissions at state-funded institutions. This debate culminated in the recent Supreme Court ruling regarding admissions procedures at the University of Michigan, which endorsed admissions rules that took into account race as a qualifying characteristic.

Unfortunately, positive studies on this issue have been scarce relative to the high profile of the debate. According to Holzer and Neumark (1999), theoretical studies of the efficiency of affirmative action on education are "virtually nonexistent." Some have argued that affirmative action is merely a patronage program and necessarily results in "mediocracy"rather than meritocracy. In the debate on affirmative action practices in college admissions, a major criticism is that affirmative action designed to create diversity comes at the cost of academic quality. A competing view, offered by those opposed to affirmative action, is that it weakens school applicants' incentives to achieve academic excellence. For instance, Justice Thomas wrote in his opinion in *Grutter v. Bollinger* that "there is no incentive for the black applicant to continue to prepare for the LSAT once he is reasonably assured of achieving the requisite score." Supporters of this practice tend to emphasize the importance of diversity and the positive influence of diversity on the pedagogical environment (Steele, 1990). These views, however, do not fully recognize the incentive structure behind affirmative action admissions rules. It is unclear how a college-admissions rule affects high school students' incentives to achieve academic excellence, which adds to their human capital stock and future productivity.

The process of college admissions by and large resembles a contest in which contestants exert costly effort in order to win a limited number of prizes. In the context of college admissions, to compete for a limited number of seats in the incoming class, college candidates have to present their academic credentials (such as GPA and SAT score) to the admissions officer. To win the seat, high school students have to invest in their human capital, which

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improves the academic performance, whereas the academic investments are costly and nonrefundable regardless of the outcome—for example, the tuition, the money spent on books, the salary paid to tutors, the time and energy, and so on. All of these features may be approximated by an all-pay auction mechanism.

We propose a simple theoretical framework that models the process of college admissions as an all-pay auction, to investigate two major questions: Is there any theoretical rationale for an affirmative action admissions rule? How do such rules affect college candidates' incentives to invest in academic effort? Two candidates—one a minority and the other a nonminority—simultaneously choose their academic efforts (human capital investments) to compete for a seat in a college. We show that an academic quality–oriented college prefers to adopt an admissions rule that scales up the test score of the minority relative to the nonminority. Although this rule is designed purely to maximize the expected academic quality of the incoming class, it turns out to favor the minority and create ethnic diversity. We show that the unique equilibrium (affirmative action) admissions rule creates a positive "cross-group interaction" between college candidates' incentives to make educational effort. As a consequence, a pro-minority rule levels the playing field and leads both candidates to exert higher academic effort. The results therefore reconcile the commonly assumed conflicts between academic quality and ethnic diversity. Paradoxically, however, we show that the nonminority candidate responds to the pro-minority admissions rule more aggressively than the minority, which tends to widen the existing racial test-score gap.

A growing literature has emerged that investigates the effect of affirmative action on agents' incentives to invest in human capital. Most of these studies are built on the theory of statistical discrimination, such as Phelps (1972). In a job assignment model, Coate and Loury (1993) find that affirmative action, represented by a mandated equal assignment rate, exerts mixed effects on minority workers' incentives. By contrast, Moro and Norman (2003) find a negative externality between the two groups' incentives: affirmative action may increase the minority workers' incentive to invest in learning but diminish the nonminority's. Furstenburg (2003) explicitly models affirmative action in the context of college admissions. He shows that a college may adopt an affirmative action admissions rule to enhance the academic quality of its class. He also identifies a negative externality that parallels Moro and Norman (2003), which implies that affirmative-action narrows the racial test score gap.

In contrast to this strand of the literature, we adopt a contest-theoretic approach to model the college admissions process, which yields a positive "cross-group" interaction. A handful of theoretical studies have recognized the resemblance between college admissions and contests. For example, Fernandez and Gali (1999) compare the efficiency of tournaments (placement exams such as SAT) with markets as allocative mechanisms. Amegashie and Wu (2004) model college admissions process as all-pay auctions and examine the selection effects of this system. However, neither of these studies concerns itself with affirmative action. In a laboratory setting, Schotter and Weigelt (1992) find that affirmative action may increase the total output of a tournament. A recent study by Fryer and Loury (forthcoming) shares some of the features of my model. They use a tournament model to investigate the categorical redistribution in a winner-take-all market and show that optimally designed tournaments naturally involve "handicapping."

