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Incentivizing R&D: Prize or subsidies?

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ABSTRACT

This paper studies the optimal design of R&D contests. A "sponsor" (e.g. the US Department of Defense or the World Health Organization) wants to improve the quality of the winning products. To do so, it partitions its budget between two schemes: an inducement prize and efficiency-enhancing subsidies to the firms competing in the contest. Prizes and subsidies have different functions, and they provide complementary incentives. In the optimally designed contest, subsidies increase while the prize decreases, if the innovation process is more challenging. Further, sensible conditions are identified under which the optimal contest implements either a "handicapping" scheme (by preferentially subsidizing the "underdog") or a "national champion" scheme (by favoring the "favorite"). Our analysis yields a number of useful implications and sheds light on an array of R&D incentive schemes, such as the DoD's design competitions and vaccine development incentives.

1. Introduction

R&D contests are often sponsored by governments, firms, non-profit organizations, and even wealthy individuals, to mobilize focused effort towards various valuable missions. In a typical R&D contest, the sponsor promises a (usually fixed) monetary reward for the success of a pre-defined project. For centuries, R&D inducement prizes have inspired various scientific and technological breakthroughs, including marine technologies, locomotive engine designs, aeronautical experimentations, and even food preservation solutions.

In the modern industrial world, R&D contests are increasingly important for incentivizing innovative effort. The U.S. Department of Defense (DoD), for example, has conventionally sponsored numerous design and technical competitions to stimulate private investment in defense technology. The DoD recently posted an award of $1 million for a lighter, more wearable power system for military use. GE announced recently a $200 million "open innovation challenge" that invites inventors, entrepreneurs, and startups to compete to develop the next-generation of power grid technologies. Procter and Gamble (P&G) has often solicited new chemical synthetics from both external and internal research entities by posting contingent rewards.

The success of various R&D contests in spurring innovations has further invigorated interest in this mechanism. In 1999, the U.S. National Academy of Engineering recommended to Congress that more extensive experimentation be carried out on federally-sponsored contests. In response, the NSF has consistently increased the proportion of its budget that is devoted to sponsoring inducement prizes.[3] The 2008 Republican presidential nominee John McCain proposed an ambitious $300 million prize to develop a car battery that would "leapfrog" the current plug-in hybrids technology. This widespread interest naturally leads to an interest in enhancing our knowledge of the relevant factors that determine the efficiency of the design and implementation of such contests. The NSF, for instance, admits to a lack of experience and theoretical support in administrating R&D contests.[4] The U.S. Congress has mounted legislative pressure on various federal agencies, such as DoD and NSF, to further explore the design of R&D contests. The current paper aims to contribute to this effort.

Besides posting an inducement prize, the sponsor of an R&D contest often assists competing parties financially to improve their competence. The DoD, for instance, frequently provides "implicit

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subsidies" to shortlisted firms, which "help them win its design competitions" (Lichtenberg, 1990).[5] In the JSF (Joint Strike Fighter) competition, Lockheed Martin and Boeing were awarded $750 million each to develop their prototypes. Lichtenberg (1990) empirically shows that financial subsidies substantially improve the productivity of private military R&D. Major pharmaceutical companies not only reward biomedical startups or in-house research teams for successful innovations, but also provide "jump-start" funding to facilitate their research.

The economics literature, however, has provided little in the way of formal modelling to further our understanding as to how contingent prizes and up-front financial assistance (subsidies) could either substitute or complement each other functionally to catalyze R&D success.[6] This ambiguity prompted Lichtenberg (1988) and others to ask: "*Why does the government provide a subsidy for private military R&D, in addition to establishing prizes for innovation*", if a more generous prize purse is an effective substitute for subsidies? This paper sets out to address this question. Specifically, we explore how a sponsor, with limited resources, chooses between the use of a prize and/or subsidies to foster a required innovation.

