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Law, social responsibility, and outsourcing☆

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ABSTRACT

Previous research into law and corporate social responsibility mostly assumes that the vertical structure of production is exogenous. Here, we allow a brand to choose between vertical integration and outsourcing. With outsourcing, the brand avoids some liability and responsibility, but loses direct control over the producer's infringement of law or code of conduct. Infringement increases with production, so the brand tailors production to guide the producer's infringement. The elasticity of demand for the product affects the degree to which, under outsourcing, the brand will increase production to induce the producer to reduce cost through infringement. If the demand is sufficiently elastic relative to the social harm caused by infringement, the optimal policy is to reduce avoidance such that the brand chooses vertical integration. However, if the demand is sufficiently inelastic relative to the social harm, then the optimal policy is to increase avoidance such that the brand chooses outsourcing.

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1. Introduction

Unilever is one of the world's top consumers of palm oil and palm kernel oil, which it uses to manufacture soaps, cleaners, margarine, and ice cream. As a corporation, it has committed to procuring all agricultural raw materials from sustainable sources by the year 2020. However, palm kernel oil derivatives may be shipped over long distances and difficult to trace. Unilever is addressing the challenge by vertical integration: "investing € 69 million in a palm kernel oil processing plant in Indonesia and considering similar joint venture investments in processing crude palm oil derivatives elsewhere *to help us achieve traceable supplies*" (Unilever, 2013, 42) [emphasis added].

Rana Plaza, an eight-storey building in Bangladesh, was designed for shops and offices, but illegally extended and converted for industrial production. In April 2013, the building collapsed, killing over 1000 workers. Factories in Rana Plaza manufactured clothing for clients including Benetton and Joe Fresh. The Institute for Global Labour and Human Rights called on Joe Fresh to protect workers in contract factories. Nobel Peace Prize winner, Muhammad Yunus, asked global fashion brands to treat Bangladesh workers *as their own employees* (BBC News, 2013; Forbes, 2013).

Here, motivated by the above examples, we study the distribution of legal liability and social responsibility across vertically related organizations. Issues of liability and responsibility arise in any chain of production, especially those that transcend international borders (Lyon and Maxwell, 2008; Kitzmueller and Shimshack, 2012; Lee and Tang, 2018).

In the strategic choice between vertical integration and outsourcing, a brand faces a fundamental trade-off. Under vertical integration, the brand bears the full brunt of legal liability and corporate social responsibility, but the brand fully controls the harmful actions that give rise to liability and responsibility. We call the harmful actions, such as clearing virgin forest, using unsafe factories, and imposing excessive overtime, "infringement". By contrast, under outsourcing, the brand only bears *vicarious* legal liability and social responsibility – to the extent that it is responsible for infringement by the producer. However, with outsourcing, the brand cannot directly control the producer's infringement.

This paper contributes to understanding of the vertical structure of production and policy towards liability and responsibility for social harms caused by production in three ways. First, we show that, under outsourcing, the brand tailors production to guide the producer's infringement. Infringement reduces production costs, and so, increases with the scale of production. The brand sets production scale so as to maximize profit given that the producer will then choose infringement conditional on the scale of production. The producer ignores the penalty on the brand, and hence, conditional on production, infringement is higher under outsourcing.

Second, we analyze how the brand's choice between vertical integration and outsourcing depends on the degree to which it can avoid liability or responsibility for the producer's infringement. The higher the degree of avoidance (equivalently, the lower the degree of vicarious liability and responsibility), the more likely is the brand to outsource production.

Third, we characterize the optimal standard of vicarious liability and responsibility according to the elasticity of demand for the product and the social harm caused by infringement. The elasticity of demand affects the degree to which, under outsourcing, the brand will increase production to induce the producer to reduce the cost of production through infringement. If the demand is elastic, then the brand will be relatively more sensitive to the cost of production, and, under outsourcing, it will set production relatively high to induce more infringement and lower production cost. Under vertical integration, the brand will incur both brand and producer penalties, and so, choose infringement to be lower relative to production. Hence, society prefers vertical integration if demand is elastic relative to the social harm, and prefers outsourcing if demand is relatively inelastic.

Overall, we contribute to a better understanding of managerial strategy and public policy with regard to legal liability and social responsibility when the vertical structure of production is endogenous. Our work is related to issues of liability when one party is possibly liable for the actions of others. One instance is product liability of manufacturers where consumers can engage in harmful actions (Hay and Spier, 2005). Another is vicarious liability, a legal concept that distinguishes between the harmful actions of an employee (employer is liable) vis-a-vis independent contractor (employer is not liable) (Sykes, 1984; Arlen and Macleod, 2005; Kraakman, 2013).

In the work closest to ours, Brooks (2002) used a numerical example to discuss how vicarious liability affects the choice of vertical structure. In his setting, production is assumed to be exogenous with binary infringement levels, and the analysis abstracts away welfare analysis. In contrast, our model emphasizes contracting between principal (brand) and agent (manufacturer), with each independently choosing production and infringement respectively to maximize their profit. We further analyze social welfare and draw the relevant managerial and policy implications.

Our research also contributes to the economics of corporate social responsibility (CSR) for labor rights, sustainability, and business practices in general. CSR activists tend to assume that imposing more responsibility is always better (BBC Panorama, 2010; CNET, 2014). However, our analysis points to an inherent non-convexity: the social welfare maximum is the maximum of welfare under vertical integration and outsourcing. As in any non-convex problem, moving toward a local optimum may be sub-optimal. We identify conditions under which a more stringent standard of CSR does increase welfare to the global optimum. In independent work, Orsdemir et al. (2016) also analyze the effect of vertical integration on responsible sourcing.

2. Setting

Consider two functions of a business, the downstream function of branding and the upstream function of production. Let the brand sell some item at a retail price, p. The item yields benefit, $B(Q)$, to consumers that depends on the scale of production, Q. The marginal benefit diminishes with scale, $B'(Q) > 0$ and $B''(Q) < 0$.

For simplicity, we assume that the brand is a monopoly, incurs no costs of retailing, and sets the retail price, $p = B'(Q)$. The item is produced at a cost that decreases in the degree, $x \in [0, 1]$, to which the producer *infringes* laws or regulations or socially responsible practices. Specifically, the producer's cost of production is $[1 - x]C(Q)$, where $C'(Q) > 0$, $C''(Q) > 0$.

Infringement of legal liability or social responsibility causes social harm, $H(Q, x)$, which increases with the scale of production, $\partial H(Q, x)/\partial Q > 0$, and infringement, $\partial H(Q, x)/\partial x > 0$. The harm exceeds and increases faster than the cost saving, $H(Q, x) \geq xC(Q)$ and $\partial H(Q, x)/\partial x \geq C(Q)$. The producer is subject to enforcement at an exogenous rate that increases with the degree of infringement, $\mu m(x)$, where $\mu \in [0, 1]$, $m(x) \in [0, 1]$, $m'(x) > 0$, and $m''(x) > 0$.

We analyze two alternative structures of production: vertical integration and outsourcing. Under vertical integration, the brand chooses the degree of infringement, x. In the event of enforcement, the brand suffers a penalty, $F_B + F_P$, which may be a monetary penalty, loss of future profit or reputation, or cost of shifting production to another location.

By contrast, under outsourcing, the brand stipulates the scale of production, Q, and sets a wholesale price, w, per unit of production, while the producer independently decides the degree of infringement, x.[1] In the event of enforcement, the producer suffers a penalty, F_P, and the brand suffers a penalty, $[1 - \alpha]F_B$, where the factor, $\alpha \in [0, 1]$, characterizes the extent to which the brand can *avoid* legal liability and social responsibility for the producer's infringement. Equivalently, $[1 - \alpha]$ represents the degree to which the brand is *vicariously* liable or responsible for the producer's infringement.

Thus, outsourcing reduces the penalty on the brand from $F_B + F_P$ to $[1 - \alpha]F_B$. The penalties and the standard of vicarious liability and social responsibility may be imposed by consumers, third party stakeholders, or the government. (Examples of increases in α are the increased pressure on manufacturers such as Unilever and Joe Fresh to be socially responsible and Mr Muhammad Yunus's call for fashion brands to treat Bangladesh workers like their employees.)

This model of outsourcing corresponds to the relations of large multinational brands like Unilever and Joe Fresh with their upstream producers and suppliers. The upstream industry is fiercely competitive, and so, the brands have power to stipulate terms of production and supply. Under outsourcing, the brand cannot contract with the producer over the degree of infringement, because such contracts are either ethically repugnant or simply illegal (just imagine contracts over how much primary forest to burn or how unsafe to make the factory). Likewise, the producer cannot insure the brand against the penalty for infringement – besides such a contract being unethical or illegal, the

[1] In the Appendix A, Remark 2 shows that the assumption of a constant wholesale price is without loss of generality, essentially because the wholesale price does not affect the producer's choice of infringement.

producer might not have the financial resources. The brand can only indirectly influence the producer's choice of infringement through the production scale and wholesale price.

In the discussion below, we focus on interior solutions, $Q^* > 0$ and $x^* < 1$. However, the proofs take account of possible boundary solutions, $x^* = 1$.

3. Vertical integration

Under vertical integration, the brand's profit is

$$\Pi_{int} = p(Q)Q - [1 - x]C(Q) - \mu m(x)[F_B + F_P]. \tag{1}$$

The brand chooses the degree of infringement, x, and scale of production, Q, according to the first-order conditions,

$$\frac{\partial \Pi_{int}}{\partial x} = C(Q) - \mu m'(x)[F_B + F_P] = 0, \tag{2}$$

and

$$\frac{\partial \Pi_{int}}{\partial Q} = p'(Q)Q + p(Q) - [1 - x]C'(Q) = 0. \tag{3}$$

Clearly, the profit function reflects a complementarity between compliance, $1 - x$, and enforcement (both the rate of enforcement, μ, and penalties on the brand and producer for infringement, F_B and F_P). If the enforcement rate or penalties are higher, the marginal return to complying with law and regulations will be higher. Intuitively, the optimal degree of compliance (infringement) will increase (decrease) in the strength of enforcement. Further, an increase in infringement will reduce the cost of production, thus x and Q are complements, and their decrease or increase should be mutually reinforcing. Formally,[2]

Proposition 1. *Under vertical integration, the scale of production, Q^*_{int}, and degree of infringement, x^*_{int}, decrease, while the price, p^*_{int}, increases in the enforcement rate, μ, and penalties on the brand and producer for infringement, F_B and F_P.*

4. Outsourcing

Under outsourcing, the brand stipulates the scale, Q, to the producer and the wholesale price, w, for each unit of production. Given the scale and wholesale price, the producer must first decide whether to engage in production and, if so, choose the degree of infringement, x, to maximize profit.