We model racial inequality by assuming that attending college creates differential returns across college candidates. The inferior return on the minority's investment in human capital may result from various factors. First of all, the investment in human capital may be unfairly rewarded in the labor market. On the other hand, the unfair labor market may not be real but may exist out of perception. If the minority holds pessimistic expectation toward reward in the labor market, the incentive to invest on human capital is also impaired. Although most studies assume that the minority and nonminority bear differential human capital investment costs, my model does not lose its bite on the potential difference in this regard, because higher learning costs simply reduce the net return.

This article is organized as follows. Section II sets up the model. Section III shows the equilibrium outcome and discusses the incentive content of affirmative action. Section IV concerns the impacts of admissions rule on minority representation and the racial test-score gap. Section V presents a concluding remark.

II. A MODEL OF COLLEGE ADMISSIONS

This model involves two college candidates indexed by $i = M, N$, who compete for one seat at a college. One candidate, M, is minority, whereas the other candidate, N, is nonminority. The admissions game proceeds as follows. At the beginning of the game, the college announces its admissions rule. The screening is primarily based on candidates' scores in a standardized college-entrance test. Upon observing the admissions rule, college candidates determine how much academic effort to spend preparing for the test. The academic efforts are converted to their scores, q_M and q_N, in the test. Finally, the college observes their test scores and admits one of them into the incoming class according to the rule announced before.

The College

The college is concerned with the academic quality of its student body. A better-qualified incoming class builds up the college's reputation and increases its value. The objective of the college's admissions office is to maximize the expected academic quality of the admitted student, which is represented by the expected test score (Q) of the accepted candidate.

The admissions decision is primarily based on candidates' test scores. However, the college may take into account a candidate's identity as a qualifying characteristic. The college has the flexibility to assign a weight $\alpha_i \in (0, \infty)$ to a candidate i's test score. As a consequence, candidate i receives a rating $\alpha_i q_i$ in the college's assessment system. Candidate is admitted if i's rating is higher than the competitor's, that is, $\alpha_i q_i > \alpha_j q_j$. In the event that they tie, the seat is randomly assigned to one of them. We normalize the weight assigned to candidate N's test score to be 1 and the weight for candidate M to be $\alpha \equiv \alpha_M/\alpha_N \in (0, \infty)$.

The admissions rule is parameterized by α, and the college chooses the optimal α to maximize Q. The value of α represents the *ex ante* preference of the college between candidates. When $\alpha > 1$, the minority candidate's test score weighs relatively more than that of the nonminority counterpart, which represents an affirmative-action admissions mechanism. When $\alpha < 1$, the admissions rule is biased against the minority candidate. When $\alpha = 1$, no characteristic other than test score matters in the admissions decision, which represents a "color-blind" admissions scheme.

College Candidates

A candidate i privately values the admission at $V_i \in (0, \infty)$, which represents the additional benefit that i receives by attending college, such as higher income and social recognition. By my assumptions, we have $V_N > V_M > 0$. A candidate i exerts academic effort e_i to improve test score q_i in the standardized entrance test, which is taken by the college as the primary screening criterion. A higher score increases the likelihood that one is admitted, whereas such a score reduces the competitor's chance. We assume that no innate ability difference exists across candidates and that they are endowed with an identical linear test-score production technology, given by $q_i = e_i$. This linear test score production function enables us to interchange the notation q and e.

