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The beauty of "bigness": On optimal design of multi-winner contests

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Abstract

This paper examines the variation in total effort expended by participants when prizes are awarded in a grand contest as opposed to a number of subcontests. When contestants are homogeneous, under a mild and plausible condition (regular contest technology), a grand contest generates more effort than any set of subcontests. When no restrictions are placed on the contest technology, the results further demonstrate an "increasing-return-to-scale" property such that each individual responds to a proportional increase in the number of contestants and the number of each prize by increasing individual effort. Therefore, when a collection of identical subcontests forms a grand contest, the total effort always increases and the grand contest leads to a higher rent-dissipation rate. Our results apply to a wide variety of competitive activities, such as high-profile sports (e.g., diving and gymnastics in the Olympic Games), the internal labor market and the "quota" system for public resource allocation.

JEL classification: C72; D72

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1. Introduction

A contest is a situation under which economic agents expend costly and non-refundable resources in order to win a limited number of prizes. Many competitive activities in everyday life can be viewed as contests or tournaments, including the competition for career promotions within firms and political elections. Because of their ubiquity, contests have attracted a great deal of attention from economic scholars, and a huge body of literature has developed that explores the strategic behaviors of rent seekers across a variety of contexts.

It has been widely acknowledged in this literature that the rules of a contest critically influence the contestants' incentive to exert costly effort. Consequently, a contest organizer must strategically design the organizing rules of the contest such that they best serve his/her interests. As argued by Gradstein (1998), "…contest structures are the outcome of careful design processes, implemented with the view of attaining a variety of objectives, one of which is the maximization of the effort expended by the contenders." Much research has been conducted on the design of contests that seek to maximize the contestants' effort. Conversely, there are a number of situations in which such rent-

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seeking expenditures are not desirable, and in fact constitute social waste. In such situations, the question is reversed, and it becomes important to organize contest rules to reduce rent-dissipation.

Another common assumption made in the contest literature is that a participant competes against all others for a single prize ("winner-take-all"). Again, these assumptions do not hold true for many actual competitive events. For instance, the government telecommunication regulator may issue several operating licenses[1]; more than one statesman is elected to parliament in an election; a firm sets aside a number of bonus packages to reward top performing workers; and employees may compete to fill multiple vacancies higher up the organizational hierarchy. In all these examples, multiple prizes are awarded, but each contestant may receive no more than one. While the design of *winner-take-all* contests has been thoroughly investigated, only a handful of scholarly papers concern themselves with the optimal structure of multi-winner contests.[2]

The outcomes of a variety of competitive events (such as art performances) can be influenced by autonomous effort and multiple other factors. Specifically, this paper focuses on contests in which "noise" factors, such as randomness of performance and imperfections in performance measurement and evaluation, affect the decision on the winners. In the economic literature, such contests are conventionally termed "imperfectly discriminatory contests."[3] We consider one particular question pertaining to multi-winner imperfectly discriminatory contest design, which is: how can a given set of heterogeneous prizes be distributed to a fixed pool of contestants to maximize the overall effort? Specifically, the purpose of this paper is to study how the total effort supplied by contestants changes when a "**grand**" contest is allowed to be split into a set of parallel "**subcontests**." In a "grand contest," which represents the simplest organizing rule, each contestant competes against all others, and the whole set of prizes is open to all the contestants in the pool. The best performers are awarded prizes according to their rank within the pool.[4] By way of contrast, the contest organizer may strategically split the grand contest into a set of "subcontests," such that a contestant in each of the subgroups competes for a smaller set of prizes. Whether a contestant wins a prize depends on his/her rank within the group. The "split" or "divisionalized" contest is just as common as the grand contest. An example of such a contest, as Moldovanu and Sela (2006) point out, is the competition for promotions at lower ranks of an organizational hierarchy. These can be seen as a set of regional or divisional subcontests.

In our paper, the contest organizer is endowed with the full flexibility to divide the pool of contestants into any feasible set of subgroups, and to match a subset of the prizes to each group. For instance, a firm could either open all bonus packages to competing workers regardless of departments or divisions (grand contest), or split the workforce and dedicate a subset of these rewards to each working unit (split contests). The question naturally asked in such a scenario is: which organizing rule better motivates contestants? More generally, we can ask: which organizing institution demands more effort from its contestants—the "grand" or the "split" contest? Does the "grand" contest necessarily dominate "split" contests, or does there exist a generalizable optimal division rule that matches each subgroup of contestants to a subset of prizes?

To address these questions, we adopt the multiple-winner nested contest model as suggested by Clark and Riis (1996, 1998a). This framework is built upon ratio-form contest success functions, and it allows a block of prizes to be awarded to winners. It therefore mimics a setting of multiple-prize contests with sufficient "noise" (imperfectly discriminatory contests).[5] We establish a mild sufficient condition (**regular** contest technology defined in Section 3),

[1] See Yates and Heckelman (2001).

[2] Multi-winner contest models that involve noisy winning rules have been studied by Clark and Riis (1996, 1998a, 1998b), Amegashie (2000), Yates and Heckelman (2001), Szymanski and Valletti (2005), and Fu and Lu (2007a, 2007b). Multiple-prize competitions in which the winners are determined by their total effort or output ranking have been investigated by Glazer and Hassin (1988), Barut and Kovenock (1998), Moldovanu and Sela (2006), and Moldovanu et al. (2007).

[3] "Perfectly discriminatory contest" (all-pay auction) represents an alternative contest model in which the prize goes to the individual who puts in the greatest effort.

[4] For over fifty years, the shortlisting rules applied in the Olympic Games and World Championships for diving and gymnastics exemplify the use of such grand contests. Instead of dividing the pool of qualifying athletes in each stage into different groups, each athlete is required to compete against all others in a common pool, with a fixed number of top performers advancing to the next stage.

[5] Fu and Lu (2007b) show that this multiple-winner nested contest model is equivalent to a multi-winner noisy ranking model. Details are provided in Section 4.2.

under which the grand contest dominates all split contests in terms of the total effort induced, regardless of the prevailing division rule.[6] This also means that the grand contest leads to a higher level of overall rent-dissipation.

To further explore the logic underlying this result, we develop an "increasing-return-to-scale" property that holds under more general settings (without the **regularity** constraint on the contest technology). In particular, consider two seemingly equivalent contests with one hundred contestants and ten prizes: the first consists of ten identical subcontests, each awarding a single prize to ten potential recipients; the other is a grand competition that awards ten prizes to a pool of a hundred potential recipients. They seem to be identical, but how and why do they differ from each other? The *increasing-return-to-scale* property stipulates that when the number of contestants and the number of prizes at each rank get scaled up, each contestant increases the level of equilibrium effort, and the total effort of the contest increases more than proportionally. This property uncovers the competing forces that lead to our observation: the positive impact of an increase in the prize purse on a contestant's incentives to supply effort always more than offsets the negative impact of a complementary increase in the number of contestants. Thus, we argue that the "scale" or the "size" of a contest indeed matters, and that a "bigger" contest elicits more effort.

The remainder of the paper proceeds as follows. We set up the model and present general equilibrium solutions in Section 2. Section 3 provides a detailed analysis of our research questions. Section 4 discusses the implications, intuition and limitations of our results. The paper concludes at Section 5.

2. Preliminaries: a multi-winner contest

Let $C(N, K, \mathbf{V})$ denote a multi-winner contest with $N \geqslant 2$ identical risk-neutral contestants competing for $K \in \{1, 2, \dots, N\}$ prizes, where the vector $\mathbf{V}$ represents the ordered set of prizes $\mathbf{V} \triangleq (V_1, \dots, V_K)$, with $V_1 \geqslant \cdots \geqslant V_K > 0$. The N contestants simultaneously choose their effort outlays in their attempt to compete for the K prizes, and each of them is eligible for a maximum of one prize.

In the case where $K = 1$, a single winner receives the prize V. We consider a ratio-form contest success function, which is axiomatized by Skaperdas (1996). The probability of a contestant i winning the prize is written as

$$P_i(X) = \frac{f(x_i)}{\sum_{j=1}^{N} f(x_j)}, \tag{1}$$

given the contestants' effort profile $X = (x_1, x_2, \dots, x_N)$. Wärneryd (2001) names $f(\cdot)$ the *impact function*, which indicates a contestants' production technology in the contest. We assume that the function $f(\cdot)$ is strictly increasing, twice differentiable and weakly log-concave. A contestant i chooses his/her effort x_i to maximize the expected payoff

$$\pi_i = P_i(X)V - x_i,$$

given other contestants' effort $X_{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_N)$.