Consider the producer. Its profit is

$$\pi = wQ - [1 - x]C(Q) - \mu m(x)F_P, \tag{4}$$

[2] Please refer to the Appendix A for proofs of the results.

and it will engage in production if and only if it earns non-negative profit,

$$\pi = wQ - [1-x]C(Q) - \mu m(x)F_P \geq 0, \tag{5}$$

which is its participation condition.

Given non-negative profit, and conditional on Q, the producer chooses infringement, x. Differentiating (4), the first-order condition is

$$\frac{d\pi}{dx} = C(Q) - \mu \frac{dm}{dx} F_P = 0, \tag{6}$$

or

$$\frac{dm}{dx} = \frac{C(Q)}{\mu F_P}. \tag{7}$$

Let the inverse function of dm/dx be $\nu(\cdot)$, so (7) becomes

$$x = \nu\left(\frac{C(Q)}{\mu F_P}\right). \tag{8}$$

Differentiating (6),

$$\frac{d^2\pi}{dx^2} = -\mu F_P m''(x) < 0,$$

so, the producer's profit function is concave in x.

Hence, (7) characterizes the equilibrium degree of infringement, $x(Q)$, as a function of the scale of production. Comparing (2) with (7), we infer that, conditional on the scale of production, the brand under vertical integration chooses a lower degree of infringement than the producer under outsourcing. The essential reason is that, under outsourcing, the producer ignores the penalty on the brand.

Next, we analyze the brand's choice of scale and wholesale price given that the producer independently chooses the degree of infringement. The brand's profit is

$$\Pi_{out} = p(Q)Q - wQ - [1-\alpha]\mu m(x)F_B.$$

The brand maximizes Π_{out} subject to the producer's participation condition, (5), and choice of infringement, (7).

To maximize profit, the brand should set the wholesale price, w, such that the producer just breaks even, $\pi = 0$. Substituting in (5), this means

$$wQ = [1-x]C(Q) + \mu m(x)F_P.$$

Hence, the brand's profit simplifies to

$$\Pi_{out} = p(Q)Q - [1-x]C(Q) - \mu m(x)\big[[1-\alpha]F_B + F_P\big]. \tag{9}$$

Compared with the brand's profit under vertical integration, (1), the brand can avoid part, α, of the expected penalty, $\mu m(x)F_B$, and so, reduces its total cost.

Differentiating (9), the brand stipulates the scale of production according to

$$\frac{\partial \Pi_{out}}{\partial Q} = p'(Q)Q + p(Q) - [1-x]C'(Q) + C(Q)\frac{dx}{dQ} - \mu m'(x)\big[[1-\alpha]F_B + F_P\big]\frac{dx}{dQ}. \tag{10}$$

Compared with the brand's choice of production scale under vertical integration, (3), the key difference is that the stipulated scale influences the producer's choice of infringement. As Proposition 2 below shows, if the brand increases scale, the producer increases infringement, $dx/dQ > 0$, and thereby reduces the cost of production by $C(Q)\cdot dx/dQ$, but also raises the expected penalty by $\mu m'(x)\big[[1-\alpha]F_B + F_P\big]\cdot dx/dQ$.

Next, we show that the producer will increase infringement with the scale that the brand stipulates, $dx/dQ > 0$. Further, we show that increases in the extent, α, to which the brand can avoid liability and responsibility by outsourcing will increase the equilibrium scale of production and infringement.

Proposition 2. *Under outsourcing, the producer's degree of infringement, $x(Q)$, increases in the scale of production, Q. In equilibrium, both the scale of production, Q^*_{out}, and infringement, x^*_{out}, increase, while the price, p^*_{out}, decreases in the brand's avoidance of liability and responsibility by outsourcing, α.*

Proposition 2 highlights an essential economy of scale in the producer's choice of infringement, x. Referring to Eq. (4), the expected penalty is a fixed cost that does not vary with the scale of production, Q, but the return to infringement increases with the scale. Hence, the larger is the scale of production, the more the producer chooses to infringe laws and regulations and social responsibility.

Proposition 2 also shows that, under outsourcing, infringement and scale are essentially complementary with the extent, α, to which outsourcing enables the brand to avoid liability and responsibility. Referring to Eq. (9), the higher is the extent to which the brand can avoid liability and responsibility, the greater is the marginal return from infringement. Hence, in equilibrium, x and Q increase with α.

By contrast, the effect of enforcement, μ and F_P, on the equilibrium is ambiguous. Consider an increase in the rate of enforcement, μ. The effect of the increased enforcement on the brand's choice of scale, Q, depends on three factors. One is whether the increased enforcement affects the expected penalty on the brand or producer relatively more. Another factor is the sensitivity of the producer's choice of infringement to the increased enforcement, i.e., the shape of $m(\cdot)$. The third factor is the cost of production, $C(Q)$. The higher is the cost of production, the more the producer gains by infringement, and hence, the less the producer will adjust infringement in response to the increased enforcement. (The analysis of the effect of an increase in the penalty, F_P, is similar.) Formally, we have

Remark 1. In equilibrium, the scale of production, Q^*_{out}, and infringement, x^*_{out}, decrease in the rate of enforcement, μ, if and only if

$$C(Q)\frac{m'''(x)}{[m''(x)]^2} > \mu F_P\left[1 - \frac{F_P}{[1-\alpha]F_B}\right]. \quad (11)$$

5. Integration vis-a-vis outsourcing

We are interested in how differences in institutions, as characterized by the standard of vicarious liability and responsibility, $[1-\alpha]$, for outsourced production, affect the brand's decision on vertical integration. Accordingly, we need to compare the effect of α on the brand's profit maxima under vertical integration and outsourcing.

Proposition 3 shows that the brand's actual choice between vertical integration and outsourcing is monotone in the extent of avoidance, α. Outsourcing enables the brand to shift part of the responsibility and liability to the producer, which suffers a smaller expected penalty from enforcement than the brand. However, under outsourcing, the producer chooses the degree of infringement. The brand cannot directly control the degree of infringement, and can only indirectly influence it through the scale of production. Overall, if the brand can sufficiently avoid liability and responsibility, then the reduction in expected penalty outweighs the loss of control over the infringement, and the brand prefers outsourcing.

Proposition 3. *There exists some extent of avoidance, $\tilde{\alpha}$, such that the brand prefers outsourcing if and only if $\alpha > \tilde{\alpha}$.*

Referring to Fig. 1, the essence of Proposition 3 is that the brand's profit increases with α under outsourcing (the higher is α, the more the brand can avoid liability and responsibility through outsourcing), but does not vary with α under vertical integration. Further with $\alpha = 0$, the brand obviously prefers vertical integration, while with $\alpha = 1$, it prefers outsourcing. Hence, there exists some intermediate $\tilde{\alpha}$ at which the brand is indifferent between outsourcing and integration.

We have shown that the brand's choice between vertical integration and outsourcing is monotone in the extent, α, to which the brand can avoid liability and responsibility by outsourcing. If the brand can avoid liability and responsibility by at least $\tilde{\alpha}$, then it will choose outsourcing. This result provides a perspective in terms of legal liability and social responsibility that complements theories of outsourcing based on costs of contracting and monitoring (Grossman and Hart, 1986; Hart and Moore, 1990). Our findings apply equally to international outsourcing and outsourcing within national jurisdictions. The only essential ingredient is that the brand can avoid some liability and responsibility by outsourcing.

Proposition 2 above shows that, under outsourcing, both the production scale and infringement increase in the extent, α, to which the brand can avoid liability and

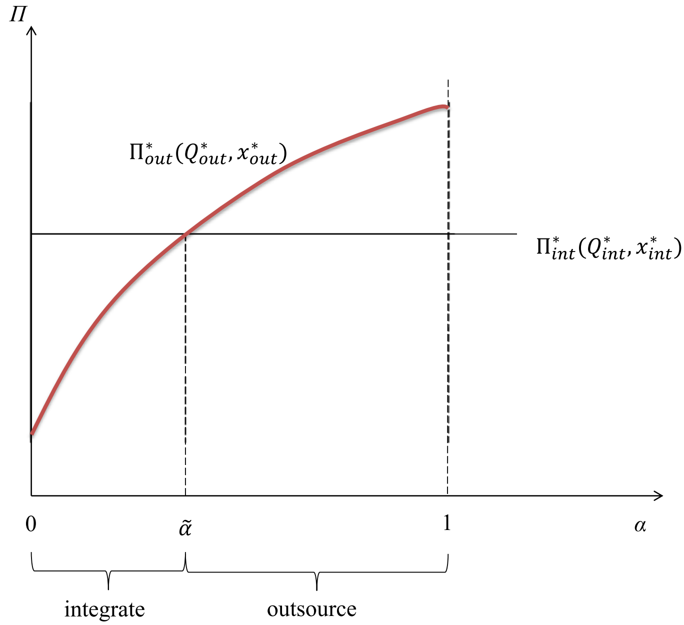

Fig. 1. Brand profit: Outsourcing vis-a-vis vertical integration.

responsibility by outsourcing. By contrast, under vertical integration, the production scale and infringement obviously do not vary with α.

Can we then conclude that the production scale and infringement are monotone in α? The answer is no, because α not only affects the production scale and infringement *within* one given structure of production (outsourcing), but also affects the brand's choice *between* vertical structures of production (integration and outsourcing). Lemma 1 formalizes this analysis.

Lemma 1. *Assuming that the brand's profit under outsourcing,* Π_{out}*, is single-peaked in the scale of production,* Q_{out}*,*

(i) *There exists* α_x *such that* $x^*_{out} \leq x^*_{int}$ *if and only if* $\alpha \leq \alpha_x$*,*
(ii) *If the penalty on the brand,* F_B*, is sufficiently large or the rate of enforcement satisfies* $m'''(\cdot) \leq 0$*, there exists* α_Q *such that* $Q^*_{out} \leq Q^*_{int}$ *if and only if* $\alpha \leq \alpha_Q$*,*
(iii) $\alpha_x \leq \alpha_Q$.

The condition for Lemma 1, single-peakness, is weaker than concavity. Fig. 2 illustrates three critical levels of avoidance: $\tilde{\alpha}$, above which the brand chooses outsourcing over vertical integration; α_x, above which the degree of infringement is higher under outsourcing, and α_Q, above which the scale of production is higher under outsourcing.