Although academic quality accrues to the college's value, these candidates value only the benefit they may obtain by attending college, whereas the academic effort they expend is costly. We assume that the academic effort incurs a unit cost on its margin. Hence, a candidate i receives $V_i - e_i$ as payoff if admitted, whereas i receives $-e_i$ if rejected. Candidate M and N's payoff functions, respectively, are as follows.

$$(1) \qquad \pi_M = \begin{cases} V_M - e_M & \text{if } \alpha e_M > e_N, \\ V_M/2 - e_M & \text{if } \alpha e_M = e_N, \\ -e_M & \text{if } \alpha e_M < e_N; \end{cases}$$

$$(2) \qquad \pi_N = \begin{cases} V_N - e_N & \text{if } e_N > \alpha e_M, \\ V_N/2 - e_N & \text{if } e_N = \alpha e_M, \\ -e_N & \text{if } e_N < \alpha e_M. \end{cases}$$

The asymmetry in V_i may stem from two major sources. As discussed before, because of the existence or the perception of racial inequality in the labor market, the minority candidate may expect a lower return from college education than a nonminority counterpart may expect and therefore undervalue the higher education. Alternatively, the differential return to college education may also arise if the minority bears a higher marginal cost to achieve academic excellence, which is assumed in a number of previous studies. Higher learning costs simply reduce the minority candidate's return from education. Due to the linear payoff structure, a model that assumes asymmetric costs is a monotonic transformation of ours and produces equivalent results.

III. THE EQUILIBRIUM

To solve the admissions game in a backward fashion, we first examine college candidates' strategies after they learn the admissions rule. Then we find the college's choice that best addresses its objective, taking into account candidates' responses.

College Candidates' Strategies

In my framework, the admissions process is abstracted as an all-pay auction where college candidates enter their test scores as bids. The equilibrium of a standard complete-information all-pay auction has been thoroughly investigated by Hillman and Riley (1989), and Baye et al. (1996). My approach is close to these two studies but allow the auctioneer (the college) to unequally weigh bidders (college candidates)' bids; that is, $\alpha \neq 1$.

As a standard property of complete-information all-pay auctions, only mixed strategy equilibrium may exist in the admissions contest. The form of the equilibrium in the subgame of admissions contest hinges on the value of policy parameter α. When $\alpha < V_N/V_M$, candidate N possesses an advantage against candidate M. Because candidate M never bids more than valuation V_M, candidate N can ensure winning if N exerts an effort $e_N = \alpha V_M$, as well as receiving a positive payoff $V_N - \alpha V_M$. By way of contrast, if $\alpha > V_N/V_M$, the college's preferential admissions rule more than offsets candidate N's initial advantage. Consequently, candidate M is able to secure the seat and extract positive rent as long as M bids V_N/α, which is less than V_M. To ease the notation in my analysis, we define $\theta \equiv V_N/V_M > 1$.

Let $F_M = F_M(e_M)$ and $F_N = F_N(e_N)$ denote candidate M and N's equilibrium effort distribution functions, respectively. Using standard technique—such as that of Hillman and Riley (1989) and Baye et al. (1996)—we show the following holds in the subgame of admissions contest.

PROPOSITION 1. *For any $\alpha \in (0, \theta]$, there exists a unique Nash equilibrium. Candidate N continuously randomizes effort over the whole support $[0, \alpha V_M]$, whereas candidate M continuously randomizes effort over the support $(0, V_M]$ and places a probability mass at zero with a size $(V_N - \alpha V_M)/V_N$. The equilibrium effort distribution functions are given by*

$$F_N(e_N) = e_N/\alpha V_M, \tag{3}$$

$$F_M(e_M) = (V_N - \alpha V_M + \alpha e_M)/V_N. \tag{4}$$

For any $\alpha \in [\theta, \infty)$, there exists a unique Nash equilibrium. Candidate M continuously randomizes effort over the whole support $[0, V_N/\alpha]$, whereas candidate N continuously randomizes effort over the support $(0, V_N]$ and places a probability mass at zero with a size $(V_M - V_N/\alpha)/V_M$. The equilibrium effort distribution functions are given by