In the case where $K > 1$, we adopt a multi-winner nested contest as suggested by Clark and Riis (1996, 1998a), but assume a general impact function $f(x)$ as defined above. We follow the tradition of Clark and Riis (1998a) in letting this contest model conveniently mimic a sequential lottery process. Contestants simultaneously submit their effort entries X. A block of K prizes are to be given away. The K recipients are then selected by K consecutive draws, as each contestant is allowed to receive no more than one prize. Thus, once a contestant is selected to win a prize, he/she is immediately removed from the pool of candidates up for the next draw. Ω_k denotes the set of remaining contestants for the kth draw, with $k \leqslant K$. The conditional probability that a contestant $i \in \Omega_k$ wins the kth prize is given by

$$p_i(x_i, X_{-i}; \Omega_k) = \frac{f(x_i)}{\sum_{j \in \Omega_k} f(x_j)}. \tag{2}$$

We should emphasize that the aforementioned sequential lottery process is a convenient "visualization" of a simultaneous selection mechanism. The model (2) does not necessarily require the implementation of a sequential prize allocation mechanism. Instead, Fu and Lu (2007b) establish that this model can be underpinned by a (simultaneous) noisy-ranking tournament. More details will be provided in Section 4.2 when we discuss the intuition of our results in light of this hidden ranking system.

[6] We assume that the more valuable prizes have priority in the allocation in each contest (grand or split contest). Such a method of prize allocation was found to be optimal for inducing effort by Clark and Riis (1998a) and Fu and Lu (2007a).

Let the K ordered prizes be allocated to the K winners by the order of the draws such that a winner selected in the kth draw receives V_k. $P_{i,k}(x_i, X_{-i})$ denotes the probability that contestant i is selected in the kth draw. Note that $P_{i,k}(x_i, X_{-i}) = \sum_{\forall \Omega_k}[\Pr(\Omega_k) I(i \in \Omega_k) p_i(x_i, X_{-i}; \Omega_k)]$, where $\Pr(\Omega_k)$ is the probability that the remaining contestants up for the kth draw are Ω_k, and $I(i \in \Omega_k)$ is an index function. $I(i \in \Omega_k) = 1$ if $i \in \Omega_k$, and $I(i \in \Omega_k) = 0$ if $i \notin \Omega_k$. Given that the effort expended by other contestants is X_{-i}, a contestant i chooses his/her effort x_i to maximize

$$\pi_i = \sum_{k=1}^{K} [V_k P_{i,k}(x_i, X_{-i})] - x_i, \tag{3}$$

given other contestants' effort X_{-i}.

Denote by x the equilibrium effort each contestant exerts in a symmetric Nash equilibrium of the contest $C(N, K, \mathbf{V})$, and denote by $E \triangleq Nx$ the total effort the N contestants make in the contest. With a symmetric effort x, we have

$$\frac{\partial P_{i,k}(x, \ldots, x)}{\partial x_i} = \frac{(1 - \sum_{g=0}^{k-1} \frac{1}{N-g})}{N} \frac{f'(x)}{f(x)}, \quad 1 \leqslant k \leqslant K.$$

An interior symmetric equilibrium x requires

$$\sum_{k=1}^{K} \frac{\partial [V_k P_{i,k}(x, \ldots, x)]}{\partial x_i} = \sum_{k=1}^{K} \left[\frac{V_k(1 - \sum_{g=0}^{k-1} \frac{1}{N-g})}{N} \frac{f'(x)}{f(x)} \right] = 1$$

$$\Leftrightarrow \frac{f(x)}{f'(x)} = \sum_{k=1}^{K} \left[\frac{V_k(1 - \sum_{g=0}^{k-1} \frac{1}{N-g})}{N} \right].$$

Define $H(x) \equiv \frac{f(x)}{f'(x)}$. Because $f(x)$ is log-concave, we have $H'(x) > 0$. We thus establish the first order condition for the symmetric equilibrium

$$H(x) - \frac{1}{N} \sum_{k=1}^{K} \left[V_k \left(1 - \sum_{g=0}^{k-1} \frac{1}{N-g} \right) \right] = 0. \tag{4}$$

The solution of (4) constitutes a symmetric pure-strategy equilibrium if and only if contestants enjoy a nonnegative expected payoff, i.e., $\pi = (\sum_{k=1}^{K} V_k) - NH^{-1}(\frac{1}{N} \sum_{k=1}^{K} [V_k(1 - \sum_{g=0}^{k-1} \frac{1}{N-g})]) \geqslant 0$. In the following analysis, we assume that this sufficient condition holds. Note that when the impact function takes the form $f(x) = ax^r, a > 0$, a sufficient condition for $\pi \geqslant 0$ is $r \in (0, 1 + \frac{1}{N-1}]$.

Proposition 1. *In the symmetric pure-strategy Nash equilibrium of a N-person, K-prize contest $C(N, K, \mathbf{V})$, each contestant makes an effort*

$$x = H^{-1} \left(\frac{1}{N} \sum_{k=1}^{K} \left[V_k \left(1 - \sum_{g=0}^{k-1} \frac{1}{N-g} \right) \right] \right), \tag{5}$$

where $H^{-1}(\cdot)$ is the inverse function of $H(\cdot)$. The total effort the N contestants make in the contest is then given by

$$E = NH^{-1} \left(\frac{1}{N} \sum_{k=1}^{K} \left[V_k \left(1 - \sum_{g=0}^{k-1} \frac{1}{N-g} \right) \right] \right). \tag{6}$$

3. The analysis: dominance of grand contest

We name the contest $C(N, K, \mathbf{V})$ the **grand contest**, where the N contestants compete against all others for the K prizes. We denote the grand contest by C_g. To exclude uninteresting cases, we require the grand contest to be a "nontrivial" one, which induces nonzero effort.

Definition 1. A contest $C(N, K, \mathbf{V})$ is nontrivial if and only if $\max\{N - K, V_1 - V_K\} > 0$.

This restriction is very intuitive. The only case where the contest induces zero effort is that $V_k \equiv V, \forall k \in \{1, 2, \ldots, K\}$ and $K \geqslant N$, i.e., the number of identical prizes is greater than or equal to the number of contestants.

We allow the contest organizer to have the flexibility to split the grand contest C_g into $M \geqslant 1$ subcontests $C_m \triangleq C(N_m, K_m, \mathbf{V}^m)$, $m = 1, \ldots, M$, with $N = \sum_{m=1}^{M} N_m$, $K = \sum_{m=1}^{M} K_m$ and $N_m \geqslant K_m \geqslant 1, \forall m \in \{1, \ldots, M\}$. In subcontest C_m, a contestant competes against $N_m - 1$ opponents for the K_m ranked prizes $\mathbf{V}^m \triangleq (V_1^m, V_2^m, \ldots, V_{K_m}^m)$ with $V_1^m \geqslant \cdots \geqslant V_{K_m}^m$. Every element of $\mathbf{V}^m$ must be in $\mathbf{V}$, and every element of $\mathbf{V}$ must go to one and only one $\mathbf{V}^m$. When $M = 1$, the contest organizer retains the grand contest. We assume that in each contest (either a grand contest or a subcontest), the prizes with higher values are allocated with priority. According to Clark and Riis (1998a) and Fu and Lu (2007a), this is a necessary condition for effort maximization.

We denote by E_m the total equilibrium effort the N_m contestants exert in each subcontest C_m, with $m \in \{1, \ldots, M\}$. By Proposition 1, we obtain[7]

$$E_m = N_m H^{-1}\left(\frac{1}{N_m}\left[\sum_{k=1}^{K_m} V_k^m - \sum_{k=1}^{K_m}\left(V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m - g}\right)\right]\right). \tag{7}$$

In particular, the total equilibrium effort E_g from the grand contest C_g is given by

$$E_g = N H^{-1}\left(\frac{1}{N}\left[\sum_{k=1}^{K} V_k - \sum_{k=1}^{K}\left(V_k \sum_{g=0}^{k-1} \frac{1}{N - g}\right)\right]\right). \tag{8}$$

Thus, to maximize the total effort exerted by the N contestants (given the K prizes available), the contest organizer would have to solve the following constrained maximization problem:

$$\max_{\{(\mathbf{V}^m, K_m, N_m), m \in \{1, \ldots, M\}\}} E \triangleq \sum_{m=1}^{M} E_m, \tag{9}$$

s.t.:

$$N_m \geqslant K_m \geqslant 1, \quad \forall m \in \{1, \ldots, M\}, \tag{10}$$

$$\sum_{m=1}^{M} N_m = N, \qquad \sum_{m=1}^{M} K_m = K, \quad M \geqslant 1. \tag{11}$$

Consider a simple example with a grand contest $C_g \triangleq C(10, 4, \mathbf{V})$ with $\mathbf{V} = (1, 1, 1, 1)$ and a linear impact function $f(x) = x$. The grand contest C_g can be split into two subcontests $C_1 \triangleq C(7, 3, \mathbf{V}^1)$ and $C_2 \triangleq C(3, 1, \mathbf{V}^2)$, where $\mathbf{V}^1 = (1, 1, 1)$ and $\mathbf{V}^2 = 1$. From Proposition 1, we have the individual effort given by $x_g = 0.287$, $x_1 = 0.291$ and $x_2 = 0.222$, respectively, in these three contests. Once the grand contest is split in such a way into C_1 and C_2, contestants allocated to C_1 increase their individual effort, while contestants in C_2 reduce their individual effort. Nevertheless, when it comes to the total effort induced, we see that $\sum_{m=1}^{2} E_m = 2.704 < E_g = 2.87$, which indicates that the grand contest C_g dominates the set of two split contests.