We consider a classic "best of simultaneous submissions" contest. A sponsor (such as the DoD) is interested in an innovative technology, so she posts an inducement prize and calls for submissions from R&D firms. Two firms compete on the quality of their "products" (prototype, industrial design, technical solution, etc.), and the firm that delivers the finest submission wins a prize.[7] The sponsor attempts to maximize the (expected) quality of the winning product. Subject to a fixed budget, she optimally chooses a portfolio that is composed of a winner's purse (e.g., cash or a procurement contract), as well as subsidies to firms, which serve to improve the competence of these firms.[8]

The quality of a firm's product is (randomly) determined by its research effort (labor input) and its research capacity (physical and human capital stock, e.g. laboratory equipment, scientists and proprietary technical knowledge). While a more generous prize encourages firms to increase their effort and thus promotes their progress, a subsidy has subtler effects. First, it has a (direct) *output-amplifying* effect: it upgrades the recipient firm's research capacity, and the more productive firm will be able to produce more out of a given level of effort.[9] At the same time, a subsidy has an (indirect) *incentive* effect, as it alters firms' incentives to supply effort and the overall effect is ambiguous. There is thus a tension between these two effects when a subsidy is to be given away. Furthermore, the balance between the various forces depends critically on the characteristics of the recipients when the two firms are heterogenous. The complexity surrounding the ultimate function of subsidies further compels us to explore how a sponsor should allocate subsidies to the "right" recipients.

Our analysis provides a complete account of the optimal R&D contest, and illuminates the subtle interaction between prizes and subsidies. We demonstrate that prizes and subsidies function differently and are complements, not substitutes, for each other. More generous subsidies augment the positive impact of a prize in promoting R&D progress, while lesser subsidies weaken it. Our result responds to the question raised by Lichtenberg (1988), and provides a rationale for supplementing prize incentives by providing subsidies in various R&D contests.

The trade-off between a prize and subsidies depends on the technical characteristics of the research project pursued in the contest. Specifically, when the development process involves a higher level of "*difficulty*" or uncertainty, the optimal contest provides more subsidies and a less generous prize purse. The result yields ample practical implications for the design of R&D incentive schemes (e.g., vaccine development incentives).

We continue our analysis by exploring how subsidies should be allocated between heterogeneous firms.[10] Conventional wisdom states that a level playing field spurs competition and further incentivizes firms' effort. This rationale tends to support a "handicapping" scheme that favors weaker firms. However, our framework accommodates both the "handicapping" policy and the "national champion" policy (which preferentially subsidizes the stronger firm) as optimal designs. The efficient subsidy allocation plan depends not only on the initial endowment conditions of the firms, but also on various characteristics of the technological environment. Sensible sufficient conditions under which the optimal contest preferentially subsidizes either the "underdog" or "favorite" are identified. In particular, the sponsor could prefer a "national champion" when (1) a subsidy technically *complements* a firm's own endowment,[11] and (2) the development process involves substantial "difficulty".

Finally, we explore an additional aspect of an innovation contest's reward structure. The various R&D contests in practice, e.g. the DoD's battery design competition and GE's open innovation challenge, have usually promised a uniform prize to the winner, irrespective of the winner's identity. A sponsor's ability to provide identity-dependent prizes can often be restricted in practice. We study this issue theoretically and include an additional element into the design problem. The contest's sponsor is allowed to promise heterogenous contingent rewards to firms. We show that the optimal structure requires a uniform prize in our setting: the ex post reward (prize) depends only on the winner's performance, not on its identity, although the ex ante subsidy may discriminate between firms.

Our paper furthers the literature on the design of R&D tournaments/contests. Despite the relatively small size of this strand of literature (see Che and Gale, 2003), various factors have been identified that could determine the efficiency of such contests. Taylor (1995), Fullerton and McAfee (1999) and Che and Gale (2003) contend that shortlisting improves the performance of R&D tournaments/contests, and they explore optimal shortlisting mechanisms. Terwiesch and Xu (2008) argue that engaging a larger number of contenders improves the diversity of the scientific solutions the contest could solicit, although it disincentivizes each participant. A handful of studies have concerned themselves with the optimal prize structures in R&D contests. For instance, Denicolò and Franzoni (2010) investigate the ramifications of the winner-take-all principle in innovation races under different market conditions. Fullerton et al. (2002) and Schöttner (2008) compare the fixed-prize tournament rule with flexible reward systems. They demonstrate contrasting results, with the former advocating flexible prizes, and the latter supporting fixed prizes. Clark and Riis (2007)