$$F_N(e_N) = (V_M - V_N/\alpha + e_N/\alpha)/V_M, \tag{5}$$

$$F_M(e_M) = \alpha e_M/V_N. \tag{6}$$

Proposition 1 characterizes the unique mixed-strategy Nash equilibrium of the admissions contest for any given $\alpha \in (0, \infty)$.[1] Proposition 1 states that when $\alpha < \theta$, the minority candidate always has a positive probability to "drop out"; that is, $e_M = 0$. This probability decreases with α and reduces to zero once $\alpha = \theta$. Intuitively, when $\alpha \in (0, \theta]$, a greater α increases the marginal return of candidate M's academic effort, which improves the incentive to expend academic effort and participate in the competition. By contrast, when $\alpha \in (\theta, \infty)$, only the nonminority candidate drops out of the competition with positive probability. Intuitively, an admissions rule with $\alpha > \theta$ excessively favors the minority candidate. Thus, a greater α further dampens the incentive of the initially advantaged nonminority candidate and drives one to drop out.

1. It seems unnatural to have college candidates randomize their academic efforts. My results, however, can be interpreted in light of the remarkable Harsanyi purification theorem. According to the theorem, the original game with perfectly known payoffs can be viewed as a limit of a sequence of games with payoff perturbation. If the random perturbation is revealed only to the player, then, for almost every realization of the payoff, the incomplete-information game yields a unique pure-strategy equilibrium that approximates the mixed-strategy equilibrium of the original game. In other words, the original mixed equilibrium can be considered as a limit of the pure-strategy equilibrium of any "close-by" perturbed game. Thus, we may understand my model without "forcing" college candidates to randomize. The seeming randomization of efforts in the equilibrium can result from the perturbed payoffs among candidates. Although one player takes pure action, the other player may still view his or her action as being drawn from a distribution because of the uncertainty associated with his or her "type" (payoff) (Reny et al., 2002).

The College: The Equilibrium (Affirmative Action) Admissions Rule

In the first stage of the game, the college picks its admissions rule, represented by the value of α. Having characterized the equilibrium plays of college candidates for any admissions rule, we may explicitly find out the college's choice that maximizes the expected test score of the admitted student, which is given by

$$(7) \qquad Q = E[\Pr(e_N > \alpha e_M)e_N + \Pr(\alpha e_M > e_N)e_M] = E[F_M(e_N/\alpha)e_N] + E[F_N(\alpha e_M)e_M].$$

By Proposition 1, when $\alpha \in (0, \theta]$, we have

$$(8) \qquad Q = \int_0^{\alpha V_M} [(V_N - \alpha V_M + e_N)/V_N] \cdot (e_N/\alpha V_M)de_N + \int_0^{V_M} (e_M/V_M) \cdot (\alpha e_M/V_N)de_M$$
$$= \alpha V_M(V_N - \alpha V_M)/2V_N + (\alpha^2 V_M^2 + \alpha V_M^2)/3V_N$$
$$= (V_M/V_N) \cdot [\alpha(V_N - \alpha V_M)/2 + \alpha V_M/3 + \alpha^2 V_M/3].$$

When $\alpha \in [\theta, \infty)$, we have

$$(9) \qquad Q = \int_0^{V_N} (e_N/V_N) \cdot (e_N/\alpha V_M)de_N + \int_0^{V_N/\alpha} [(V_M - V_N/\alpha + e_M)/V_M] \cdot (\alpha e_M/V_N)de_M$$
$$= V_N^2/3\alpha V_M + [(\alpha V_M - V_N)/2V_M] \cdot (V_N/\alpha^2) + V_N^2/3\alpha^2 V_M$$
$$= (V_N/\alpha^2 V_M) \cdot [(\alpha V_M - V_N)/2 + V_N/3 + \alpha V_N/3].$$

In summary,

$$(10) \qquad Q = \begin{cases} (V_M/V_N) \cdot [\alpha(V_N - \alpha V_M)/2 + \alpha V_M/3 \\ \quad + \alpha^2 V_M/3] \quad \text{if } \alpha \in (0, \theta], \\ (V_N/\alpha^2 V_M) \cdot [(\alpha V_M - V_N)/2 + V_N/3 \\ \quad + \alpha V_N/3] \quad \text{if } \alpha \in [\theta, \infty]. \end{cases}$$

By my assumption, it is in the college's sole discretion to choose any $\alpha \in (0, \infty)$ that best fits its objective. As discussed before, affirmative action takes place if the chosen policy parameter α exceeds 1. We show in the appendix that Q, the expected test score of the admitted student, is continuous on α and strictly increases with α when $\alpha \in (0, \theta]$, while strictly decreases when $\alpha \in [\theta, \infty)$. Hence, the following obtains.