However, when the contest organizer has full flexibility in splitting the pool of contestants, and in matching a subset of the prizes to each subgroup, a contest can be structured and split in numerous ways. An interesting question arises: is the dominance of the grand contest we observed from this example merely an artifact of this particular setting, or does it stem from a regularity that applies in a general context? Also, if such a regularity does exist, to what extent does it hold? In addition, while all prizes in the above example carry the same value, our theoretical setting allows for a heterogeneous prize structure. When the prizes bear uneven values, the contest organizer is endowed with additional flexibility, which adds tremendous additional complexity to the optimization problem.

[7] We assume the existence of a symmetric pure-strategy equilibrium for every possible subcontest. When $f(x) = x^r$, this is guaranteed if $r \in (0, 1 + \frac{1}{N-1}]$.

Definition 2. A contest is **regular** if $H^{-1}(\cdot)$ is weakly concave, where $H(\cdot) \equiv \frac{f(\cdot)}{f'(\cdot)}$.

To answer the questions we raised above, we consider the class of contest technologies that satisfy the **regularity** condition.[8]

To compare $\sum_{m=1}^{M} E_m$ with E_g, we first present the following key result, which summarizes an interesting property of the sequence $\{V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m-g}\}_{k=1}^{K_m}$, $m \in \{1, \dots, M\}$.

Proposition 2. *Suppose $N \geqslant K \geqslant 2$, $N_m \geqslant K_m \geqslant 1$, where $1 \leqslant m \leqslant M$, $M \geqslant 2$ and $\sum_{m=1}^{M} N_m = N$ and $\sum_{m=1}^{M} K_m = K$. We have*

$$\sum_{m=1}^{M}\sum_{k=1}^{K_m}\left(V_k^m \sum_{g=0}^{k-1}\frac{1}{N_m-g}\right) > \sum_{k=1}^{K}\left(V_k \sum_{g=0}^{k-1}\frac{1}{N-g}\right). \tag{12}$$

Please refer to the appendix for the proof of Proposition 2. The main idea of the proof is as follows. The sum of each K_m-term sequence $\{V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m-g}\}_{k=1}^{K_m}$ can be written as the sum of another K_m-term sequence $\theta^m \triangleq \{\theta_k^m\}_{k=1}^{K_m}$, i.e., $\sum_{k=1}^{K_m}(V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m-g}) = \sum_{k=1}^{K_m} \theta_k^m$, while the sum of the K-term sequence $\{V_k \sum_{g=0}^{k-1} \frac{1}{N-g}\}_{k=1}^{K}$ can also be written as the sum of another K-term sequence $\beta \triangleq \{\beta_k\}_{k=1}^{K}$, i.e., $\sum_{k=1}^{K}(V_k \sum_{g=0}^{k-1} \frac{1}{N-g}) = \sum_{k=1}^{K} \beta_k$.[9] Define S to be the set composed of all the K elements in sequence β, and $\widetilde{S}$ to be the set composed of all the $\sum_{m=1}^{M} K_m$ $(= K)$ elements of $\bigcup_{m=1}^{M} \theta^m$. In the proof, we constructively establish a one-to-one correspondence between S and $\widetilde{S}$, such that every element in S is smaller than or equal to its counterpart in $\widetilde{S}$. The economic implication of this result will be elaborated in Section 4.2. Using the properties of Proposition 2, we obtain the following result.

Theorem 1. *With a **regular** contest technology, the grand contest induces strictly more total effort than any set of subcontests, i.e. $E_g > \sum_{m=1}^{M} E_m$, $\forall M \geqslant 2$.*

Proof. The proof proceeds with three steps.

Step 1. We claim $\sum_{m=1}^{M} E_m \leqslant NH^{-1}(\{\frac{1}{N}[\sum_{m=1}^{M}\sum_{k=1}^{K_m} V_k^m - \sum_{m=1}^{M}\sum_{k=1}^{K_m}(V_k^m \sum_{g=0}^{k-1}\frac{1}{N_m-g})]\})$.

From (7), we rewrite $\sum_{m=1}^{M} E_m$ as

$$\sum_{m=1}^{M} E_m = N\sum_{m=1}^{M}\left\{\frac{N_m}{N}H^{-1}\left(\frac{1}{N_m}\left[\sum_{k=1}^{K_m}V_k^m - \sum_{k=1}^{K_m}\left(V_k^m\sum_{g=0}^{k-1}\frac{1}{N_m-g}\right)\right]\right)\right\}.$$

Because $H^{-1}(\cdot)$ is weakly concave, by Jensen's inequality, we establish

$$\sum_{m=1}^{M}\left\{\frac{N_m}{N}H^{-1}\left(\frac{1}{N_m}\left[\sum_{k=1}^{K_m}V_k^m - \sum_{k=1}^{K_m}\left(V_k^m\sum_{g=0}^{k-1}\frac{1}{N_m-g}\right)\right]\right)\right\}$$
$$\leqslant H^{-1}\left(\left\{\frac{1}{N}\left[\sum_{m=1}^{M}\sum_{k=1}^{K_m}V_k^m - \sum_{m=1}^{M}\sum_{k=1}^{K_m}\left(V_k^m\sum_{g=0}^{k-1}\frac{1}{N_m-g}\right)\right]\right\}\right).$$

Thus, we obtain

$$\sum_{m=1}^{M} E_m \leqslant NH^{-1}\left(\left\{\frac{1}{N}\left[\sum_{m=1}^{M}\sum_{k=1}^{K_m}V_k^m - \sum_{m=1}^{M}\sum_{k=1}^{K_m}\left(V_k^m\sum_{g=0}^{k-1}\frac{1}{N_m-g}\right)\right]\right\}\right). \tag{13}$$

[8] This property is not necessary for the results in Section 4.1.

[9] Please refer to the proof in Appendix A or (19) and (20) for detailed definitions of θ^m and β.

Step 2. We claim $\frac{1}{N}[\sum_{m=1}^{M}\sum_{k=1}^{K_m}V_k^m-\sum_{m=1}^{M}\sum_{k=1}^{K_m}(V_k^m\sum_{g=0}^{k-1}\frac{1}{N_m-g})]<\frac{1}{N}[\sum_{k=1}^{K}V_k-\sum_{k=1}^{K}(V_k\sum_{g=0}^{k-1}\frac{1}{N-g})]$. This inequality directly follows from Proposition 2 and the fact $\sum_{m=1}^{M}\sum_{k=1}^{K_m}V_k^m=\sum_{k=1}^{K}V_k$.

Step 3. $\sum_{m=1}^{M}E_m<E_g$.
Because $H^{-1}(\cdot)$ is strictly increasing, we have

$$\sum_{m=1}^{M}E_m\leqslant NH^{-1}\left(\frac{1}{N}\left[\sum_{m=1}^{M}\sum_{k=1}^{K_m}V_k^m-\sum_{m=1}^{M}\sum_{k=1}^{K_m}\left(V_k^m\sum_{g=0}^{k-1}\frac{1}{N_m-g}\right)\right]\right)$$
$$<NH^{-1}\left(\frac{1}{N}\left[\sum_{k=1}^{K}V_k-\sum_{k=1}^{K}\left(V_k\sum_{g=0}^{k-1}\frac{1}{N-g}\right)\right]\right)=E_g. \qquad \square \tag{14}$$

Theorem 1 establishes a fairly mild sufficient condition (**regular** contest technology) for the dominance of the grand contest: A grand contest always elicits strictly more effort than the split "smaller" subcontests, no matter how the pool of contestants are divided or how the set of prizes are split across subcontests. Alternatively, simply merging "smaller" contests always creates more competition and induces contestants to exert more effort, no matter how these "smaller" contests are constructed.