Lemma 1 establishes sufficient conditions for $\alpha_Q > 0$, so that there exists a range of avoidance such that the production scale is higher under vertical integration and another

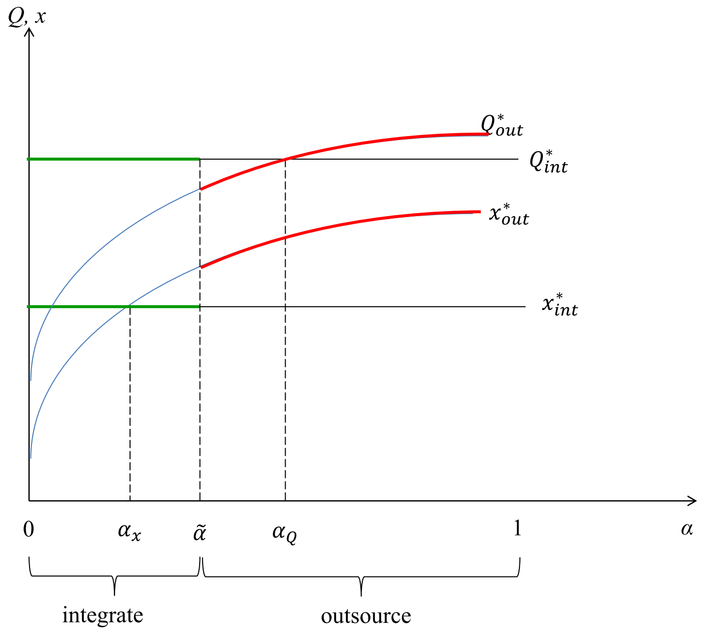

Fig. 2. Infringement and production scale: Outsourcing vis-a-vis vertical integration.

range such that the production scale is higher under outsourcing. However, it is possible that $\alpha_x = 0$, which means that infringement is always higher under outsourcing than vertical integration. Essentially, if the expected penalty on the producer is sufficiently high, both infringement under vertical integration and outsourcing will be very low, and indeed, so low that infringement under outsourcing cannot be less than under vertical integration.[3]

Lemma 1 enables us to analyze how changes in the extent of avoidance affect the production scale and infringement. Referring to Fig. 2, three scenarios are possible:

Proposition 4. *Depending on the relative magnitudes of the three cut-offs, $\tilde{\alpha}$, α_x, and α_Q, we have*

(I) $\tilde{\alpha} > \alpha_Q$*: A slight reduction in α around $\tilde{\alpha}$ discretely reduces both the scale of production and degree of infringement.*

(II) $\alpha_x < \tilde{\alpha} \leq \alpha_Q$*: A slight reduction in α around $\tilde{\alpha}$ leads to no change or a discrete increase in the scale of production, and a discrete reduction in the degree of infringement.*

[3] It is intuitive that $\alpha_x < \alpha_Q$. Suppose that $\alpha = \alpha_Q$, and so, the scale of production is the same under outsourcing and vertical integration. Now, under outsourcing, the producer chooses infringement without considering the expected penalty on the brand. Hence the producer will choose a higher level of infringement than the vertically-integrated brand, $x^*_{out} > x^*_{int}$, which means that $\alpha > \alpha_x$. Thus, we have $\alpha_x < \alpha_Q$.

(III) $\tilde{\alpha} < \alpha_x$ *and* $\alpha_x > 0$*: A slight reduction in* α *around* $\tilde{\alpha}$ *discretely raises both the scale of production and degree of infringement.*

Referring to Fig. 2, consider a reduction in α from above to below $\tilde{\alpha}$ in scenario (iii). For $\alpha > \tilde{\alpha}$, the brand chooses outsourcing, so, both the production scale and infringement increase in α. As α crosses from above to below $\tilde{\alpha}$, the brand switches from outsourcing to vertical integration, and hence, both the production scale and infringement increase discretely. Evidently, neither the production scale nor infringement are monotone in α.

Moreover, in this scenario, a reduction in α results in *more* infringement, which seems quite counter-intuitive. The reason is the effect of α on the brand's choice between vertical structures of production (integration vis-a-vis outsourcing).

6. Welfare

Having characterized the equilibrium behavior of brand and producer, we now turn to social welfare. Do more stringent standards of vicarious liability and responsibility (lower α) raise welfare? What is the socially optimal level of vicarious liability and responsibility for outsourced production?

Taking a utilitarian approach, we stipulate social welfare as consumer benefit less the cost of production and less the harm caused by infringement,

$$W(Q,x) = B(Q) - [1-x]C(Q) - H(Q,x). \tag{12}$$

Generally, it is difficult to characterize the optimal standard of vicarious liability and responsibility. Under outsourcing, welfare varies with avoidance through the equilibrium scale and infringement. Even if, under outsourcing, welfare is a well-behaved function of α, there remains the challenge of comparing the maximum welfare under outsourcing, W_{out}, with the welfare under vertical integration, W_{int}. Any change in avoidance around $\tilde{\alpha}$ that induces the brand to switch between outsourcing and vertical integration would discretely affect production scale, infringement, and welfare. Thus, maximizing welfare is an inherently non-convex problem.

To analyze the optimal extent of avoidance, we first suppose that $\alpha > \tilde{\alpha}$, in which case, the brand chooses outsourcing. Differentiating (12), the effect of avoidance on welfare,

$$\begin{aligned}\frac{dW_{out}}{d\alpha} &= B'(Q)\frac{dQ}{d\alpha} - [1-x]C'(Q)\frac{dQ}{d\alpha} + C(Q)\frac{dx}{dQ}\frac{dQ}{d\alpha} - \frac{\partial H(Q,x)}{\partial Q}\frac{dQ}{d\alpha} \\ &\quad - \frac{\partial H(Q,x)}{\partial x}\frac{dx}{dQ}\frac{dQ}{d\alpha} \\ &= \left\{[B'(Q) - [1-x]C'(Q)] + \left[C(Q)\frac{dx}{dQ} - \frac{\partial H(Q,x)}{\partial Q} - \frac{\partial H(Q,x)}{\partial x}\frac{dx}{dQ}\right]\right\}\frac{dQ}{d\alpha}.\end{aligned} \tag{13}$$

By (8), in equilibrium, there is a unique reflexive correspondence between infringement, x, and scale, Q. Hence, we can re-arrange (13) to

$$\frac{dW_{out}}{d\alpha} = [B'(Q) - [1-x]C'(Q)]\frac{dQ}{d\alpha} + \left[C(Q) - \frac{\partial H(Q,x)}{\partial x} - \frac{\partial H(Q,x)}{\partial Q}\frac{dQ}{dx}\right]\frac{dx}{d\alpha}. \quad (14)$$

This equation characterizes the fundamental trade-off in the effect of the policy instrument, avoidance, α, on welfare. If α is higher, the brand would raise the scale of production, Q, and the producer would then increase infringement, x.

In (14), the first set of right-hand side terms, $B'(Q) - [1-x]C'(Q)$, represents the effect on the benefit and cost of producing for the marginal consumer through the scale of production, Q. This effect is positive, essentially because the brand equilibrates the marginal revenue with the private marginal cost, but the marginal benefit exceeds the marginal revenue.

The higher scale induces the producer to increase infringement, x. In (14), the second set of right-hand side terms, $C(Q) - \partial H(Q,x)/\partial x - \partial H(Q,x)/\partial Q \cdot dQ/dx$, represents the effect of avoidance on the private cost of production and social harm through its indirect impact on infringement. The cost of production is lower by $C(Q)$. Social harm is directly higher by $\partial H(Q, x)/\partial x$, and indirectly higher by $\partial H(Q, x)/\partial Q \cdot dQ/dx$ through the effect of infringement on scale. By assumption, social harm increases faster than the cost reduction, $\partial H(Q, x)/\partial x \geq C(Q)$. Hence, the second set of terms on the right-hand side of (14) is negative.

We impose the following regularity condition, which assumes that the social harm caused by infringement is proportionate to the cost of production.

Assumption 1. $H(Q,x) = hxC(Q)$, with $h \geq 1$.

Recall from Proposition 2 and (8) that x is an implicit function of Q. Hence, the social harm, $H(Q, x(Q)) \equiv hx(Q)C(Q)$. Then, generally, by (12), welfare simplifies to

$$W(Q, x(Q)) = B(Q) - C(Q) - \frac{h-1}{h}H(Q, x(Q)). \quad (15)$$

Under vertical integration, welfare, W_{int}, does not vary with α. By (14), under outsourcing, welfare, W_{out}, varies with α according to

$$\frac{dW_{out}}{d\alpha} = \left\{[B'(Q) - C'(Q)] - \frac{h-1}{h}\frac{dH(Q, x(Q))}{dQ}\right\}\frac{dQ}{d\alpha}. \quad (16)$$

In equilibrium under outsourcing, given α, denote the brand's choice of production scale by the function, $Q(\alpha)$. Then, it is straightforward to show

Lemma 2. *Assuming that the social harm caused by infringement, $H(Q, x(Q))$, is nonconcave in Q, then welfare under outsourcing, $W_{out}(Q, x(Q))$, is single-peaked in α, and, if $\frac{dW_{out}}{dQ}|_{Q=Q(0)} > 0$ and $\frac{dW_{out}}{dQ}|_{Q=Q(1)} < 0$, there exists a unique $\alpha^* \in (0, 1)$ that maximizes welfare under outsourcing.*

Lemma 2 depicts that $W_{out}(Q, x(Q))$ can be maximized by a unique α^*. To characterize the (second-best) welfare maximum, however, we must compare the resultant maximum with the welfare under vertical integration: The maximum at α^* may not be feasible, as α^* could have led the brand to switch to integration.

Linear-quadratic setting

To gain tractability and more specific insight into policy, we focus on a specific parametrization. Let the retail demand be linear,

$$p(Q) = a - bQ \tag{17}$$

with $b > 0$, production be subject to constant returns to scale,

$$C(Q) = cQ, \tag{18}$$

and the enforcement rate be quadratic,

$$\mu m(x) = \mu x^2. \tag{19}$$

Under vertical integration, substituting from (17)–(19) in (2) and (3), the brand's first-order condition for scale of production is

$$a - 2bQ - \left\{ 1 - \frac{cQ}{2\mu[F_P + F_B]} \right\} c = 0, \tag{20}$$

and the first-order condition for infringement is

$$cQ - 2\mu x[F_P + F_B] = 0. \tag{21}$$

Similarly, under outsourcing, substituting in (10), the brand's first-order condition for scale of production,

$$a - 2bQ - \left\{ 1 - \frac{cQ}{2\mu F_P} \left[1 - [1 - \alpha] \frac{F_B}{F_P} \right] \right\} c = 0, \tag{22}$$

and, by (7), the producer chooses infringement according to

$$cQ - 2\mu x F_P = 0. \tag{23}$$

To focus on interior solutions, assume that

Assumption 2.

$$a \geq c \text{ and } b \geq \max\left\{\frac{c^2}{4\mu[F_P + F_B]}, \left[1 - [1-\alpha]\frac{F_B}{F_P}\right]\frac{c^2}{4\mu F_P}\right\}. \tag{24}$$

Then, it is straightforward to show that

Lemma 3. *Under vertical integration, the equilibrium is*

$$\begin{aligned} Q^*_{int} &= \frac{2\mu[F_P + F_B][a-c]}{4b\mu[F_P + F_B] - c^2}, \\ x^*_{int} &= \frac{ac - c^2}{4b\mu[F_P + F_B] - c^2}, \\ \Pi^*_{int} &= \frac{\mu[F_P + F_B][a-c]^2}{4b\mu[F_P + F_B] - c^2}; \end{aligned} \tag{25}$$

while, under outsourcing, the equilibrium is

$$\begin{aligned} Q^*_{out}(\alpha) &= \frac{2\mu F_P[a-c]}{4b\mu F_P - \left[1 - [1-\alpha]\frac{F_B}{F_P}\right]c^2}, \\ x^*_{out}(\alpha) &= \frac{ac - c^2}{4b\mu F_P - \left[1 - [1-\alpha]\frac{F_B}{F_P}\right]c^2}, \\ \Pi^*_{out}(\alpha) &= \frac{\mu F_P[a-c]^2}{4b\mu F_P - \left[1 - [1-\alpha]\frac{F_B}{F_P}\right]c^2}; \end{aligned} \tag{26}$$

and

$$\tilde{\alpha} = \alpha_Q = \frac{F_B}{F_P + F_B} \text{ and } \alpha_x = 1 - \frac{4b\mu F_P}{c^2}. \tag{27}$$

By (27) and Assumption 2, $\alpha_x < \alpha_Q = \tilde{\alpha}$, so, this parametrization fits Proposition 4, scenario (II). In this scenario, a reduction in α to below $\tilde{\alpha}$ which would lead to vertical integration and discretely raise welfare. Yet, it is possible that the maximum welfare with outsourcing associated with the optimal extent of avoidance, α^* would be even higher.