[5] Under this DoD "Independent R&D" program, participating contractors are allowed to recover a fixed (capped) amount of the total expenditure that they use on federal missions. The subsidy is considered "implicit" as it does not directly offset marginal costs. Lichtenberg (1990) has shown that many firms spend much more than the subsidized amounts.

[6] Previous literature on contest design largely studied optimal allocation of budget across prizes. For example, Moldovanu and Sela (2001) study optimal allocation of prizes in all-pay auction, Fu and Lu (2009) study optimal allocation of prizes in multi-stage Tullock contests.

[7] Taylor (1995), Fullerton and McAfee (1999) and Che and Gale (2003) have demonstrated across contexts that it is optimal to shortlist only two contestants in R&D contests.

[8] The research subsidies can be used by the recipients, for example, to buy new instruments and hire more researchers.

[9] For instance, an upgraded laboratory allows more experiments to be conducted in parallel.

[10] Ex ante asymmetry across contestants is widely acknowledged in the literature. For example, Siegel (2011) studies bidding equilibria in multi-prize all-pay auctions while allowing for heterogeneity across bidders.

[11] Formal definitions of "substitutes" and "complements" are provided in Section 4.2 where the optimal subsidy allocation is analyzed.

study selection R&D tournaments, where the sponsor attempts to select the ablest R&D firm. They show that selection efficiency can be improved if the sponsor allows contestants to choose reward schedules. Cohen et al. (2008), Terwiesch and Xu (2008), and Kaplan and Wettstein (2010) study the optimal prize schedule of a R&D contest where the prize is allowed to depend on the winner's actual performance. Kaplan and Wettstein (2010) further allow the contest sponsor to offer differential rewards to asymmetric contestants. The existing tournament/contest literature has, to a large extent, ignored the possible utility a sponsor may gain from supplementing prize incentives with subsidies, and has yet to provide a rationale for the widespread adoption of subsidies in R&D contests.[12,13] Our paper attempts to fill in this gap.

The contest/tournament literature (e.g., Che and Gale, 2003; Morgan and Wang, 2010, among others) has conventionally espoused the logic of efficient handicapping, and contended that the optimal rule should favor the weaker firm in order to level the playing field. Che and Gale (2003) show that imposing a bidding cap (which handicaps the stronger firm) better incentivizes both firms. This paper allows the sponsor to moderate the competitive balance by distributing subsidies between firms. Further upsetting the balance (by preferentially subsidizing the initially stronger firm) may end up being the optimum. This paper complements the literature in this aspect. Our paper thus echoes Schwarz and Severinov's (2010) finding that it may be optimal for decision-makers to allocate all of their resources to the most promising alternative in career investment tournaments.

The remainder of this paper is structured as follows. Section 2 presents the setup of the model. Section 3 executes the formal analysis; while Section 4 provides further discussion on contest design based on the equilibrium results. Section 5 concludes the paper.

2. Model

A sponsor, who is interested in an innovative technology, invites two R&D firms to carry out this project. The two firms, indexed by $i=1, 2$, develop and submit their products to the sponsor. The firm that develops a higher-quality product is entitled to a prize $\Gamma_0>0$.

The sponsor has full discretion to divide her budget M into three parts: direct subsidies to the two firms (s_1 and s_2) and an inducement winning prize (Γ_0), with $s_1, s_2, \Gamma_0 \geq 0$.[14] She chooses her portfolio (s_1, s_2, Γ_0) to maximize the expected quality of the winning product.

2.1. Timing of moves

A two-stage game is considered. The sequence of moves is as follows. First, the sponsor announces the rule of the contest, which is represented by the profile (s_1, s_2, Γ_0). Second, observing (s_1, s_2, Γ_0), firms simultaneously commit to their R&D efforts x_1 and x_2 . The nature of R&D effort x_i will be elaborated upon in Section 2.2.