THEOREM 1. *In the unique equilibrium of the game, an academic quality–oriented college adopts an (affirmative action) admissions rule with $\alpha^* = \theta > 1$, which (uniquely) maximizes the expected test score of the admitted student.*

By Theorem 1, affirmative action endogenously arises as the unique outcome of the game. Theorem 1 establishes that the equilibrium admissions rule that best addresses the interest of the college turns out to take the form of affirmative action, although we do not explicitly assume that the college concerns the ethnic diversity of its student body. It follows that the academic quality of the college tends to be compromised if affirmative action is banned in admissions practice and the college adopts a color-blind admissions rule ($\alpha = 1$).

The Incentive Effects of Affirmative Action

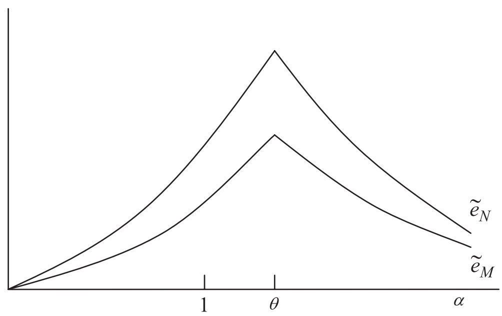
We show that the unique equilibrium admissions rule, which is designed to foster academic quality, turns out to favor the (weaker) minority candidate and allows the admissions not to be awarded to the better-scoring candidate. To provide more intuition for the seemingly counterintuitive results, we consider how candidates' effort (test score) strategies respond to the change in the admissions rule. We define $\tilde{e}_M$ and $\tilde{e}_N$ to be the expected academic efforts expended by candidate M and N, respectively. By Proposition 1, when $\alpha \in (0, \theta]$, we have

$$(11) \qquad \tilde{e}_M = \int_0^{V_M} (\alpha e_M/V_N)de_M = \alpha V_M^2/2V_N,$$

and

$$(12) \qquad \tilde{e}_N = \int_0^{\alpha V_M} (e_N/\alpha V_M)de_N = \alpha V_M/2.$$

FIGURE 1
Expected Efforts



When $\alpha \in [\theta, \infty)$, we have

$$(13) \quad \tilde{e}_M = \int_0^{V_N/\alpha} (\alpha e_M / V_N) de_M = V_N / 2\alpha,$$

and

$$(14) \quad \tilde{e}_N = \int_0^{V_N} (e_N / \alpha V_M) de_N = V_N^2 / 2\alpha V_M.$$

PROPOSITION 2. *The equilibrium (affirmative action) admissions rule—that is, $\alpha^* = \theta$—uniquely maximizes both candidates' expected academic efforts.*

We illustrate by Figure 1 how candidates' academic efforts respond to varying α. Both $\tilde{e}_M$ and $\tilde{e}_N$ are strictly increasing functions of α when $\alpha \in (0, \theta]$ but strictly decreasing functions of α when $\alpha \in [\theta, \infty)$. Increasing α encourages both candidates to expend more effort until it reaches θ. By contrast, once α exceeds θ, a greater α makes both candidates reduce their efforts. Proposition 2 therefore obtains.

My result brings forth a different flavor than that of models based on statistical discrimination theory, such as those proposed by Moro and Norman (2003) and Furstenburg (2003). Moro and Norman argue that with affirmative action in place, the initially discriminated group has a stronger incentive to invest for skills, whereas the initially dominant group has a weaker incentive. Furstenburg finds a similar effect in regard to how affirmative action influences the cross-group human capital distribution. In contrast to the negative cross-group externality, my model unveils a different aspect of the cross-group interaction resulting from affirmative action. We find that college candidates' incentives to exert academic effort may positively interact with each other.