To investigate, we first must understand whether the welfare-maximizing standard of vicarious liability and responsibility under outsourcing, $[1-\alpha^*]$, is feasible in the sense that, if $\alpha = \alpha^*$, then the brand would choose outsourcing. The following result addresses this issue.

Lemma 4. $\alpha^* \leq \tilde{\alpha}$ *if and only if*

$$b \leq \hat{b} \equiv \frac{c^2[h[F_P + F_B] - F_B]}{2\mu F_P[F_P + F_B]}. \tag{28}$$

Lemma 4 shows that, if the demand is sufficiently elastic relative to the social harm caused by infringement, then $\alpha^* < \tilde{\alpha}$. Then, if $\alpha = \alpha^*$, the brand would choose vertical integration, implying that the welfare maximum with outsourcing is not feasible. However, we caution this still does not imply vertical integration is optimal – because welfare with α just above $\tilde{\alpha}$, which leads to outsourcing, might still exceed that with vertical integration.

Substituting from (17), (18), (25), and (26) in (15), welfare under vertical integration,

$$W_{int} = \int_0^{Q^*_{int}} \{[a - bQ]dQ - cQ^*_{int} - [h-1]x^*_{int}cQ^*_{int}\}dQ$$
$$= \frac{[a-c]^2 2\mu[F_P + F_B]}{[4b\mu[F_P + F_B] - c^2]^2}\left[3b\mu[F_P + F_B] - hc^2\right], \quad (29)$$

and welfare under outsourcing,

$$W_{out}(\alpha^*) = \int_0^{Q^*_{out}} \{[a - bQ]dQ - cQ - [h-1]x^*(\alpha^*)cQ\}dQ$$
$$= \frac{[a-c]^2 \mu F_P}{2[b\mu F_P + c^2[h-1]]}. \quad (30)$$

Generally, the optimal policy depends on the conditions of demand and social harm. Propositions 5 and 6 show that vertical integration is optimal if demand is sufficiently elastic relative to the social harm, while outsourcing is optimal if demand is sufficiently inelastic.

Proposition 5. *If demand is sufficiently elastic relative to the social harm that* $b \le \hat{b}$, *the socially optimal standard of vicarious liability and responsibility is* $[1 - \tilde{\alpha}]$, *which induces vertical integration.*

Fig. 3a illustrates Proposition 5. The maximum welfare under outsourcing – at the optimal standard of vicarious liability and responsibility – exceeds that under integration, $W_{out}(\alpha^*) > W_{int}$. However, by Lemma 4, if $b \le \hat{b}$, then $\alpha^* \le \tilde{\alpha}$, which means that outsourcing is not feasible if the standard of vicarious liability and responsibility is $[1 - \alpha^*]$. This standard is so high that the brand chooses vertical integration.

Intuitively, demand being elastic means that consumers are price-sensitive. Then the brand will be relatively more sensitive to the cost of production, and under outsourcing, it will set production relatively high to induce high infringement, and so, reduce the cost of production. By contrast, under vertical integration, the brand will incur both brand and producer penalties, and so, choose lower production and infringement. Hence, welfare is higher with vertical integration.

If the demand is relatively inelastic such that $b > \hat{b}$, then, by Lemma 4, $\alpha^* > \tilde{\alpha}$, which implies that the optimal standard of vicarious liability and responsibility under outsourcing, $[1 - \alpha^*]$, is feasible in the sense that at that level, the brand chooses outsourcing. However, as Fig. 3b and 3c illustrate, the maximum welfare under outsourcing may

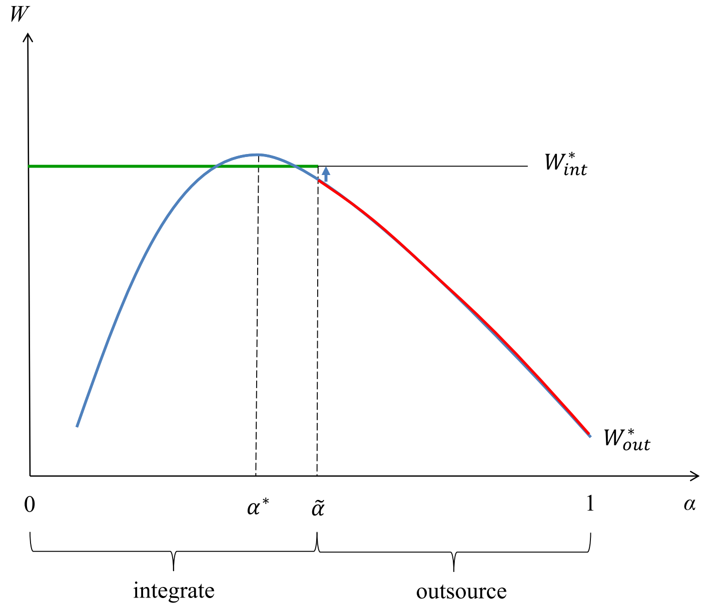

Fig. 3a. Welfare: $\alpha^* \leq \tilde{\alpha}$.

either exceed or fall below that with vertical integration. The optimal policy depends on comparing $W_{out}(\alpha^*)$ with W_{int}, and we find that:

Proposition 6. *If demand is sufficiently inelastic relative to the social harm that*

$$4[F_P + F_B]\left[3b\mu[F_P + F_B] - hc^2\right]\left[b\mu F_P + c^2[h-1]\right] < F_P\left[4b\mu[F_P + F_B] - c^2\right]^2, \quad (31)$$

which is satisfied when b is sufficiently large, the socially optimal standard of vicarious liability and responsibility is $[1 - \alpha^*]$, which induces outsourcing.

It is straightforward to show that, if the demand, b, satisfies (31), then it also satisfies $b > \hat{b}$. Accordingly, by Lemma 4, the optimal standard of vicarious liability and responsibility under outsourcing is feasible. Further, referring to Fig. 3c, Proposition 6 shows that, if b satisfies (31), the maximum welfare under outsourcing exceeds that with vertical integration. Hence, the optimal policy is to set the standard of vicarious liability and responsibility at $[1 - \alpha^*]$ and induce the brand to outsource production. That is, the social optimum stipulates a less stringent standard than would induce vertical integration, $\alpha^* > \tilde{\alpha}$.

Intuitively, a relatively more inelastic demand allows the brand to charge a higher price, thereby making it be less sensitive to the cost of production. Under outsourcing, it will have less incentive to increase production to stimulate infringement, and so, reduce the cost of production. By contrast, under vertical integration, the brand will incur both brand and producer penalties, and so, set relatively lower production and infringement. Hence, welfare is higher with outsourcing.

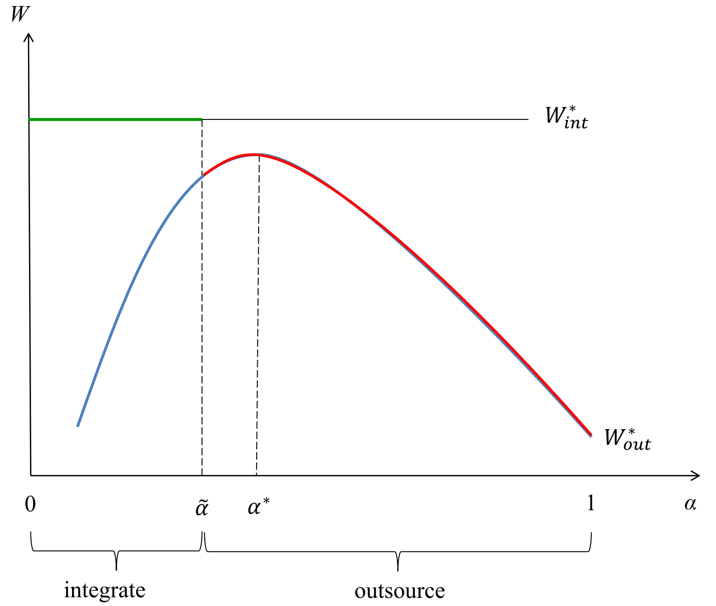

Fig. 3b. Welfare: $\alpha^* > \tilde{\alpha}$ and $W_{out}(\alpha^*) < W_{int}$.

While the main policy results (Propositions 5 and 6) were derived under particular assumptions, we emphasize that the results are qualitatively robust to more general settings. The intuition is not restricted to that in the linear-quadratic setting, although the specific quantitative analyses might vary with the modeling environment (as elaborated in an earlier version of this paper (Fu et al., 2016)).

7. Concluding remarks

Our analysis provides insight into compliance with laws and social responsibility when the vertical structure of production (integration or outsourcing) is endogenous. Infringement of law and responsibility reduces production costs, and so, increases with the scale of production. Under outsourcing, the brand tailors the scale of production to manage the producer's choice of infringement. Maximizing welfare requires comparing welfare under vertical integration and outsourcing, and so, is an inherently non-convex problem. Vertical integration is optimal if demand is elastic relative to the social harm caused by infringement, while outsourcing is optimal if demand is sufficiently inelastic.