2.2. R&D competition between firms

We consider a standard "best of simultaneous submissions" tournament. The quality q_i of a firm i's product is randomly drawn from a distribution with cumulative distribution function $[F(q_i)]^{g(\theta_i,\, s_i)x_i^r}$, with $g(\theta_i, s_i)>0$ and $r\in(0, 1]$. The function $F(q_i)$ is a continuous cumulative distribution function on a support $\left[\underline{q}, \overline{q}\right]$. A higher R&D effort x_i implies that a higher q_i is more likely to be realized, and that the firm i is more likely to leapfrog its rival. This framework has been adopted by a large number of studies. An intuitive interpretation of this family of models can be seen in Fullerton and McAfee (1999), and Baye and Hoppe (2003) and Fu and Lu (2011).[15]

The function $g(\theta_i, s_i)$ measures a firm i's research capacity, which is determined by its physical and human capital stock. θ_i reflects the firm's initial endowment (e.g. the conditions of existing laboratory equipment, the quality of available scientists, and knowledge or experience in relevant areas, etc.), while s_i represents the additional financial resources provided through a subsidy. The function $g(\theta_i, s_i)$ satisfies the following properties.

Condition 1. *Both $\theta_i$and $s_i$contribute to a firm i's research capacity, i.e., $g_{\theta_i}>0$, $g_{s_i}>0$, and their marginal impacts decrease, i.e., $g_{\theta_i\theta_i}\leq 0$, $g_{s_is_i}\leq 0$.*

A financial subsidy adds to a firm's R&D capacity. For example, the firm could spend the subsidy upgrading laboratory equipment, hiring additional scientists, and acquiring relevant intellectual assets. Additional physical or human capital stock boosts a firm's productivity. For instance, additional laboratory hardware allows the firm to conduct more parallel experiments. Assume without loss of generality $\theta_1\geq\theta_2$. That is, firm 1 is ex ante better endowed.

We conveniently call $g(\theta_i, s_i)x_i^r$ a firm i's *(direct) R&D output* (e.g. scientific knowledge). Higher output facilitates the firm's success in winning the R&D race. The production function $g(\theta_i, s_i)x_i^r$ takes a multiplicative form. It reflects the complementarity between a firm's physical or human capital stock and its labor input (effort) in carrying out R&D.

A firm's R&D effort x_i represents mainly its non-pecuniary (labor) input devoted to the targeted project, which is usually unverifiable (Taylor, 1995). It could include time, energy and opportunities that were sacrificed to pursue this project. x_i^r depicts how a firm i's R&D effort contributes to its output for a given research capacity. The parameter r literally measures the elasticity of a firm's production function to additional effort. It indicates the effectiveness of R&D effort in this particular innovation activity. The size of r is determined by the technical characteristic of the innovation project and is common to both firms. Its implication is further elaborated upon in Section 4.1.

Two remarks are in order. First, the competition depicted by our model does not coincide with the standard scientific grant application process (e.g., NSF grant applications). Grant applicants compete based on the quality of their proposals and the significance of their research questions, while grants are given away as "prizes" that reward the strongest proposals. In our model, firms race towards a pre-defined innovation project. The winner is rewarded with a prize Γ_0. Instead of being merit-based, research subsidies are provided non-contingently. Second, in our setting, the role played by a subsidy s_i closely mirrors the "implicit subsidy" scheme in the DoD's design competitions. The DoD allows contractors to "recover" a fixed (capped) amount of certain types of expenditure that were incurred specifically for participating in the competition, but does not directly offset marginal cost (see Lichtenberg, 1988).

[12] One notable exception is provided by Siegel (2011), who allows the contest designer to provide subsidies to players in a deterministic contest (all-pay auction) setting.