Consider the case $\alpha \in (0, \theta]$, where both $\tilde{e}_M$ and $\tilde{e}_N$ increase with α. This result reflects two effects. One is a direct effect. Intuitively, a greater α increases the marginal return of the minority candidate's academic effort and therefore encourages one to expend more effort. The other is an indirect effect. As the minority candidate expends more effort, the nonminority candidate is forced to increase effort in response to a more aggressive competitor. As a consequence, an increase in α within this range improves both candidates' incentives.

By contrast, once α exceeds the critical value θ, this cross-group interaction reverses the effects of affirmative action on candidates' incentives. In this instance,the predominant preference awarded to the minority dampens the nonminority candidate's incentive. The minority candidate is therefore allowed to reduce effort in the face of a less-competitive rival.

This particular type of strategic interaction gives rise to my findings. The academic quality of the incoming class (the expected test score of the admitted candidate) is maximized if and only if the admissions rule perfectly offsets the initial advantage of the nonminority candidate; that is, $\alpha^* = \theta$ $(= V_N/V_M)$. The fully leveled playing field escalates the competition between the two candidates and invites both of them to exert more academic efforts. This overall gain in academic efforts makes the expected score of the winner rise, even though the better-scoring candidate may not necessarily be accepted.

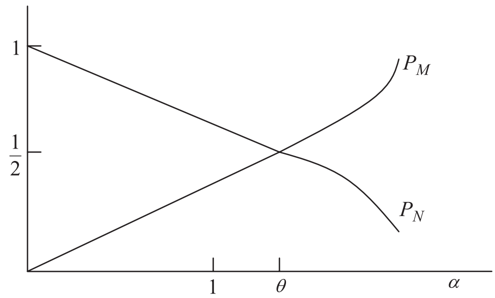
IV. DISCUSSION

Given the equilibrium specified here, it is now possible to explore the ramifications of affirmative action in regard to other important issues. We first examine how the equilibrium (affirmative action) admissions rule affects the minority representation in the college. We then apply my results to investigate whether affirmative action widens or narrows the long-existing racial test-score gap.

Minority Representation and Diversity

My main result, Theorem 1, rationalizes the widespread practice of race-conscious preferential admissions in selective colleges. However, one major argument of those who

FIGURE 2
Likelihoods of Acceptance



advocate affirmative action is that it enhances the minority representation in higher education. My model provides insights in this regard. We consider the minority representation in terms of the expected winning probability of the minority candidate, denoted by P_M. We denote by P_N the expected winning probability of the nonminority candidate. We illustrate candidates' likelihoods of winning as functions of the admissions policy parameter α.

Figure 2 shows that P_M strictly increases with α, whereas P_N strictly decreases with α. Minority representation does increase under the equilibrium (affirmative action) admissions rule, as compared to the case where affirmative action is banned and only color-blind admissions ($\alpha = 1$) take place. This result is not surprising, given that a huge bulk of empirical evidence has revealed that affirmative action has significantly enhanced the minority enrollment in colleges. Yet it is interesting to note that the downward-sloped curve of P_N and the upward-sloped curve of P_M intersect in the unique equilibrium of the game with $\alpha^* = \theta$.

THEOREM 2. *"Equal chance": The equilibrium admissions rule—that is, $\alpha^* = \theta$—uniquely equalizes the expected probabilities of winning between the minority and nonminority candidates; that is, $P_M(\theta) = P_N(\theta) = 1/2$.*

Theorem 2 states that the admissions rule designed to improve the academic quality of a college naturally results in "equal chance" between the minority and nonminority college candidates. The college's demand for academic quality is not in conflict with the interest in a diversified student body ("equal representation") but rather coincides with it. Hence, my results reconcile the seeming tension between academic quality and diversity (equity).