The essential parameter in our analysis is the standard, $[1 - \alpha]$, of vicarious legal liability and social responsibility on the brand for outsourced production. This parameter is determined by consumers, third-party stakeholders, and government. For instance, following the Rana Plaza disaster, Nobel Laureate Muhammad Yunus called for global fashion brands to treat Bangladesh workers as their own employees. The pressure of

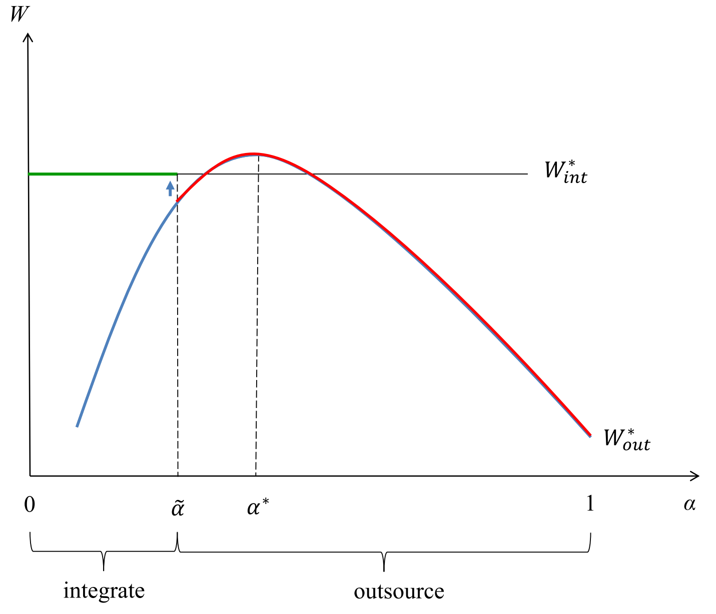

Fig. 3c. Welfare: $\alpha^* > \tilde{\alpha}$ and $W_{out}(\alpha^*) \geq W_{int}$.

social responsibility was so strong that Unilever vertically integrated into the production of palm kernel oil.

However, it is not generally optimal to toughen the standard of vicarious liability and social responsibility (reduce α) to the point that the brand chooses vertical integration. Vertical integration ensures that the brand fully internalizes the penalties for infringement and is optimal if the expected penalties equal the social harm, $\mu m(x)[F_B + F_P] = H(Q, x)$. However, enforcement might not be optimal due to conflicts in jurisdiction. As Unilever and Rana Plaza illustrate, the brand may be in one jurisdiction and the producer in another, and the authorities in the two jurisdictions might not coordinate their enforcement in an optimal way. Realistically, enforcement authorities fail to coordinate internationally and even within nations. In the second-best setting where enforcement is not optimal, tougher standards and vertical integration need not be optimal.

This paper focuses on society's choice of the standard of vicarious liability and social responsibility and the brand's trade-off between vertical integration and outsourcing. Our analysis assumes that society can influence the vertical structure, production, and infringement only through the standard, which is a constant, and that, under outsourcing, the brand can only influence the producer's decision on infringement *indirectly* through the scale of production.

Future research could relax these assumptions and allow the standard of vicarious liability or social responsibility and the penalties to be conditioned on the scale of production, i.e., let α, F_B, and F_P be functions of Q. With the additional degree(s) of freedom, welfare would be higher. In particular, if the penalty on the producer can be contingent on production, the producer will essentially be taxed for each unit of output that involves infringement. Then, there will not be an economy of scale in infringement and society can directly regulate infringement.[4]

Another direction for future research would consider the brand's monitoring of the producer and imposing sanctions for infringement (subject to producer's limited liability). This generalization, however, is quite complicated. The problem is three-dimensional, involving the brand's optimal choice of production scale, monitoring intensity, and wholesale price subject to the manufacturer's choice of infringement. For tractability, it seems necessary to impose regularity conditions on the monitoring technology and sanctions. Yet, the equilibrium is sensitive to the specific assumptions. We feel that the only practical approach is to analyze vertical structure and monitoring separately. Accordingly, an important direction for further work is to study the brand's trade-off between production scale and monitoring intensity under outsourcing.

Yet another direction for future research is to investigate the impact of asymmetric information between consumers and the brand over sourcing. By upstream vertical integration, the brand not only ensures better control over the supply chain but also provides a credible signal to consumers of its commitment to organic production, green practices, and fair trade (Hainmueller et al., 2015; Dragusanu et al., 2014). To the extent that consumers care about how things are produced and not just the finished product, this presents another dimension of competition between brands. Besides the well-known dimensions of price and product quality, future research could study competition among brands on the "quality of production".

Our analysis focused on upstream vertical integration. Yet, another direction for future research is the downstream distribution of legal liability and social responsibility. In July 2014, the General Counsel of the U.S. National Labor Relations Board ruled that McDonald's, as franchisor, is "jointly liable" for the employment practices of its franchisees. Labor activists hailed this decision, which breaks new ground in employment law, while industry decried it as "rip[ping] apart a proven and time-tested business model". Commentators disagree on whether franchisors would take a bigger role in managing their franchisees or step back even further to avoid joint liability (New York Times, 2014). Clearly, these are issues worth researching.

Appendix A. Proofs

Proof of Proposition 1. We use the techniques of supermodularity (Vives, 2000; Van Zandt, 2002) to prove the result for x and Q with respect to μ, while omitting the proofs with respect to F_P and F_B, as they are similar. Let $\phi(\mu) =$

[4] An earlier version of this paper (Fu et al., 2016) considers extensions of the analysis to penalties for infringement depending on the scale of production, and the demand for the product decreasing in the degree of infringement.

$\text{argmax}_{x,Q}\{\Pi_{int}(x, Q, \mu) : x \in [0, 1], Q \in R_+\}$ be the solution correspondence for the brand's profit maximization.

Partially differentiating (2) with respect to Q,

$$\frac{\partial^2 \Pi_{int}}{\partial x \partial Q} = C'(Q) > 0.$$

Accordingly, Π_{int} has strictly increasing differences in (x, Q). Partially differentiating (2) with respect to μ,

$$\frac{\partial^2 \Pi_{int}}{\partial x \partial \mu} = -m'(x)[F_B + F_P] < 0,$$

and so, Π_{int} has strictly decreasing differences in (x, μ).

Similarly, partially differentiating (3) with respect to μ,

$$\frac{\partial^2 \Pi_{int}}{\partial Q \partial \mu} = 0,$$

and so, Π_{int} has decreasing differences in (Q, μ).

Then, by Van Zandt (2002), Theorem 3, the maximal and minimal selections of $\phi(\mu)$ are decreasing functions. Equivalently, for $\mu' < \mu''$, the maximum solution, $\bar{\phi}(\mu') \geq \bar{\phi}(\mu'')$, and the minimum solution, $\underline{\phi}(\mu') \geq \underline{\phi}(\mu'')$. Hence, x and Q decrease in μ. Since the price decreases in the production scale, $p'(Q) < 0$, it follows that p increases in μ. □

Proof of Proposition 2. We first show that $x(Q)$ increases in the scale of production, Q. Partially differentiating (4) with respect to x,

$$\frac{\partial \pi}{\partial x} = C(Q) - \mu m'(x) F_P,$$

and partially differentiating again with respect to Q,

$$\frac{\partial^2 \pi}{\partial x \partial Q} = C'(Q) > 0.$$

Hence, the producer's profit, π, has strictly increasing differences in (x, Q), and so, by supermodularity, $x(Q)$ increases in Q.

For the relationship between (Q^*_{out}, x^*_{out}) and the extent of avoidance, α, we again use supermodularity. By (8), $\nu(\cdot)$ is the inverse of dm/dx, hence

$$m'\left(\nu\left(\frac{C(Q)}{\mu F_P}\right)\right) = \frac{C(Q)}{\mu F_P}, \tag{A.1}$$

$$\nu'\left(\frac{C(Q)}{\mu F_P}\right) = \frac{1}{m''(x)} > 0, \tag{A.2}$$

since $m''(\cdot) > 0$, and

$$\nu''\left(\frac{C(Q)}{\mu F_P}\right) = -\frac{m'''(x)}{[m''^3}. \tag{A.3}$$

Substituting from (8) in the brand's profit function, (9),

$$\Pi_{out} = p(Q)Q - \left[1 - \nu\left(\frac{C(Q)}{\mu F_P}\right)\right]C(Q) - \mu m\left(\nu\left(\frac{C(Q)}{\mu F_P}\right)\right)[[\ 1-\alpha]F_B + F_P]. \tag{A.4}$$

Let $\varphi(\alpha) = \text{argmax}_Q \Pi_{out}(Q, \alpha : Q \in R_+)$. Partially differentiating (A.4) with respect to Q,

$$\frac{\partial \Pi_{out}}{\partial Q} = p'(Q)Q + p(Q) - \left[1 - \nu\left(\frac{C(Q)}{\mu F_P}\right)\right]C'(Q) + \nu'\left(\frac{C(Q)}{\mu F_P}\right)\frac{C'(Q)}{\mu F_P}C(Q)$$
$$- \mu m'\left(\nu\left(\frac{C(Q)}{\mu F_P}\right)\right)\cdot \nu'\left(\frac{C(Q)}{\mu F_P}\right)\frac{C'(Q)}{\mu F_P}[[\ 1-\alpha]F_B + F_P], \tag{A.5}$$

which characterizes the brand's choice of production scale.

Partially differentiating (A.5) with respect to α,

$$\frac{\partial^2 \Pi_{out}}{\partial Q \partial \alpha} = m'\left(\nu\left(\frac{C(Q)}{\mu F_P}\right)\right)\cdot \nu'\left(\frac{C(Q)}{\mu F_P}\right)\frac{F_B}{F_P}C'(Q) > 0,$$

after substituting from (A.1) and (A.2). This proves that Π_{out} has strictly increasing differences in (Q, α).