[13] A handful of papers allow for direct transfers between the sponsor and contestants. Taylor (1995), Fullerton and McAfee (1999) and Fu and Lu (2010) allow the contest sponsor to charge entry fees to restrict entries. Moldovanu et al. (2009) allow the contest organizer to impose financial punishments to incentivize contestants. The current paper assumes limited liability and allows only nonnegative resource transfers to competing parties.

[14] It is assumed throughout the paper that the R&D firms are subject to limited liability, which thus requires that the monetary transfer s_i from the sponsor be nonnegative.

[15] This framework was first proposed and applied by Tan (1992), Piccione and Tan (1996).

R&D effort incurs a unitary marginal cost to a firm.[16] Each firm i chooses R&D effort x_i to maximize its expected payoff

$$\pi_i\left(x_i, x_j\right) = Pr\left(q_i > q_j | x_i, x_j\right) \ \Gamma_0 - x_i, \ i = 1, 2.$$

2.3. Sponsor's optimization problem

Denote by q_{max} the quality of winning product. For a given effort profile (x_1, x_2), q_{max} is distributed according to the cumulative distribution function $[F(q_{max})]^{[g(\theta_i, s_i)x_i^r + g(\theta_j, s_j)x_j^r]}$. Hence, maximizing $E(q_{max})$ is equivalent to maximizing $[g(\theta_i, s_i)x_i^r + g(\theta_j, s_j)x_j^r]$, which is called *overall output* hereafter since it ultimately determines the performance of the contest.

3. Analysis

In this section, we characterize the subgame perfect equilibrium of the two-stage game.

3.1. Equilibrium R&D effort

Each firm i wins if and only if $q_i > q_j$. The probability of the event is then given by

$$\begin{aligned} Pr\left(q_i > q_j | x_i, x_j\right) &= \int_{\underline{q}}^{\overline{q}} [F(q_i)]^{g(\theta_j, s_j)x_j^r} dF(q_i)^{g(\theta_i, s_i)x_i^r} \\ &= \frac{g(\theta_i, s_i)x_i^r}{g(\theta_i, s_i)x_i^r + g\left(\theta_j, s_j\right)x_j^r}. \end{aligned} \tag{1}$$

Each firm i chooses its R&D effort x_i to maximize its expected payoff

$$\begin{aligned} \pi_i\left(x_i, x_j\right) &= Pr\left(q_i > q_j | x_i, x_j\right) \ \Gamma_0 - x_i \\ &= \frac{g(\theta_i, s_i)x_i^r}{g(\theta_i, s_i)x_i^r + g\left(\theta_j, s_j\right)x_j^r}\Gamma_0 - x_i, \ i = 1, 2. \end{aligned}$$

Standard techniques lead to the well-known results in contest literature. In the unique equilibrium, each firm exerts an R&D effort

$$x^*(s_1, s_2, \Gamma_0) = \frac{rg(\theta_i, s_i)g\left(\theta_j, s_j\right)}{\left[g(\theta_i, s_i) + g\left(\theta_j, s_j\right)\right]^2}\Gamma_0. \tag{2}$$

A more capable firm does not have to exert more effort than its rival. The asymmetry between the firms is reflected in the differing levels of the equilibrium technical output $g(\theta_i, s_i)x_i^r$, which guarantees that the more efficient firm stands a better chance of winning the competition.

3.2. Optimal budget allocation

The sponsor maximizes $H(s_1, s_2, \Gamma_0) \equiv [g(\theta_i, s_i) + g(\theta_j, s_j)]x^{*r}$. Differentiating $H(s_1, s_2, \Gamma_0)$ with respect to s_i and Γ_0 yields

$$\frac{\partial H(s_1, s_2, \Gamma_0)}{\partial s_i} = H(s_1, s_2, \Gamma_0)\frac{g_{s_i}(\theta_i, s_i)}{g(\theta_i, s_i) + g\left(\theta_j, s_j\right)}\left[1 - r + r\frac{g\left(\theta_j, s_j\right)}{g(\theta_i, s_i)}\right], i, j = 1, 2, i \neq j, \tag{3}$$

and

$$\frac{\partial H(s_1, s_2, \Gamma_0)}{\partial \Gamma_0} = H(s_1, s_2, \Gamma_0)\frac{r}{\Gamma_0}. \tag{4}$$

Before exploring the optimal budget allocation profile $(s_1^*, s_2^*, \Gamma_0^*)$, the roles played by these strategic instruments are first examined.