Racial Test Score Gap

This framework allows us to examine how affirmative action admissions affect the racial test-score gap. My results show that affirmative action creates stronger incentive for both candidates to acquire educational benefit. However, what remains is whether the preferential admissions rule helps the minority candidate catch up with the nonminority in education attainment. The racial test-score gap has long been existing. A number of empirical studies, such as that by Neal and Johnson (1996), have found that racial gaps in test score or skills may account for a significant portion of the racial wage differential. Understanding racial test-score gap may substantially contribute to social policymaking that attempts to reduce racial inequality. In fact, should the racial test-score gap be closed, the race-conscious preferential admissions would no longer be a compelling means to enhance the minority representation in higher education. Nevertheless, do affirmative action admissions narrow the gap or widen it?

We consider the test score gap as the expected test score differential between the nonminority candidate and the minority candidate. We set the case of no affirmative action as the natural benchmark. In the benchmark case, the college does not have the freedom to practice preferential admissions and simply adopts a color-blind admissions rule with $\alpha = 1$. Define $K(\alpha) = \tilde{e_N} - \tilde{e_M}$ as the expected test score gap between the nonminority candidate and the minority candidate. By equations 11 and 12, the equilibrium test-score gap $K^* \equiv K(\theta)$ is given by

$$(15) \qquad K^* = \theta V_M/2 - \theta V_M^2/2V_N = (V_N - V_M)/2.$$

The test score gap in the benchmark case is given by

$$(16) \qquad K(1) = (V_M/V_N) \cdot [(V_N - V_M)/2].$$

PROPOSITION 3. *Racial test-score gap widens under the equilibrium (affirmative action) admissions rule as compared to the case of color-blind admissions* ($\alpha = 1$).

K^* is greater than $K(1)$ because $V_M < V_N$. We show that affirmative action admissions rule ($\alpha^* = \theta > 1$) results in a greater test score differential. Affirmative action improves both candidates' incentive to acquire more education, yet the nonminority candidate N responds more aggressively, even though the minority candidate is the targeted beneficiary of this policy practice. In short, my results imply that the preferential admissions rule alone does not close racial test-score gap but widens it. This finding is testable and has been evidenced by empirical observations: "Since 1988, the racial gap in college admissions tests has actually become wider, and there is no compelling evidence that any improvement is in the offing"; see Austen-Smith and Fryer (forthcoming).

V. CONCLUDING REMARK

This study sets forth a stylized theoretical framework for examining the incentive effects of affirmative action in college admissions. Although diversity is the most commonly stated rationale by policymakers who support affirmative action, we have shown that an affirmative action admissions rule may endogenously arise in equilibrium even when colleges are solely interested in fostering academic quality. We find that the equilibrium rule designed to maximize the academic quality of the college achieves equal representation and improves the incentives of both minorities and nonminorities to invest in academic effort (human capital). The results reconcile the perceived tension between academic quality and diversity and rationalize the prevalent and persistent practice of affirmative-action admissions procedure in selective institutions.

My finding confirms the conventional wisdom that placing a handicap on the stronger contestant escalates the competition and boosts performance. This reveals an important feature of the incentive structure behind preferential admissions procedures. The positive strategic interaction between two candidates enables the affirmative action practice to induce both of them to increase their efforts, as well as level the playing field.

The main result has strong policy implications. It is essential for a policymaker to understand the incentive structure underlying an affirmative-action policy proposal. Even though preferential admissions can be a powerful incentive mechanism that enhances the value of a college, it does not narrow but widen the racial test-score gap. We predict that affirmative action alone will not help reduce racial inequality in education attainment. The policy maker needs additional policy tools for achieving this objective. One such alternative might be programs designed to reduce the marginal cost of academic effort by minorities—such as scholarships, special classes, and additional funding toward public schools in minority communities. The model suggests that such practices may maximize the quality of the college and achieve equal representation while eliminating the test score gap.

This study leaves tremendous room for future extensions. First of all, the emphasis is the partial equilibrium incentive effect of affirmative action at the college admissions level and does not consider general equilibrium effects in the labor market. It would be interesting to extend the model in this manner to examine how affirmative action at the college admissions level affects future productivity and social welfare. Second, my approach involves a single college and mainly applies to selective institutions. Another interesting extension would be to allow multiple colleges of different tiers to compete for a fixed pool of students and to examine how the structure of the education market contributes to the formation of colleges' admissions rules.