Thus, for $\alpha_1 < \alpha_2$ and for $Q_1 \in \varphi(\alpha_1)$ and $Q_2 \in \varphi(\alpha_2)$, we have $Q_1 < Q_2$. Since the price decreases in the production scale, it follows immediately that the corresponding prices, $p_1 > p_2$. Further, by (8) and (A.2), the producer's choice of infringement is

$$x_1 = \nu\left(\frac{C(Q_1)}{\mu F_P}\right) < \nu\left(\frac{C(Q_2)}{\mu F_P}\right) = x_2.$$

□

Proof of Remark 1. Partially differentiating (A.5) with respect to μ,

$$\frac{\partial^2 \Pi_{out}}{\partial Q \partial \mu} = -\ \nu'\left(\frac{C(Q)}{\mu F_P}\right)\left[\frac{C(Q)}{\mu^2 F_P}\right]C'(Q)$$
$$-\ \nu''\left(\frac{C(Q)}{\mu F_P}\right)\left[\frac{C(Q)}{\mu^2 F_P}\right]\left[\frac{C'(Q)C(Q)}{\mu F_P}\right] - \nu'\left(\frac{C(Q)}{\mu F_P}\right)\frac{C'(Q)C(Q)}{\mu^2 F_P}$$

$$
\begin{aligned}
&- m'\left(\nu\left(\frac{C(Q)}{\mu F_P}\right)\right)\nu'\left(\frac{C(Q)}{\mu F_P}\right)\frac{C'(Q)[[1-\alpha]F_B]+F_P]}{\mu F_P}\\
&+ \mu m''\left(\nu\left(\frac{C(Q)}{\mu F_P}\right)\right)\nu'\left(\frac{C(Q)}{\mu F_P}\right)\frac{C(Q)}{\mu^2 F_P}\nu'\left(\frac{C(Q)}{\mu F_P}\right)\frac{C'(Q)[[1-\alpha]F_B+F_P]}{\mu F_P}\\
&+ \mu m'\left(\nu\left(\frac{C(Q)}{\mu F_P}\right)\right)\nu''\left(\frac{C(Q)}{\mu F_P}\right)\frac{C(Q)}{\mu^2 F_P}\cdot\frac{C'(Q)[[1-\alpha]F_B+F_P]}{\mu F_P}\\
&+ \mu m'\left(\nu\left(\frac{C(Q)}{\mu F_P}\right)\right)\nu'\left(\frac{C(Q)}{\mu F_P}\right)\frac{C'(Q)[[1-\alpha]F_B+F_P]}{\mu^2 F_P}
\end{aligned}
$$

Substituting from (A.1)–(A.3), the above simplifies to

$$
\frac{\partial^2\Pi_{out}}{\partial Q\partial\mu} = -\frac{C(Q)C'(Q)}{\mu^2 F_P}\left[\frac{[1-\alpha]F_B\cdot C(Q)m'''(x)+\mu F_P\cdot[m''^2[F_P-[1-\alpha]F_B]}{\mu F_M^2[m''^3}\right].
$$

Now,

$$
\frac{C(Q)C'(Q)}{\mu^2 F_P} > 0,
$$

and $\mu F_M^2[m''(x)]^3 > 0$, since, by assumption, $m''(\cdot)>0$. Hence, $\partial^2\Pi_{out}/\partial Z\partial\mu<0$, i.e., Π_{out} has decreasing differences in (Q, μ) if and only if

$$
[1-\alpha]F_B\cdot C(Q)m'''(x)+\mu F_P\cdot[m''^2[F_P-[1-\alpha]F_B]>0,
$$

which simplifies to (11). □

Remark 2. The brand will choose the same scale of production whether it pays the producer by a linear or non-linear wholesale pricing scheme.

Proof. Suppose that the brand pays the producer according to $w(Q)$. Consider the producer. Its profit is

$$
\pi = w(Q) - [1-x]C(Q) - \mu m(x)F_P, \tag{A.6}
$$

and it will engage in production if and only if it earns non-negative profit,

$$
\pi = w(Q) - [1-x]C(Q) - \mu m(x)F_P \geq 0, \tag{A.7}
$$

which is its participation condition.

Given non-negative profit, and conditional on Q, the producer chooses infringement, x. Differentiating (A.6), the first-order condition simplifies to (6). The producer's choice of infringement is independent of the pricing scheme, $w(\cdot)$, essentially because the price depends only on Q and does not vary with x.

The brand's profit is

$$\Pi_{out} = p(Q)Q - w(Q) - [1-\alpha]\mu m(x)F_B. \tag{A.8}$$

The brand maximizes Π_{out} subject to the producer's participation condition, (A.7), and choice of infringement, (6).

Since the wholesale price does not affect the producer's infringement, the brand should set the wholesale price to optimize the participation condition, so that the producer just breaks even, $\pi = 0$. Substituting in (A.7), this means

$$w(Q) = [1-x]C(Q) + \mu m(x)F_P.$$

Substituting in (A.8), the brand's profit simplifies to (9). □

Proof of Proposition 3. Referring to Fig. 1, the proof comprises four steps: (i) The brand's profit under vertical integration, $\Pi_{int}(Q^*_{int}, x^*_{int})$, does not vary with α; (ii) The brand's profit under outsourcing, $\Pi_{out}(Q^*_{out}, x^*_{out})$, increases in α; (iii) If $\alpha = 0$, the brand earns less with outsourcing, and (iv) If $\alpha = 1$, the brand earns more with outsourcing. Then, by continuity, there exists some $\tilde{\alpha}$ such that

$$\Pi_{out}(Q^*_{out}, x^*_{out}|\tilde{\alpha}) = \Pi_{int}(Q^*_{int}, x^*_{int}|\tilde{\alpha}),$$

and such that $\Pi_{out}(Q^*_{out}, x^*_{out}|\alpha) \geq \Pi_{int}(Q^*_{int}, x^*_{int}|\alpha)$ if and only if $\alpha > \tilde{\alpha}$.

(i) This result is obvious from (1).

(ii) Next, we prove that $\Pi_{out}(Q^*_{out}, x^*_{out})$ is increasing in α. For any α_i, let $(Q^*_i, x(Q^*_i))$ maximize $\Pi_{out}(Q, x(Q)|\alpha_i)$. Suppose otherwise, that $\alpha_1 < \alpha_2$, and

$$\Pi_{out}(Q^*_1, x(Q^*_1)|\alpha_1) \geq \Pi_{out}(Q^*_2, x(Q^*_2)|\alpha_2). \tag{A.9}$$

By construction, $(Q^*_2, x(Q^*_2))$ maximizes $\Pi_{out}(Q, x(Q)|\alpha_2)$, hence

$$\Pi_{out}(Q^*_2, x(Q^*_2)|\alpha_2) \geq \Pi_{out}(Q^*_1, x(Q^*_1)|\alpha_2).$$

Substituting in (A.9),

$$\Pi_{out}(Q^*_1, x(Q^*_1)|\alpha_1) \geq \Pi_{out}(Q^*_1, x(Q^*_1)|\alpha_2).$$

Substituting from (9), and simplifying, we have

$$-\mu m(x(Q^*_1))[[\,1-\alpha_1]F_B + F_P] \geq -\mu m(x(Q^*_1))[[\,1-\alpha_2]F_B + F_P],$$

which implies that $\alpha_1 \geq \alpha_2$, which is a contradiction. Thus, we infer that $\Pi_{out}(Q^*_{out}, x^*_{out})$ must be increasing in α.

(iii) Suppose that $\alpha = 0$. Then, by (9), under outsourcing, the brand's profit simplifies to

$$\Pi_{out} = p(Q)Q - [1-x]C(Q) - \mu m(x)[F_B + F_P] = \Pi_{int},$$

using (1). However, under outsourcing, the brand is subject to the constraint that the producer chooses infringement, while under vertical integration, the brand can freely choose infringement. So, the brand's profit under outsourcing cannot exceed the profit under vertical integration,

$$\Pi_{out}(Q^*_{out}, x^*_{out}) \leq \Pi_{int}(Q^*_{int}, x^*_{int}).$$

(iv) Suppose that $\alpha = 1$. Then, by (9), under outsourcing, the brand's profit simplifies to $\Pi_{out} = p(Q)Q - [1-x]C(Q) - \mu m(x)F_P$. Comparing with (1), we infer that, for any (Q, x), the profit, $\Pi_{out}(Q, x) \geq \Pi_{int}(Q, x)$. Hence, in particular,

$$\Pi_{out}(Q^*_{int}, x^*_{int}) \geq \Pi_{int}(Q^*_{int}, x^*_{int}). \tag{A.10}$$

Under outsourcing, suppose that the brand chooses production scale, Q^*_{int}, and, by (6), the producer chooses infringement, $x_{out}(Q^*_{int})$. Then, by (9), the brand will earn profit,

$$\Pi_{out}(Q^*_{int}, x_{out}(Q^*_{int})) = p(Q^*_{int})Q^*_{int} - [1 - x_{out}(Q^*_{int})]C(Q^*_{int}) - \mu m(x_{out}(Q^*_{int}))F_P.$$

By the definition, (4), given production, Q^*_{int}, the producer maximizes profit with infringement, $x_{out}(Q^*_{int})$, and so,

$$[1 - x_{out}(Q^*_{int})]C(Q^*_{int}) + \mu m(x_{out}(Q^*_{int}))F_P \leq [1 - x^*_{int}]C(Q^*_{int}) + \mu m(x^*_{int})F_P.$$

Substituting above, we have

$$\Pi_{out}(Q^*_{int}, x_{out}(Q^*_{int})) \geq \Pi_{out}(Q^*_{int}, x^*_{int}).$$

Now, by definition, (Q^*_{out}, x^*_{out}) maximizes the brand's profit under outsourcing, so,

$$\Pi_{out}(Q^*_{out}, x^*_{out}) \geq \Pi_{out}(Q^*_{int}, x_{out}(Q^*_{int})) \geq \Pi_{out}(Q^*_{int}, x^*_{int}).$$

Combining the above with (A.10), we have

$$\Pi_{out}(Q^*_{out}, x^*_{out}) \geq \Pi_{int}(Q^*_{int}, x^*_{int}),$$

which completes the proof. □

Proof of Lemma 1. Using (8), by (2),

$$x^*_{int} = \min\left\{\nu\left(\frac{C(Q^*_{int})}{\mu[F_P + F_B]}\right), 1\right\}, \tag{A.11}$$

and by (6),

$$x^*_{out} = \min\left\{\nu\left(\frac{C(Q^*_{out})}{\mu F_P}\right), 1\right\}. \quad \text{(A.12)}$$

We first prove part (ii), then (i) and (ii).

Part (ii)

Consider two cases – where $x^*_{int} < 1$ and $x^*_{int} = 1$.

Case 1: $x^*_{int} = \nu(\frac{C(Q^*_{int})}{\mu[F_P+F_B]}) < 1$.

The proof comprises four steps, (a)–(d).

(a) Under vertical integration, the production scale, Q^*_{int}, does not vary with α. This is immediate from (1).

(b) Under outsourcing, the production scale, Q^*_{out}, increases in α. This is proved in Proposition 2.