The weak concavity of $H^{-1}(\cdot)$ not only represents a technical restriction, but reflects a key aspect of the economic nature of the underlying contest technology. By Proposition 1, $H^{-1}(\cdot)$ represents the equilibrium individual effort function, which is given by $x=H^{-1}(\frac{1}{N}[\sum_{k=1}^{K}V_k(1-\sum_{g=0}^{k-1}\frac{1}{N-g})])$. When $H^{-1}(\cdot)$ is weakly concave, it implies that while the equilibrium individual effort x strictly increases with the value of prize V_k, the marginal return from the viewpoint of the contest organizer (i.e., the marginal return in term of effort supply) diminishes as the value of the prize increases. Intuitively, given a regular technology, a more generous prize purse could stimulate contestants to step up their effort, but this positive incentive effect would gradually diminish as the amount of the prize purse rises.

Therefore, the **regularity** condition (i.e., the weak concavity of $H^{-1}(\cdot)$) is by no means a strong or artificial restriction. In particular, the **regularity** condition is satisfied by the class of power functions $f(x)=ax^r$, $a>0$, $r>0$, which have been commonly assumed in the literature. Hence, the result of Theorem 1 holds for a wide class of contest settings.

Note that contestants in different subcontests may respond differently after a grand contest is split. Some of them may have to exert more effort, while others exert less, depending on the particular structures of the subcontests. However, the "gain" of effort in some subcontests must come at the cost of "losses" in the others, and the "loss" must more than offset the "gain." As a consequence, the total effort unambiguously decreases as the grand contest is split.

The equilibrium rent-dissipation rate for a contest is defined as the ratio of total effort induced and the total prizes. Theorem 1 directly implies the following corollary.

Corollary 1. *With a* ***regular*** *contest technology, the grand contest generates a higher rent-dissipation rate than any set of split contests.*

4. Discussion

In this paper, we show that a grand contest elicits more effort than any split version of the contest under a mild regularity condition on $H^{-1}(\cdot)$. In addition to its contribution to the literature on single-stage multi-winner contests, our results shed light on the design of multi-stage elimination contests. Most existing studies on elimination contests assume that contestants are divided into groups in each preliminary stage, with only a single winner proceeding to the next stage from each group. Amegashie (2000) studies the incentive effect of differing "shortlisting" procedures in two-stage imperfectly-discriminatory contests. Assuming a linear contest technology $f(x)=x$, he shows that in order to select a given number of finalists, it would be beneficial for the contest organizer to first run a grand contest in the preliminary stage, instead of splitting the preliminaries into a set of even subcontests. Our result generalizes this important insight and we seek to extend this line of thought to a context that allows for a general contest technology and a more flexible division of the grand contest. Our result provides a foundation for the elimination rule assumed by Fu and Lu (2007a) who require that each remaining contestant in a multi-stage contest compete against all other survivors in each preliminary stage. Hence, the competition in every preliminary stage is modeled as a multi-winner "grand" contest. Theorem 1 thus directly confirms the optimality of this "pooling competition" rule assumed by Fu and Lu (2007a).

Theorem 1 directly implies that, when any collection of contests are combined into a grand contest, the total effort would strictly increase. In particular, more effort would be elicited when a number of identical contests are merged. This implies that the equilibrium level of individual effort positively responds to the "scale" of the contest: each contestant supplies more effort when the number of prizes increases in proportion to the number of contestants. As will be shown in Section 4.1, this interesting property does not demand the regularity condition on $H^{-1}(\cdot)$. It holds for more general contest technologies instead.

4.1. The "replication" of contests

For any multi-winner contest $C_0 \triangleq C(N, K, \mathbf{V})$, a t-fold replication of it is defined as $C^t \triangleq C(tN, tK, \mathbf{V}^t)$.[10] Here $\mathbf{V}^t \triangleq (V_1\mathbf{1}, V_2\mathbf{1}, \dots, V_K\mathbf{1})$ where $\mathbf{1} = (1, 1, \dots, 1)$ is a t-element row vector. C^t thus resembles a "status" contest where the tN contestants compete for K status with t positions in each rank. Theorem 1 immediately implies that the individual effort in C_0 is lower than that of C^t under the regularity condition. Next, we show that the dominance of C^t over C_0 does not rely on this restriction.

By (5), the individual effort in contest C_0 is

$$x_0 = H^{-1}\left(\frac{1}{N}\sum_{k=1}^{K}\left\{V_k\left[1 - \sum_{g=0}^{k-1}\frac{1}{N-g}\right]\right\}\right). \tag{15}$$

By way of contrast, in contest C^t, the individual equilibrium effort x^t would be given by

$$x^t = H^{-1}\left(\frac{1}{tN}\sum_{k=1}^{K}\left\{V_k\left[t - \sum_{h=1}^{t}\sum_{g=0}^{(k-1)t+h-1}\frac{1}{tN-g}\right]\right\}\right). \tag{16}$$

Proposition 3. *A contestant exerts strictly more effort in the t-fold replicated contest C^t, i.e., $x^t > x_0$.*

Proof. Because $H^{-1}(\cdot)$ is strictly increasing in its argument, to verify this claim, we only need to show

$$\frac{1}{N}\sum_{k=1}^{K}\left\{V_k\left[1 - \sum_{g=0}^{k-1}\frac{1}{N-g}\right]\right\} < \frac{1}{tN}\sum_{k=1}^{K}\left\{V_k\left[t - \sum_{h=1}^{t}\sum_{g=0}^{(k-1)t+h-1}\frac{1}{tN-g}\right]\right\}.$$

By the fact $\frac{1}{N}\sum_{k=1}^{K} V_k = \frac{1}{tN}\sum_{k=1}^{K} V_k t$, this inequality would hold if and only if

$$t\sum_{k=1}^{K}\left\{V_k\sum_{g=0}^{k-1}\frac{1}{N-g}\right\} > \sum_{k=1}^{K}\left\{V_k\sum_{h=1}^{t}\sum_{g=0}^{(k-1)t+h-1}\frac{1}{tN-g}\right\}. \tag{17}$$

We have this is implied by Proposition 2. □

Proposition 3 contends that the t-fold replicated contest induces more individual effort as compared to a set of identical small contests even without the regularity condition of Definition 2. In the symmetric equilibrium, a contestant behaves more competitively in the "bigger" contest, although he/she has the same chance to receive each prize in either setting. Our analysis therefore sheds light on a more fundamental question: How does the structure of a

[10] Wärneryd (2001) defines a contest with rN contestants competing for a prize of the value rV as the r-fold replication of the contest with N contestants competing for a prize of the value V. In our context, which may involve more than one winner, we borrow the terminology "r-fold replication" of the original contest, but it represents a different setting from Wärneryd (2001). Moldovanu and Sela (2006) in a perfectly-discriminatory contest setting compare two contest architectures. The grand one has n contestants competing for one prize worth 1; while the t-parallel architecture has $t > 1$ subcontest, in each of which n/t contestants compete for one prize worth $1/t$. In contrast to these papers, we allow the number of prizes and the number of contestants to vary, but keep constant the value of each single prize.

multi-winner contest affect the contestants' incentives to expend effort? We argue that the equilibrium level of effort expended in a contest exhibits *increasing-return-to-scale* as follows.

Theorem 2. *In a multi-winner contest, when the number of contestants and the number of each prize V_k are scaled up by a common integer factor t,*

(i) *each contestant increases his/her equilibrium effort*;
(ii) *the total effort increases by more than t times.*

The following implication naturally arises from Theorem 2.

Corollary 2. *When the number of contestants and the number of prizes increase proportionally, the rent-dissipation rate strictly increases.*

Our results show that a "bigger" contest would demand more effort from each contestant even if the number of prizes increases in proportion to the number of contestants. Theorem 2 yields important insights into economic studies on contests. It implies that contestants behave differently when the "scale" of the contest varies. Thus, the results obtained from relatively small contest settings may not naturally extend to large-scale contests.