APPENDIX

PROOF OF THEOREM 1

Proof. When $\alpha \in (0, \theta]$,

$$(17) \qquad \begin{aligned} \partial Q/\partial\alpha &= \partial\{(V_M/V_N)\cdot[\alpha(V_N-\alpha V_M)/2 \\ &\quad + \alpha V_M/3 + \alpha^2 V_M/3]\}/\partial\alpha \\ &= (V_M/6V_N)\cdot(3V_N + 2V_M - 2\alpha V_M). \end{aligned}$$

Because $\alpha < \theta$, (17) $\geq (V_M/6V_N)\cdot(3V_N + 2V_M - 2V_N) = (V_M/6V_N)\cdot(V_N + 2V_M) > 0$.

When $\alpha \in [\theta, \infty)$,

$$(18) \qquad \begin{aligned} \partial Q/\partial\alpha &= \partial\{(V_N/\alpha^2 V_M)\cdot[(\alpha V_M - V_N)/2 \\ &\quad + V_N/3 + \alpha V_N/3]\}/\partial\alpha \\ &= (V_N/\alpha^2 V_M)\cdot(V_N/3\alpha - V_N/3 - V_M/2). \end{aligned}$$

Because $\alpha \geq \theta$, (18) $\leq (V_N/\alpha^2 V_M)\cdot(V_M/3 - V_N/3 - V_M/2) < 0$.

So far we show that Q monotonically increases with α when $\alpha \in (0, \theta]$ and decreases with α when $\alpha \in [\theta, \infty)$. In addition, because $\lim_{\alpha\uparrow\theta} Q = \lim_{\alpha\downarrow\theta} Q = (V_N + V_M)/3$, we conclude that Q is continuous on α. It follows that Q is uniquely maximized by $\alpha^* = \theta$. ■

PROOF OF PROPOSITION 2

Proof. $\lim_{\alpha\uparrow\theta}\tilde{e}_M = \lim_{\alpha\downarrow\theta}\tilde{e}_M = V_M/2$; $\lim_{\alpha\uparrow\theta}\tilde{e}_N = \lim_{\alpha\downarrow\theta}\tilde{e}_N = V_N/2$. So $\tilde{e}_M$ and $\tilde{e}_N$ are both continuous.

By equations 11 to 14, $\tilde{e}_M$ and $\tilde{e}_N$ increase with α when $\alpha \in (0, \theta]$ and decrease when $\alpha \in [\theta, \infty)$. ■

PROOF OF THEOREM 2

Proof. When $\alpha \in (0, \theta]$, we have

$$(19) \qquad P_M = E[\Pr(\alpha e_M > e_N)] = E[F_N(\alpha e_M)] = \int_0^{V_M} (\alpha e_M / V_M V_N) de_M = \alpha V_M / 2V_N.$$

Hence, $P_N = E[\Pr(e_N > \alpha e_M)] = 1 - E[\Pr(\alpha e_M > e_N)] = (2V_N - \alpha V_M)/2V_N$.

When $\alpha \in [\theta, \infty)$, we have

$$(20) \qquad P_M = E[\Pr(\alpha e_M > e_N)] = E[F_N(\alpha e_M)] = \int_0^{V_N/\alpha} [V_M - V_N/\alpha + e_M)/V_M] \cdot (\alpha/V_N) de_M = (2\alpha V_M - V_N)/2\alpha V_M.$$

Hence, $P_N = E[\Pr(e_N > \alpha e_M)] = 1 - E[\Pr(\alpha e_M > e_N)] = V_N/2\alpha V_M$.

First, we show $\lim_{\alpha \uparrow \theta} P_M(\alpha) = \lim_{\alpha \downarrow \theta}(\alpha) = P_M(\alpha) = 1/2 = P_N(\alpha = 1 - P_M(\alpha)$. Hence, P_M, as well as P_N, is continuous on α. By equations 19 and 20, P_M strictly increases with α, whereas P_N strictly decreases with α, which implies $P_M \gtreqless P_N$ iff $\alpha \gtreqless \theta$. Hence, $\alpha^* = \theta$ uniquely equalizes P_M and P_N. ■

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