(c) If $\alpha = 0$, then $Q^*_{int} > Q^*_{out}$. Suppose that $\alpha = 0$. By (1),

$$\frac{\partial \Pi_{int}}{\partial Q} = p'(Q)Q + p(Q) - [1-x]C'(Q), \quad \text{(A.13)}$$

while, by (9),

$$\frac{\partial \Pi_{out}}{\partial Q} = p'(Q)Q + p(Q) - [1-x]C'(Q) - \frac{F_B}{F_P}C(Q)\frac{dx}{dQ}.$$

Hence, using (8)

$$\left.\frac{\partial \Pi_{out}}{\partial Q}\right|_{Q=Q^*_{int}} - \left.\frac{\partial \Pi_{int}}{\partial Q}\right|_{Q=Q^*_{int}} = \left[\nu\left(\frac{C(Q^*_{int})}{\mu F_P}\right) - \nu\left(\frac{C(Q^*_{int})}{\mu[F_P+F_B]}\right)\right]C'^*_{int} - \frac{F_B}{F_P}C(Q^*_{int})\frac{dx}{dQ}.$$

By definition, Q^*_{int} maximizes (1), and so,

$$\left.\frac{\partial \Pi_{int}}{\partial Q}\right|_{Q=Q^*_{int}} = 0.$$

Further, by (8),

$$\left.\frac{dx}{dQ}\right|_{Q=Q^*_{int}} = \frac{d\nu\left(\frac{C(Q)}{\mu F_P}\right)}{dQ} = \nu'\left(\frac{C(Q^*_{int})}{\mu F_P}\right)\cdot\frac{C'^*_{int})}{\mu F_P}.$$

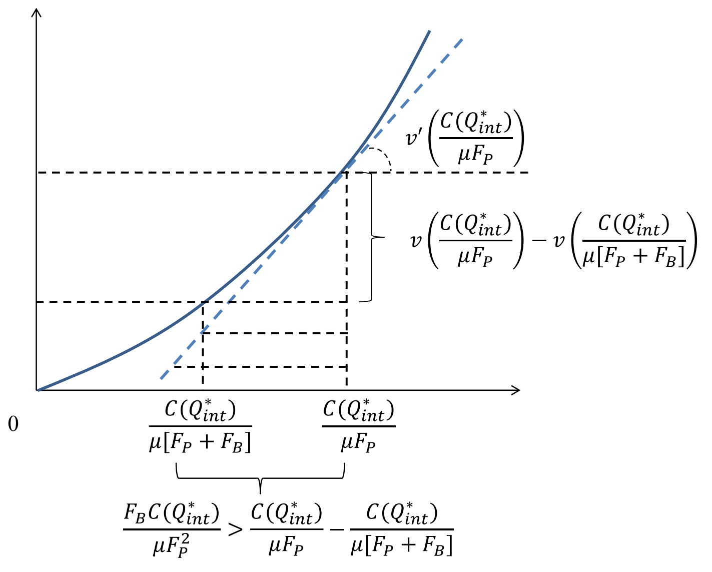

Fig. 4. Infringement function.

Substituting above,

$$\frac{1}{C'(Q^*_{int})}\frac{\partial \Pi_{out}}{\partial Q}\bigg|_{Q=Q^*_{int}} = \nu\left(\frac{C(Q^*_{int})}{\mu F_P}\right) - \nu\left(\frac{C(Q^*_{int})}{\mu[F_P+F_B]}\right) - \frac{F_B}{\mu F_P^2}C(Q^*_{int})\cdot \nu'\left(\frac{C(Q^*_{int})}{\mu F_P}\right). \quad (A.14)$$

If

$$\frac{F_B}{\mu F_P^2}C(Q^*_{int})\cdot \nu'\left(\frac{C(Q^*_{int})}{\mu F_P}\right) > \nu\left(\frac{C(Q^*_{int})}{\mu F_P}\right) - \nu\left(\frac{C(Q^*_{int})}{\mu[F_P+F_B]}\right), \quad (A.15)$$

then, by (A.14),

$$\frac{\partial \Pi_{out}}{\partial Q}\bigg|_{Q=Q^*_{int}} < 0,$$

which implies that, at $Q = Q^*_{int}$, the profit under outsourcing, Π_{out}, decreases in Q. Since the profit function Π_{out} is single-peaked in Q_{out}, the maximum of Π_{out} must be at a lower level of Q, i.e., $Q^*_{out} < Q^*_{int}$.

Referrring to Fig. 4, a sufficient condition for (A.15) is that the function, $\nu(\cdot)$, be weakly convex. Formally, if $\nu(\cdot)$ is weakly convex, then

$$\frac{F_B}{\mu F_P^2} C(Q^*_{int}) \cdot \nu'\left(\frac{C(Q^*_{int})}{\mu F_P}\right) > \left[\frac{C(Q^*_{int})}{\mu F_P} - \frac{C(Q^*_{int})}{\mu[F_P + F_B]}\right] \cdot \nu'\left(\frac{C(Q^*_{int})}{\mu F_P}\right)$$
$$\geq \nu\left(\frac{C(Q^*_{int})}{\mu F_P}\right) - \nu\left(\frac{C(Q^*_{int})}{\mu[F_P + F_B]}\right),$$

which proves (A.15). By (A.2) and (A.3), if $m'{}'(\cdot) \leq 0$, then $\nu''(\cdot) \geq 0$, i.e., $\nu(\cdot)$ is weakly convex.

Referrring to Fig. 4, another sufficient condition for (A.15) is that F_B be sufficiently large. The left-hand side of (A.15) increases in F_B, while the right-hand side is bounded above by $\nu(C(Q^*_{int})/\mu F_P)$, which decreases in F_B, since, by Proposition 1, Q^*_{int} decreases in F_B.

(d) If $\alpha = 1$, then $Q^*_{int} < Q^*_{out}$. Suppose that $\alpha = 1$. By (9),

$$\frac{\partial \Pi_{out}}{\partial Q} = p'(Q)Q + p(Q) - [1 - x]C'(Q).$$

Using (A.13) and (8),

$$\left.\frac{\partial \Pi_{out}}{\partial Q}\right|_{Q=Q^*_{int}} - \left.\frac{\partial \Pi_{int}}{\partial Q}\right|_{Q=Q^*_{int}} = \left[\nu\left(\frac{C(Q^*_{int})}{\mu F_P}\right) - \nu\left(\frac{C(Q^*_{int})}{\mu[F_P + F_B]}\right)\right] C'^{*}_{int}) > 0.$$

Now, by definition, Q^*_{int} maximizes (1), and so,

$$\left.\frac{\partial \Pi_{int}}{\partial Q}\right|_{Q=Q^*_{int}} = 0.$$

Substituting above,

$$\left.\frac{\partial \Pi_{out}}{\partial Q}\right|_{Q=Q^*_{int}} > 0,$$

which implies that, at $Q = Q^*_{int}$, the profit under outsourcing, Π_{out}, increases in Q. Assuming Π_{out} is single-peaked in Q_{out}, the maximum of Π_{out} must be at a higher level of Q, i.e., $Q^*_{out} > Q^*_{int}$.

Case 2: $x^*_{int} = 1$. In this case, we claim that for $Q^*_{out} \leq Q^*_{int}$, for all $\alpha \in [0, 1]$. Define $\hat{Q}_{out}$ as the scale of production such that

$$\nu\left(\frac{C(\hat{Q}_{out})}{\mu F_P}\right) = 1. \tag{A.16}$$

Suppose that $Q^*_{int} \geq \hat{Q}_{out}$*. By (A.2)),* $\nu(\cdot)$ *is increasing, and so,*

$$\nu\left(\frac{C(Q^*_{int})}{\mu F_P}\right) \geq 1.$$

Hence, by (8), the profit-maximizing infringement, $x_{out}(Q^*_{int}) = 1$*. Then,* $m'(x_{out}(Q^*_{int})) = m''(x_{out}(Q^*_{int})) = 0$*, and* $\nu'(Q^*_{int}) = 0$*. Substituting in (A.5), the first-order condition simplifies to*

$$\left.\frac{\partial \Pi_{out}}{\partial Q}\right|_{Q=Q^*_{int}} = p'(Q^*_{int})Q^*_{int} + p(Q^*_{int}) = 0,$$

which implies that $Q^*_{out} = Q^*_{int}$*.*

Next, suppose that $Q^*_{int} < \hat{Q}_{out}$*. By (A.16), since* $\nu(\cdot)$ *is increasing,*

$$\nu\left(\frac{C(Q^*_{int})}{\mu F_P}\right) < 1.$$

Hence, by (A.5), the first derivative,

$$\left.\frac{\partial \Pi_{out}}{\partial Q}\right|_{Q=Q^*_{int}} = p'(Q^*_{int})Q^*_{int} + p(Q^*_{int}) - [1 - x_{out}(Q^*_{int})]C'(Q^*_{int})$$
$$- [1-\alpha]C(Q^*_{int})\nu'\left(\frac{C(Q^*_{int})}{\mu F_P}\right)\frac{C'(Q^*_{int})F_B}{\mu F_P^2} < 0,$$

which, by the assumption that Π_{out} *is single-peaked in* Q_{out}*, implies that* $Q^*_{out} < Q^*_{int}$*.*

Thus, we conclude that $Q^*_{out} \leq Q^*_{int}$*, for all* α*. Then setting* $\alpha_Q = 1$ *completes the proof of part (ii).*

Part (i)

The proof is similar to the proof of (ii). If $x^*_{int} = 1$*, then setting* $\alpha_x = 1$*, the result is trivial. We now consider the case of* $x^*_{int} < 1$*.*

(a) Under vertical integration, the infringement, x^*_{int}*, does not vary with* α*, which is immediate from (1).*

(b) Under outsourcing, the infringement, x^*_{out}*, increases in* α*, as proved in Proposition 2.*

(c) If $\alpha = 1$*, then* $x^*_{int} < x^*_{out}$*. Suppose that* $\alpha = 1$*. By (ii)(d) above,* $Q^*_{out} > Q^*_{int}$*. Substituting in (A.11) and (A.12),*

$$x^*_{out} = \nu\left(\frac{C(Q^*_{out})}{\mu F_P}\right) > \nu\left(\frac{C(Q^*_{out})}{\mu[F_P + F_B]}\right) > \nu\left(\frac{C(Q^*_{int})}{\mu[F_P + F_B]}\right) = x^*_{int},$$

since $\nu(\cdot)$ *is increasing.*

(d) If $\alpha = 0$, then two cases are possible. One is where $x^*_{out} \geq x^*_{int}$ for all α, in which case, set $\alpha_x = 0$, and the result is trivial. The other case is where $x^*_{out} < x^*_{int}$. By (b) and (c) above, since x^*_{out} is continuous in α, there exists $\alpha_x > 0$ such that $x^*_{out} \leq x^*_{int}$ if and only if $\alpha \leq \alpha_x$.

Part (iii)

If $x^*_{int} = 1$, then set $\alpha_x = \alpha_Q = 1$, and the result is trivial. We now consider the case of $x^*_{int} < 1$. By definition, at $\alpha = \alpha_Q$, $Q^*_{int} = Q^*_{out} = Z$ say. Under vertical integration, by (1),

$$\frac{dm}{dx} = \frac{C(Q)}{\mu[F_B + F_P]}.$$

Using (7), we have

$$\left.\frac{dm}{dx}\right|_{x=x^*_{int}} = \frac{C(Q)}{\mu[F_P + F_B]} < \frac{C(Q)}{\mu F_P} = \left.\frac{dm}{dx}\right|_{x=x^*_{out}}.$$

Given that $m'(x) > 0$, and $m''(x) > 0$, we can conclude that $x^*_{int} < x^*_{out}$.