4.2. The beauty of "bigness": the logic

We first illustrate the logic that underlies the *increasing-return-to-scale* property. When a contest is folded, what contributes to the more competitive supply of effort—the increasing numbers of prizes, the increasing number of contestants, or a mix of these factors? To explore the logic underlying Theorem 2, we set forth an alternative but equivalent setup of multi-winner contest model (2). Fu and Lu (2007b) establish the equivalence between the multiple-winner nested contest model (2) and the following simultaneous noisy-ranking contest model that originates from McFadden's (1973, 1974) random choice framework. In this noisy-ranking setup of contest, the contest organizer observes and evaluates each contestant's output through a noisy signal (y_i) defined as

$$\log y_i = \log f(x_i) + \varepsilon_i,^{11} \quad \forall i \in \{1, 2, \ldots, N\}, \tag{18}$$

where the deterministic strictly increasing function $f(\cdot): \mathbb{R}_+ \to \mathbb{R}_+$ measures the impact or output of contestant i's effort x_i. The additive i.i.d. noise term ε_i reflects the randomness in the production process or the imperfection of the observation and evaluation process. The contest organizer prefers higher outputs and so contestants are ranked according to their perceived output (y_i) in a descending order. Each contestant receives a prize according to his or her rank and the availability of prizes. This prize distribution rule can be also seen in Glazer and Hassin (1988), Barut and Kovenock (1998), Clark and Riis (1998b) and Moldovanu and Sela (2001, 2006). The i.i.d. random components ε_i are drawn from a type I extreme-value (maximum) distribution. In this noisy ranking model, for any given effort entry X, a complete ranking of contestants immediately follows from any realization of the noise terms $\boldsymbol{\varepsilon} \triangleq (\varepsilon_i)$.

Apart from the mathematical equivalence, Fu and Lu (2007b) also reveal a hidden (simultaneous) selection mechanism underneath the nested contest (2). This selection mechanism unites a number of multi-winner competitions (including race-type tournaments where performance of contestants is measured by how fast they finish a task) and abstracts them as lottery contests (winner-take-all or multi-winner). Thus, we can interpret our results in light of this noisy-ranking approach.

We may unravel the competing effects underlying the *increasing-return-to-scale* property by asking the question: "Holding other factors constant, is it more difficult for a contestant to be ranked among the top ten out of a hundred, or the first out of ten?" This question prompts a comparison between a ten-contestant winner-take-all contest and its "ten-fold replication."

On the one hand, while the former requires him/her to beat only nine competitors to win, the latter event requires a contestant to beat at least ninety rivals. This leads us to take the position that a larger number of contestants alone

[11] It is worth noting that this model is equivalent to a multiplicative-noise ranking model $y_i = f(x_i)\tilde{\varepsilon}_i, \forall i \in \{1, 2, \ldots, N\}$, where the noise term $\tilde{\varepsilon}_i$ is defined as $\tilde{\varepsilon}_i \triangleq \exp \varepsilon_i$.

cannot contribute to **individual** effort supply: As the number of contestants is scaled up, a contestant tends to reduce the level of effort he or she supplies because of the decreased marginal return to this effort that results if the set of prizes remains the same. Indeed, while a larger pool of competitors would undoubtedly increase the total effort, individual equilibrium effort must decrease when prizes are fixed. This is strongly indicated by the equilibrium individual effort solution as given by Eq. (5): When $K = 1$ in (5), we have $x = H^{-1}(\frac{1}{N}(1 - \frac{1}{N})V_1)$, which strictly decreases with N for $N \geqslant 2$.

On the other hand, the complementary replication of prizes (together with the bigger pool of contenders) creates a positive effect on the incentives that contestants have for exerting more effort. The effort of a contestant can be rewarded as long as his/her rank is within the top ten. This fact yields higher marginal returns to the contestants' effort outlays when the contest is scaled up. It therefore leads also to a more competitive supply of effort, as the marginal cost of effort remains constant. This positive effect more than offsets the negative effect inflicted by the increasing number of contestants. Intuitively, each prize in the ten-contestant contest attracts only ten competitors, while each prize in its ten-fold replication attracts a hundred competitors. A prize creates a "broader" battlefield as the contest is scaled up, and the ten prizes in the folded contest could therefore elicit more than ten-fold effort.

This positive effect can be even more subtly evidenced in the general setting where prizes are heterogeneous and the grand contest can be unevenly partitioned. The equilibrium effort supply in a contest is determined by the subtle interaction between the number of contestants and the set of prizes. As indicated by the simple example in Section 3, when the grand contest is split, a contestant could either increase or decrease his/her effort supply, depending on his/her particular competitive environment. We show that the optimality of a grand contest is guaranteed under the regularity condition,[12] despite the flexibility of the contest organizer to construct the set of subcontests. The intuition behind this result is demonstrated as follows.

Without loss of generality, we focus on the case where the impact function takes the form $f(x) = ax^r$, $a > 0, r > 0$. This class of impact functions leads to a linear $H^{-1}(\cdot)$ function, which represents the least concavity requirement for $H^{-1}(\cdot)$. In this case, a subcontest m elicits the total effort $E_m = r[\sum_{k=1}^{K_m} V_k^m - \sum_{k=1}^{K_m}(V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m - g})]$, $m = 1, 2, \ldots, M$, while the grand contest elicits the total effort $E_g = r[\sum_{k=1}^{K} V_k - \sum_{k=1}^{K}(V_k \sum_{g=0}^{k-1} \frac{1}{N-g})]$.[13] Since E_m and E_g are linear in r, we restrict our attention to the case of $r = 1$. Thus, we have $E_m = \sum_{k=1}^{K_m} V_k^m - \sum_{k=1}^{K_m}(V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m-g})$, $m = 1, 2, \ldots, M$, and $E_g = \sum_{k=1}^{K} V_k - \sum_{k=1}^{K}(V_k \sum_{g=0}^{k-1} \frac{1}{N-g})$. While $\sum_{m=1}^{M}\sum_{k=1}^{K_m} V_k^m$ and $\sum_{k=1}^{K} V_k$ measure the total amounts of prizes in both settings respectively, $\sum_{m=1}^{M}\sum_{k=1}^{K_m}(V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m-g})$ and $\sum_{k=1}^{K}(V_k \sum_{g=0}^{k-1} \frac{1}{N-g})$ measure the total surplus all contestants receive respectively. Since $\sum_{m=1}^{M}\sum_{k=1}^{K_m} V_k^m = \sum_{k=1}^{K} V_k$, $\sum_{m=1}^{M} E_m < E_g$ is equivalent to $\sum_{m=1}^{M}\sum_{k=1}^{K_m}(V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m-g}) > \sum_{k=1}^{K}(V_k \sum_{g=0}^{k-1} \frac{1}{N-g})$, i.e., the pool of contestants receive more (overall) surplus in the collection of subcontests. To illustrate how the prizes contribute to the surplus received by these contestants in both settings, we rewrite $\sum_{k=1}^{K_m}(V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m-g})$ and $\sum_{k=1}^{K}(V_k \sum_{g=0}^{k-1} \frac{1}{N-g})$ in a convenient form.

Define $\Gamma_l^m \triangleq \sum_{k=l}^{K_m} V_k^m$, $l = 1, 2, \ldots, K_m$ and $m = 1, 2, \ldots, M$. Define $\Gamma_l^g \triangleq \sum_{k=l}^{K} V_k$, $l = 1, 2, \ldots, K$. Then we have

$$\sum_{m=1}^{M}\sum_{k=1}^{K_m}\left(V_k^m \sum_{g=0}^{k-1} \frac{1}{N_m - g}\right) = \sum_{m=1}^{M}\sum_{l=0}^{K_m-1} \frac{\Gamma_{l+1}^m}{N_m - l}, \quad \text{and} \tag{19}$$

$$\sum_{k=1}^{K}\left(V_k \sum_{g=0}^{k-1} \frac{1}{N - g}\right) = \sum_{l=0}^{K-1} \frac{\Gamma_{l+1}^g}{N - l}. \tag{20}$$

Notably, $\frac{\Gamma_{l+1}^m}{N_m-l}$ measures the contribution of Γ_{l+1}^m to the total surplus of contestants in the collection of subcontests, and $\frac{\Gamma_{l+1}^g}{N-l}$ measures the contribution of Γ_{l+1}^g to the total surplus of contestants in the grand contest. As indicated by the proof of Proposition 2, there exists a one-to-one correspondence between the K components in the RHS of (19) and those in the RHS of (20), such that each component in the RHS of (19) is greater than its counterpart in the RHS of (20). This

12 This regularity condition is not required when the subcontests are identical.

13 Please refer to Proposition 1.

correspondence thus indicates that the batch of prizes **V**, when split and placed in a set of subcontests, contribute more surplus to contestants, regardless of the division rule. In other words, prizes are "utilized" less efficiently when they are awarded in subcontests, as each of the prizes could elicit the effort entry only from a subset of these contenders.

This effect, together with the concavity of $H^{-1}(\cdot)$, leads to the dominance of the grand contest, despite the fact that contestants can behave asymmetrically (either increase or decrease their effort) when they are placed in different subcontests. The logic can be seen in the following arguments. Imagine an arbitrary division of a grand contest, and suppose that a disproportionately large prize is allocated to a subcontest of an average size. On the one hand, contestants placed in this subcontest would supply more effort than they would do in the grand one. This positive motive is unfortunately limited by the concavity of $H^{-1}(\cdot)$ (see Section 3 for the economic interpretation of the regularity condition). On the other hand, contestants in other subcontests would supply less. Given the fact $\sum_{m=1}^{M}\sum_{l=0}^{K_m-1}\frac{\Gamma_{l+1}^{m}}{N_m-l} > \sum_{l=0}^{K-1}\frac{\Gamma_{l+1}^{g}}{N-l}$, the positive effect cannot counteract the negative effect.

4.3. Asymmetric players

Our analysis so far has assumed that all contestants are identical. We make this assumption for two major purposes: First, it enables us to find a closed-form solution to our model. This goal is particularly pertinent as the literature in this area does not yet demonstrate a solution to complete-information multi-prize contests in which the contestants are asymmetric. Second, the assumption helps control for confounding factors that could dilute the main theme of the analysis that focuses on the scale effect. However, this restriction may also limit the generality of our result without verifying its robustness.

When contestants are endowed with differing talents, an additional line of freedom is added to the contest design problem. The contest organizer could strategically match the confrontations between different contestants on particular variables (e.g. weight classes in boxing matches), and divide the set of prizes correspondingly. Although a complete characterization of a general model is hard to obtain, one may imagine that the optimal contest design would depend on the distribution of talents. Despite this additional complication, simple intuition would allow one to infer that a grand contest would continue to dominate over split contests when the asymmetry of talents is less significant. In contrast, the dominance would be increasingly uncertain as the degree of asymmetry ascends.

This argument could be directly implied from the continuity of the effort supply functions with respect to the type parameter such as contestants' marginal costs,[14] and the strict dominance result we have established for symmetric contestants in Theorem 1. We illustrate this intuition using a simple example in which four contestants compete for two identical prizes $V_1 = V_2 = 1$. Let the impact function take the linear form $f(x) = x$. However, the four contestants differ in their bidding costs: A contestant of type I bears a unitary marginal cost for their effort, i.e., $C(x) = x$; while a contestant of type II bears a constant marginal cost $c > 1$, i.e., $C(x) = cx$. Thus, a type-II contestant is less capable than a type-I contestant. We further assume that two of the contestants are of type I, while the other two are of type II.

In a monotonic equilibrium of the grand contest, the type-I contestant expends an effort x, and the type-II contestant expends y. In such a scenario, the equilibrium effort (x, y) would be simultaneously determined by the following nonlinear equations:

$$\frac{y}{2(x+y)(x+2y)}\left[\frac{2x}{2(x+y)}+\frac{2x}{x+2y}\right]+\frac{y}{2(x+y)(2x+y)}\left[\frac{3x+2y}{2(x+y)}+\frac{x+y}{2x+y}\right]=1, \tag{21}$$

$$\frac{x}{2(x+y)(2x+y)}\left[\frac{2y}{2(x+y)}+\frac{2y}{2x+y}\right]+\frac{x}{2(x+y)(x+2y)}\left[\frac{2x+3y}{2(x+y)}+\frac{x+y}{x+2y}\right]=c. \tag{22}$$

In this setting, the optimal partition rule points toward splitting the contest into two subcontests, and matching two identical contestants in each of them. In this scenario, the two subcontests would elicit a total effort of $\frac{1}{2}(1+\frac{1}{c})$. Given these steps, to what extent would the grand contest continue to dominate the split contests?

We compare the total effort in the grand contest (i.e. $2(x + y)$) to that in the optimal set of subcontests (i.e., $\frac{1}{2}(1+\frac{1}{c})$). As it is difficult to analytically solve for a closed-form solution to the high-order equations (21) and (22),

[14] The continuity of the individual effort supply functions is guaranteed by the continuity of the first order conditions (such as those of (21) and (22)) with respect to the type parameter (such like marginal costs).

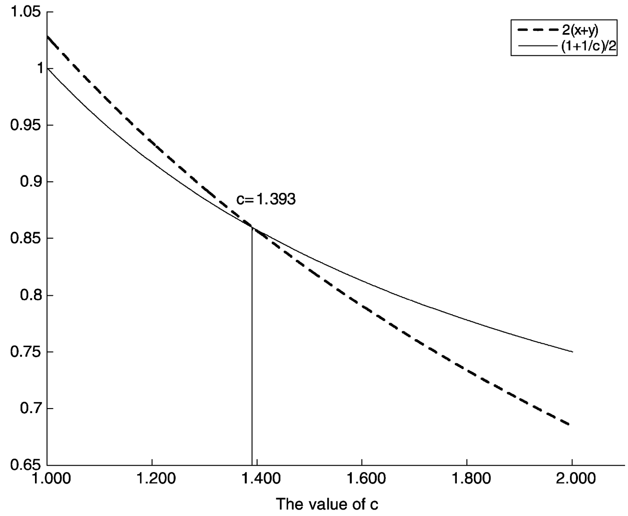

Fig. 1. Effort supplied in the grand contest and the set of (appropriately) split contests.

we conduct a numerical exercise to test our hypothesis. The results are illustrated by Fig. 1: the grand contest dominates the split contests whenever $c \leqslant 1.393$; while the comparison outcome is reversed once c exceeds the threshold. Although limited, the numerical evidence supports our hypothesis that a grand contest could elicit more effort when the ability differentials among contestants are sufficiently insignificant.

Thus, it can be said that designing effort-maximizing contests with substantially heterogeneous contestants requires greater sophistication in the matching of contestants and prizes. A more general theory is required that can adequately illuminate the subtlety of this dimension despite the technical difficulty.

5. Concluding remarks

In addition to making key theoretical contributions, our results help shed light on many real-life situations that resemble the competitive activities modeled in this paper. We conclude by illustrating the practical implications and utility of our results.

We have shown that a grand contest induces more effort than any split version of the contest when the contestants are more or less homogeneous. Thus, if the level of effort exerted by the contestants benefits the contest organizer, a grand contest would better serve the latter's interest. The rules of many sporting events exemplify this insight.

In high-profile sports competitions, such as the Olympic Games, high qualification standards ensure that players are fairly homogeneous. This is especially true for players involved in later stages of the games. In the preliminaries and semi-finals of diving and gymnastics competitions, athletes are gathered in a single pool and not divided into matched groups. The finalists are selected on the basis of their performance relative to all other players. In fact, diving competitions in the Olympic Games evolved from the use of "subcontests"[15] to the adoption of a "grand contest" in 1952, and the format of a "grand contest" has been in place ever since. Our findings demonstrate that this practice is indeed optimal, and provide further justification for its wide adoption.

Another particular situation in which we find our conclusion to be relevant is the organization of a firm's internal labor market. Given that teams are an increasingly popular means of organizing a workforce, firms may choose to distribute a number of "prizes" (vacancies on higher levels of the organizational hierarchy, bonuses, or other forms of compensation) in two ways: the prizes could be awarded to top-performing workers either within each team, or a combination of a few teams (a subcontest), or across the entire workforce (grand contest). Given what we know about such contests, how should a firm select and reward top-performing workers? Our results suggest that individual workers' intra-team rankings should be assigned lesser weights, and should be deferred to their comparative perfor-

[15] Before 1952, athletes competing in the preliminaries were divided into subgroups, and the two top-ranked athletes in each group advance to the final.

mance across all teams. A challenge arises from the fact that teams may perform different tasks, and the management may lack a universally acceptable criterion with which to evaluate individual workers. Nevertheless, as long as the outputs of different teams can be compared on some common ground such that cross-team comparison is informative regarding effort exerted by contestants, taking into account cross-team evaluations would provide workers with more incentive to exert productive effort.

By way of contrast, to the extent that rent-seeking activity is considered to be wasteful and undesirable, our results suggest that contest organizers can successfully reduce the wastage of loud lobbying by dividing a grand contest into a set of smaller subcontests. Consequently, our results could be pertinent to the non-market rationing of scarce resources. Intuitively, our results provide a rationale for "quota" systems, which have been widely used to allocate public resources. When public resources are distributed among different regions or groups in fixed quotas, economic agents in each group or region can only compete to share the given quota. Thus, the competition is downgraded to local contentions, and rent-seeking activities are reduced.[16]

When a "quota" system is implemented, prizes tend to be distributed among regions or groups in a way that is proportionate to the populations of the regions and the sizes of the groups, which is, ostensibly, a fair allocation. However, our results call into question the "fairness" of this practice: As implied by Theorem 2 ("increasing returns to scale"), those who are in a larger group or region receive less surplus than those who are in a smaller partition.