By definition, at $\alpha = \alpha_x$, the infringement, $x^*_{int} = x^*_{out}$. Suppose otherwise that $\alpha_x > \alpha_Q$. At $\alpha = \alpha_x$, $Q^*_{int} < Q^*_{out}$. Then, the proof of (i)(d) above shows that $x^*_{out} > x^*_{int}$, which is a contradiction. Thus, we conclude that $\alpha_Q > \alpha_x$. □

Proof of Lemma 2. Recall (15), welfare under outsourcing

$$W_{out} = B(Q) - C(Q) - \frac{h-1}{h} H(Q, x(Q)). \tag{15}$$

Now $B''(Q) < 0$, $C''(Q) > 0$, and, by assumption, $H(Q, x(Q))$ is non-concave. Hence, W_{out} is strictly concave in Q.

Suppose that at $\alpha = \alpha^*$,

$$\left.\frac{dW_{out}}{d\alpha}\right|_{\alpha=\alpha^*} = \frac{dW_{out}}{dQ}\frac{dQ}{d\alpha} = 0.$$

By Proposition 2, $dQ/d\alpha > 0$, hence the above implies that $dW_{out}/dQ = 0$. Further, for $\alpha < \alpha^*$, the brand will choose $Q < Q(\alpha^*)$, and since W_{out} is strictly concave in Q, the welfare will be lower. Likewise, for $\alpha > \alpha^*$, the brand will choose $Q > Q(\alpha^*)$, and since W_{out} is strictly concave in Q, the welfare will be lower. Thus, welfare is single-peaked in α.

Now, suppose that at $\alpha = 0$, $\left.\frac{dW_{out}}{dQ}\right|_{Q=Q(0)} > 0$. Then

$$\left.\frac{dW_{out}}{d\alpha}\right|_{\alpha=0} = \frac{dW_{out}}{dQ}\frac{dQ}{d\alpha} > 0,$$

since, by Proposition 2, $dQ/d\alpha > 0$. Further, suppose that at $\alpha = 1$, $\left.\frac{dW_{out}}{dQ}\right|_{Q=Q(1)} < 0$. Then

$$\left.\frac{dW_{out}}{d\alpha}\right|_{\alpha=1} = \frac{dW_{out}}{dQ}\frac{dQ}{d\alpha} < 0.$$

Accordingly, since welfare is single-peaked in α, there exists a unique $\alpha^* \in (0,\ 1)$ that maximizes welfare. □

Proof of Lemma 4. The social welfare function, under outsourcing, is

$$W(\alpha) = B(Q) - [[1-x] + hx]C(Q).$$

We then have

$$\begin{aligned}\frac{dW(\alpha)}{d\alpha} &= \{B'(Q) - [[1-x] + hx]C'(Q)\}\frac{dQ}{d\alpha} - [h-1]C(Q)\frac{dx}{d\alpha}\\ &= \{\{[a-bQ] - c[[1-x] + hx]\}2\mu F_P - [h-1]cQ\cdot c\}\frac{dk(\alpha)}{d\alpha}.\end{aligned}$$

The optimal level of avoidance, α^* under outsourcing is then determined by the first-order condition,

$$\{[a-bQ] - c[[1-x] + hx]\}2\mu F_P - [h-1]cQ\cdot c = 0.$$

Substituting from (26), $x^*_{out} = \frac{cQ}{2\mu F_P}$ in the above,

$$[[a-bQ] - c]2\mu F_P - c[h-1]cQ - [h-1]cQ\cdot c = 0,$$

or

$$[[a-bQ] - c]2\mu F_P - 2c[h-1]cQ = 0.$$

This implies that the production under outsourcing at the optimal level of avoidance,

$$Q_{out}(\alpha^*) = \frac{[a-c]\mu F_P}{b\mu F_P + c^2[h-1]}. \tag{A.17}$$

Eq. (26) states the equilibrium production under outsourcing, $Q^*_{out}(\alpha)$. To solve for the optimal level of avoidance, α^*, equate $Q_{out}(\alpha^*) = Q^*_{out}(\alpha)$,

$$\frac{[a-c]\mu F_P}{b\cdot\mu F_P + c^2[h-1]} = \frac{2\mu F_P[a-c]}{4b\mu F_P - \left[1-[1-\alpha]\frac{F_B}{F_P}\right]c^2}.$$

Simplifying,

$$2c^2[h-1] = 2b\mu F_P - \left[1-[1-\alpha]\frac{F_B}{F_P}\right]c^2,$$

or

$$\left[1-[1-\alpha]\frac{F_B}{F_P}\right]=\frac{2b\mu F_P-2c^2[h-1]}{c^2},$$

which yields

$$\alpha^*=1-\frac{c^2h-2b\mu F_P}{c^2}\frac{F_P}{F_B}. \tag{A.18}$$

Comparing (A.18) with (27), we have $\alpha^*\leq\tilde{\alpha}$ if and only if

$$1-\frac{c^2h-2b\mu F_P}{c^2}\frac{F_P}{F_B}\leq\frac{F_B}{F_P+F_B},$$

which simplifies to

$$b\leq\frac{c^2[h[F_P+F_B]-F_B]}{2\mu F_P[F_P+F_B]}.$$

□

Proof of Proposition 6. We now investigate whether the welfare under outsourcing is higher than that under integration when $\alpha^*>\tilde{\alpha}$. Generally, social welfare is

$$W=B(Q)-[1-x]cQ-hxcQ.$$

Under vertical integration,

$$\begin{aligned}W_{int}&=\int_0^{Q^*_{int}}[a-bQ]dQ-cQ^*_{int}-[h-1]x^*_{int}cQ^*_{int}\\&=aQ^*_{int}-\frac{b}{2}Q^{*2}_{int}-cQ^*_{int}-[h-1]x^*_{int}cQ^*_{int}.\end{aligned}$$

Substituting for x^*_{int} and Q^*_{int} from (25),

$$\begin{aligned}W_{int}=&[a-c]\frac{2\mu[F_P+F_B][a-c]}{4b\mu[F_P+F_B]-c^2}-\frac{b}{2}\left[\frac{2\mu[F_P+F_B][a-c]}{4b\mu[F_P+F_B]-c^2}\right]^2\\&-[h-1]\frac{c[a-c]}{4b\mu[F_P+F_B]-c^2}\frac{2\mu[F_P+F_B][a-c]}{4b\mu[F_P+F_B]-c^2}\\=&\frac{[a-c]^2 2\mu[F_P+F_B]}{[4b\mu[F_P+F_B]-c^2]^2}\left[3b\mu[F_P+F_B]-hc^2\right].\end{aligned}$$

Under outsourcing, social welfare

$$W^*_{out}=\int_0^{Q_{out}(\alpha^*)}[a-bQ]dQ-cQ_{out}(\alpha^*)-[h-1]cx_{out}(\alpha^*)Q_{out}(\alpha^*).$$

By (23) and (A.17), the production under outsourcing at the optimal extent of avoidance,

$$x_{out}(\alpha^*) = \frac{c}{2\mu F_P} Q_{out}(\alpha^*) = \frac{c}{2\mu F_P} \frac{[a-c]\mu F_P}{b\mu F_P + c^2[h-1]} = \frac{1}{2} \frac{c[a-c]}{b\mu F_P + c^2[h-1]}.$$

Substituting above, welfare is

$$\begin{aligned} W^*_{out} &= \int_0^{Q_{out}(\alpha^*)} [a - bQ]dQ - cQ_{out}(\alpha^*) - [h-1]cx_{out}(\alpha^*)Q_{out}(\alpha^*) \\ &= [a-c]\frac{[a-c]\mu F_P}{b\mu F_P + c^2[h-1]} - \frac{b}{2}\left[\frac{[a-c]\mu F_P}{b\mu F_P + c^2[h-1]}\right]^2 \\ &\quad -[h-1]\frac{1}{2}\frac{c[a-c]}{b\mu F_P + c^2[h-1]}c\left[\frac{[a-c]\mu F_P}{b\mu F_P + c^2[h-1]}\right] \\ &= \frac{[a-c]^2\mu F_P}{[b\mu F_P + c^2[h-1]]^2}\left\{[b\mu F_P + c^2[h-1]] - \frac{b}{2}\mu F_P - \frac{1}{2}[h-1]c^2\right\} \\ &= \frac{[a-c]^2\mu F_P}{2[b\mu F_P + c^2[h-1]]}. \end{aligned}$$

Accordingly, $W_{int} \le W^*_{out}$ if and only if

$$\frac{[F_P + F_B]}{[4b\mu[F_P + F_B] - c^2]^2}\left[3b\mu[F_P + F_B] - hc^2\right] - \frac{F_P}{4[b\mu F_P + c^2[h-1]]} \le 0.$$

Recall condition, (28), $b > \frac{c^2[h[F_P+F_B]-F_B]}{2\mu F_P[F_P+F_B]}$. Hence $W_{int} \le W^*_{out}$ if and only if

$$\begin{aligned} G(b,h) \equiv\ & 4[F_P + F_B]\left[3b\mu[F_P + F_B] - hc^2\right]\left[b\mu F_P + c^2[h-1]\right] \\ & -F_P\left[4b\mu[F_P + F_B] - c^2\right]^2 \le 0. \end{aligned}$$

To simplify analysis, denote $F_B = dF_P$ where $d > 0$. Then,

$$G(b,h) = 4[1+d]F_P\left[3b\mu[1+d]F_P - hc^2\right]\left[b\mu F_P + c^2[h-1]\right] - F_P\left[4b\mu[1+d]F_P - c^2\right]^2.$$

Differentiating with respect to b,

$$\begin{aligned} \frac{dG(b,h)}{db} &= 4[1+d]F_P\left\{3\mu[1+d]F_P\left[b\mu F_P + c^2[h-1]\right] + \left[3b\mu[1+d]F_P - hc^2\right]\mu F_P\right\} \\ &\quad -8\mu[1+d]F_P^2\left[4b\mu[1+d]F_P - c^2\right] \\ &= 4[1+d]\mu F_P^2\{-2b\mu[1+d]F_P + [3[1+d][h-1] - [h-2]]c^2\}. \end{aligned}$$

Denote

$$\tilde{b} \equiv \frac{3[F_P + F_B][h-1] - F_P[h-2]}{2\mu[F_P + F_B]F_P} = \frac{3[1+d][h-1] - [h-2]}{2\mu[1+d]F_P}c^2.$$

Recall condition, (28),

$$b > \frac{c^2[h[F_P + F_B] - F_B]}{2\mu F_P[F_P + F_B]} = \frac{[1+d][h-1]+1}{2\mu[1+d]F_P}c^2.$$

It is easy to show that for any $d > 0$,

$$\tilde{b} > \frac{[1+d][h-1]+1}{2\mu[1+d]F_P}c^2.$$

Thus, if $b > \tilde{b}$, then $dG(b, h)/db < 0$, i.e., $G(b, h)$ strictly decreases with b. Accordingly, we infer that if b is sufficiently large in the sense that $G(b, h) < 0$, then $W_{int} \le W^*_{out}$. □

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