As a consequence, our results also confirm the conventional wisdom that "the first in village is better than the second in Rome": achieving success among many is usually more rewarding than achieving success among few, but the former demands a greater toll or sacrifice. Thus, despite the greater opportunities for success in "Rome," it may often be in a contestant's interest to stay in the "village." A direct extension of our current study would be to allow contestants to make their participation decisions and choose from a few parallel contests, where the contest organizers have designed the incentive structures to compete for participation.[17]

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Appendix A. The proof of Proposition 2

Note that in order to prove Proposition 2, it is sufficient to prove inequality (12) for the case of $M = 2$, i.e.,

$$\sum_{k=1}^{K_1}\left(V_k^1\sum_{t=0}^{k-1}\frac{1}{N_1-t}\right)+\sum_{k=1}^{K_2}\left(V_k^2\sum_{t=0}^{k-1}\frac{1}{N_1-t}\right)>\sum_{k=1}^{K_1+K_2}\left(V_k\sum_{t=0}^{k-1}\frac{1}{N_1+N_2-t}\right), \tag{A.1}$$

where $N_1 \geqslant K_1 \geqslant 1$, $N_2 \geqslant K_2 \geqslant 1$, and $N_1 + N_2 \geqslant K_1 + K_2$.

It is worth noting that the sum of the K_m-term sequence $\{V_k^m \sum_{t=0}^{k-1} \frac{1}{N_m-t}\}_{k=1}^{K_m}$ can be rewritten as the sum of another K_m-term sequence $\theta^m \triangleq \{\frac{1}{N_m-t}\Gamma_t^m\}_{t=0}^{K_m-1}$ where $\Gamma_t^m \triangleq \sum_{k=t+1}^{K_m} V_k^m$, $t = 0, 1, \ldots, K_m - 1$, $m = 1, 2$, while the sum of the $(K_1 + K_2)$-term sequence $\sum_{k=1}^{K_1+K_2}(V_k \sum_{t=0}^{k-1} \frac{1}{N_1+N_2-t})$ can be rewritten as the sum of another $(K_1 + K_2)$-term sequence $\beta \triangleq \{\frac{1}{N_1+N_2-t}\Gamma_t\}_{t=0}^{K_1+K_2-1}$ where $\Gamma_t \triangleq \sum_{k=t+1}^{K_1+K_2} V_k$, $t = 0, 1, \ldots, K_1 + K_2 - 1$. Thus, inequality (A.1) is equivalent to

[16] In this regard, the implications of our results are in line with Wärneryd (1998) and Inderst et al. (2005), who suggest that distributional conflicts can be reduced, if jurisdictional organizations are more hierarchical, as that tends to reduce the level of competition.

[17] For instance, a worker could decide to join a larger firm or a smaller one. As documented by Brown et al. (1990), larger firms usually have to pay a compensating differential to attract workers, as their workers may enjoy their work less than those who work in smaller firms. This extension is also on the agenda for our future research.

$$\sum_{t=0}^{K_1-1}\left(\frac{1}{N_1-t}\sum_{k=t+1}^{K_1}V_k^1\right)+\sum_{t=0}^{K_2}\left(\frac{1}{N_2-t}\sum_{k=t+1}^{K_2}V_k^2\right)$$
$$>\sum_{t=0}^{K_1+K_2-1}\left(\frac{1}{N_1+N_2-t}\sum_{k=t+1}^{K_1+K_2}V_k\right). \tag{A.2}$$

Without loss of generality, we assume $\frac{1}{N_1}\sum_{k=1}^{K_1}V_k^1 \geqslant \frac{1}{N_2}\sum_{k=1}^{K_2}V_k^2$, which implies that the average gross payoff in subcontest 1 is greater than that in subcontest 2.

Consider any two decreasing sequences of nonnegative numbers $\{W_k^1\}_{k=1}^{L_1}$ and $\{W_k^2\}_{k=1}^{L_2}$. Let $\{W_k\}_{k=1}^{L_1+L_2}$ denote the decreasing sequence that combines $\{W_k^1\}_{k=1}^{L_1}$ and $\{W_k^2\}_{k=1}^{L_2}$. For any positive numbers M_i, $i=1,2$, we obtain the following useful properties.

P1: (i) $\max\{\frac{1}{M_1}\sum_{k=1}^{L_1}W_k^1, \frac{1}{M_2}\sum_{k=1}^{L_2}W_k^2\} \geqslant \frac{1}{M_1+M_2}\sum_{k=1}^{L_1+L_2}W_k$;

(ii) $\max\{\frac{1}{M_1}\sum_{k=1}^{L_1}W_k^1, \frac{1}{M_2}\sum_{k=1}^{L_2}W_k^2\} = \frac{1}{M_1+M_2}\sum_{k=1}^{L_1+L_2}W_k$ iff $\frac{1}{M_1}\sum_{k=1}^{L_1}W_k^1 = \frac{1}{M_2}\sum_{k=1}^{L_2}W_k^2$.

We attempt to establish a one-to-one matching between each term on the LHS of (A.2) to a term to its RHS such that each component of the LHS is greater than its counterpart on the LHS. The proof proceeds as follows.

By the assumption $\frac{1}{N_1}\sum_{k=1}^{K_1}V_k^1 \geqslant \frac{1}{N_2}\sum_{k=1}^{K_2}V_k^2$ and **P1**(i), we must have $\frac{1}{N_1}\sum_{k=1}^{K_1}V_k^1 \geqslant \frac{1}{N_1+N_2}\sum_{k=1}^{K_1+K_2}V_k$, that is, the first term of sequence $\{\frac{1}{N_1-t}\sum_{k=t+1}^{K_1}V_k^1\}_{t=0}^{K_1-1}$ is greater than the first term in sequence $\{\frac{1}{N_1+N_2-t}\sum_{k=t+1}^{K_1+K_2}V_k\}_{t=0}^{K_1+K_2-1}$.

Next we compare either $\frac{1}{N_1-1}\sum_{k=2}^{K_1}V_k^1$ (the second term of sequence $\{\frac{1}{N_1-t}\sum_{k=t+1}^{K_1}V_k^1\}_{t=0}^{K_1-1}$), or $\frac{1}{N_2}\sum_{k=1}^{K_2}V_k^2$ (the first term of sequence $\{\frac{1}{N_2-t}\sum_{k=t+1}^{K_2}V_k^2\}_{t=0}^{K_2-1}$), to $\frac{1}{N_1+N_2-1}\sum_{k=2}^{K_1+K_2}V_k$ (the second term of sequence $\{\frac{1}{N_1+N_2-t}\sum_{k=t+1}^{K_1+K_2}V_k\}_{t=0}^{K_1+K_2-1}$). Note that because $V_1 \geqslant V_1^1$, we must have $\sum_{k=2}^{K_1+K_2}V_k \leqslant \sum_{k=2}^{K_1}V_k^1+\sum_{k=1}^{K_2}V_k^2$, which gives

$$\frac{1}{N_1+N_2-1}\sum_{k=2}^{K_1+K_2}V_k \leqslant \frac{1}{N_1+N_2-1}\left[\sum_{k=2}^{K_1}V_k^1+\sum_{k=1}^{K_2}V_k^2\right]. \tag{A.3}$$

By **P1**(i), we must have

$$\max\left(\frac{1}{N_1-1}\sum_{k=2}^{K_1}V_k^1, \frac{1}{N_2}\sum_{k=1}^{K_2}V_k^2\right)$$
$$\geqslant \frac{1}{N_1+N_2-1}\left[\sum_{k=2}^{K_1}V_k^1+\sum_{k=1}^{K_2}V_k^2\right]$$
$$\geqslant \frac{1}{N_1+N_2-1}\sum_{k=2}^{K_1+K_2}V_k. \tag{A.4}$$

Thus, we can match either the higher one between $\frac{1}{N_1-1}\sum_{k=2}^{K_1}V_k^1$ and $\frac{1}{N_2}\sum_{k=1}^{K_2}V_k^2$ on the LHS of (A.1) to $\frac{1}{N_1+N_2-1}\sum_{k=2}^{K_1+K_2}V_k$ on its RHS such that the former is no less than the latter.

We continue this practice. Suppose that we have continued this practice for a total of k_1+k_2 rounds. That is, each of the $k_m(\geqslant 0)$ largest terms in the sequence $\{\frac{1}{N_m-t}\sum_{k=t+1}^{K_m}V_k^m\}_{t=0}^{K_m-1}$, $m=1,2$, has been matched to one of the k_1+k_2 largest terms in the sequence $\{\frac{1}{N_1+N_2-t}\sum_{k=t+1}^{K_1+K_2}V_k\}_{t=0}^{K_1+K_2-1}$.

We first consider the case $k_m < K_m$, $m=1,2$. In this case, neither of the two sequences $\{\frac{1}{N_m-t}\sum_{k=t+1}^{K_m}V_k^m\}_{t=0}^{K_m-1}$, $m=1,2$, has been exhausted. Each of them leaves a subsequence written as $\{\frac{1}{N_m-t}\sum_{k=t+1}^{K_m}V_k^m\}_{t=k_1}^{K_m-1}$. The sequence on the RHS of (A.1) thus leaves a subsequence $\{\frac{1}{N-t}\sum_{k=t+1}^{K_1+K_2}V_k\}_{t=k_1+k_2}^{K_1+K_2-1}$. Again, we can match the higher one between $\frac{1}{N_1-k_1}\sum_{k=k_1+1}^{K_1}V_k^1$ (i.e., the first term of subsequence $\{\frac{1}{N_1-t}\sum_{k=t+1}^{K_1}V_k^1\}_{t=k_1}^{K_1-1}$) and $\frac{1}{N_2-k_2}\sum_{k=k_2+1}^{K_2}V_k^2$

(i.e., the first term of subsequence $\{\frac{1}{N_2-t}\sum_{k=t+1}^{K_2} V_k^2\}_{t=k_2}^{K_2-1}$), to $\frac{1}{N_1+N_2-k_1-k_2}\sum_{k=k_1+k_2+1}^{K_1+K_2} V_k$ (i.e., the first term of subsequence $\{\frac{1}{N_1+N_2-t}\sum_{k=t+1}^{K_1+K_2} V_k\}_{t=k_1+k_2}^{K_1+K_2-1}$). Clearly, because $\sum_{k=1}^{k_1+k_2} V_k \geqslant \sum_{k=1}^{k_1} V_k^1 + \sum_{k=1}^{k_2} V_k^2$, we must have

$$\sum_{k=k_1+1}^{K_1} V_k^1 + \sum_{k=k_2+1}^{K_2} V_k^2 \geqslant \sum_{k=k_1+k_2+1}^{K_1+K_2} V_k,$$

which implies

$$\frac{1}{N_1+N_2-k_1-k_2}\left[\sum_{k=k_1+1}^{K_1} V_k^1 + \sum_{k=k_2+1}^{K_2} V_k^2\right] \geqslant \frac{1}{N_1+N_2-k_1-k_2}\sum_{k=k_1+k_2+1}^{K_1+K_2} V_k. \tag{A.5}$$

Thus, we conclude that

$$\max\left(\frac{1}{N_1-k_1}\sum_{k=k_1+1}^{K_1} V_k^1, \frac{1}{N_2-k_2}\sum_{k=k_2+1}^{K_2} V_k^2\right) \geqslant \frac{1}{N_1+N_2-k_1-k_2}\sum_{k=k_1+k_2+1}^{K_1+K_2} V_k, \tag{A.6}$$

and we can safely match $\max(\frac{1}{N_1-k_1}\sum_{k=k_1+1}^{K_1} V_k^1, \frac{1}{N_2-k_2}\sum_{k=k_2+1}^{K_2} V_k^2)$ to $\frac{1}{N_1+N_2-k_1-k_2}\sum_{k=k_1+k_2+1}^{K_1+K_2} V_k$, which continues to preserve the dominance.

We iterate this procedure until one of the sequence is exhausted. Without loss of generality, we assume that after $k_1 + K_2$ rounds of this practice, the sequence $\{\frac{1}{N_1-t}\sum_{k=t+1}^{K_1} V_k^1\}_{k=1}^{K_1}$ is not exhausted, while all the K_2 terms of the sequence $\{\frac{1}{N_2-t}\sum_{k=t+1}^{K_2} V_k^2\}_{t=0}^{K_2-1}$ have been used. Thus, we need to match each term of the $(K_1 - k_1)$-term subsequence $\{\frac{1}{N_1-t}\sum_{k=k_1+1}^{K_1} V_k^1\}_{t=k_1}^{K_1-1}$ to a term of the $(K_1 - k_1)$-term subsequence $\{\frac{1}{N_1+N_2-t}\sum_{k=t+1}^{K_1+K_2} V_k\}_{t=k_1+K_2}^{K_1+K_2-1}$.

Clearly, we must have that $V_{k_1+h}^1 \geqslant V_{k_1+K_2+h}$, for $1 \leqslant h \leqslant K_1 - k_1$, because the terms of sequence $\{V_{k_1+K_2+h}\}_{h=0}^{K_1-k_1}$ are the lowest $K_1 - k_1$ terms of decreasing sequence $\{V_k\}_{k=1}^{K_1+K_2}$, while the terms of sequence $\{V_{k_1+h}^1\}_{h=0}^{K_1-k_1}$ may not be the lowest $K_1 - k_1$ terms of decreasing sequence $\{V_k\}_{k=1}^{K_1+K_2}$. We thus have $\sum_{k=k_1+K_2+t}^{K_1+K_2} V_k \leqslant \sum_{k=k_1+t}^{K_1} V_k^1$, for any positive integer $t \leqslant K_1 - k_1$. This further guarantees $\frac{1}{N_1-k_1-t}\sum_{k=k_1+t+1}^{K_1} V_k^1 \geqslant \frac{1}{N_1+N_2-k_1-K_2-t}\sum_{k=k_1+K_2+t+1}^{K_1+K_2} V_k$ for any nonnegative integer $t \leqslant K_1 - k_1 - 1$, because $N_2 - K_2 \geqslant 0$. We therefore prove (A.2) weakly holds, i.e., the LHS is weakly greater than the RHS.

To show that (A.1) strictly holds, it is sufficient to show that there exists at least one correspondence such that the term on the LHS of (A.2) is strictly greater that its counterpart on the RHS. By assumption, we have $\frac{1}{N_1}\sum_{k=1}^{K_1} V_k^1 \geqslant \frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2$. If $\frac{1}{N_1}\sum_{k=1}^{K_1} V_k^1 > \frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2$, **P1**(ii) leads to that $\frac{1}{N_1}\sum_{k=1}^{K_1} V_k^1 > \frac{1}{N_1+N_2}\sum_{k=1}^{K_1+K_2} V_k$, i.e., the first term of sequence $\{\frac{1}{N_1-t}\sum_{k=t+1}^{K_1} V_k^1\}_{t=0}^{K_1-1}$ is strictly greater than its counterpart (the first term in sequence $\{\frac{1}{N_1+N_2-t}\sum_{k=t+1}^{K_1+K_2} V_k\}_{t=0}^{K_1+K_2-1}$). If $\frac{1}{N_1}\sum_{k=1}^{K_1} V_k^1 = \frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2$ and $K_1 \geqslant 2$, we must have $\frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2 > \frac{1}{N_1-1}\sum_{k=2}^{K_1} V_k^1$, as $\frac{1}{N_1}\sum_{k=1}^{K_1} V_k^1 > \frac{1}{N_1-1}\sum_{k=2}^{K_1} V_k^1$ by Definition 1. If $\frac{1}{N_1}\sum_{k=1}^{K_1} V_k^1 = \frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2$ and $K_1 = 1$, we must have that the counterpart of $\frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2$ (the second term of sequence $\{\frac{1}{N_2-t}\sum_{k=t+1}^{K_2} V_k^2\}_{t=0}^{K_2-1}$) is $\frac{1}{N_1+N_2-1}\sum_{k=2}^{K_2+1} V_k$ (the second term in sequence $\{\frac{1}{N_1+N_2-t}\sum_{k=t+1}^{K_1+K_2} V_k\}_{t=0}^{K_1+K_2-1}$). When $N_1 > K_1 = 1$, we have $\frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2 > \frac{1}{N_1+N_2-1}\sum_{k=2}^{K_2+1} V_k$ since $\sum_{k=1}^{K_2} V_k^2 \geqslant \sum_{k=2}^{K_2+1} V_k$. When $N_1 = K_1 = 1$, we must have $V_1^1 < V_1$ when $\frac{1}{N_1}\sum_{k=1}^{K_1} V_k^1 = \frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2$. In this case, $\sum_{k=1}^{K_2} V_k^2 > \sum_{k=2}^{K_2+1} V_k$. This implies $\frac{1}{N_2}\sum_{k=1}^{K_2} V_k^2 > \frac{1}{N_1+N_2-1}\sum_{k=2}^{K_2+1} V_k$. Based on the above arguments, there must exist at least one correspondence such that the term on the LHS of (A.2) is strictly greater that its counterpart on the RHS. □

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