Proposition 1. *All three instruments contribute to overall output $H(s_1, s_2, \Gamma_0)$, i.e. $\frac{\partial H(s_1, s_2, \Gamma_0)}{\partial \tau} > 0$, where τ denotes any of the three instruments (s_1, s_2, Γ_0).*

Proof. See Appendix A. ■

Proposition 1 states that a more generous prize purse or an additional subsidy always amplifies the overall output. As a result, the optimal allocation profile requires the budget of the sponsor to be binding.

In what follows, an approach of "*two-step optimization*" is adopted to analyze the three-dimension optimization problem. In the first step, a *sub-problem* is constructed, which concerns itself only with the efficient allocation of subsidies. We explore how a given amount of resources s should be divided between s_1 and s_2 in order to maximize $H(s_1, s_2, \Gamma_0)$, when the size of inducement prize Γ_0 is held fixed. In the second step, we investigate how to optimally allocate a total budget between subsidies and a prize purse, given that any budget available for subsidies is to be optimally split. We first obtain the following.

Lemma 1. *Under Condition 1, $H(s_1, s_2, \Gamma_0)$ is a single-peaked function of s_i, $i = 1, 2$, for any given (s, Γ_0).*

Proof. See Appendix A. ■

Lemma 1 implies that for any given Γ_0, the sub-problem is a well-behaved one. A unique allocation profile $(s_1^*(s, \Gamma_0), s_2^*(s, \Gamma_0))$ exists, which optimally divides s between s_1 and s_2. The technical property of this sub-optimization problem is characterized in Lemma A1 (see Appendix A).

This result allows us to reduce the three-dimensional optimization problem to a two-dimensional optimization problem that allocates M between s and Γ_0. Define $H(s; \Gamma_0) = \max_{\{s_1 + s2 = s\}} H(s_1, s_2, \Gamma_0)$, which is the value function of the sub-optimization problem when s is efficiently split between s_1 and s_2. We maximize the value function $H(s; \Gamma_0)$ by splitting M between s and Γ_0, instead of maximizing $H(s_1, s_2, \Gamma_0)$ directly.

Lemma 2. *Under Condition 1, the value function $H(s; \Gamma_0)$ exhibits the following properties: (i) $H(s; \Gamma_0)$ is continuous, differentiable and strictly increasing with both s and Γ_0; (ii) $H(s; \Gamma_0)$ is strictly quasi-concave in (s, Γ_0); and (iii) $H(s; \Gamma_0)$ is supermodular in (s, Γ_0), i.e., $\frac{\partial^2 H(s; \Gamma_0)}{\partial s \partial \Gamma_0} > 0$.*

Proof. See Appendix A. ■

These properties guarantee that maximizing $H(s; \Gamma_0)$ subject to the budget constraint $s + \Gamma_0 = M$ is a well-behaved program. The following result can be formally established.

Proposition 2. *Under Condition 1, for any M, a unique budget allocation plan $(s_1^*, s_2^*, \Gamma_0^*)$ exists to maximize $E(q_{max})$.*

These results facilitate our subsequent analysis. The well-behaved maximization problem allows us to focus on the local properties of $H(s_1, s_2, \Gamma_0)$ with respect to the three arguments to delineate the properties of the (unique) global optimum.[17]

[16] Firms may differ in terms of the marginal costs of their R&D activities. Cost asymmetry is not explicitly included in this model. However, the asymmetry in terms of productivity can be reflected by the difference in efficiency parameter θ_i, as a higher marginal product of effort implies that a given amount of technological output requires a lower cost.

[17] Closed-form solutions to this problem were obtained in parameterized settings, where $g(\cdot, \cdot)$ is assumed to take specific well-behaved functional forms, e.g., linear functions or Cobb–Douglas functions. Analysis of these specific cases is omitted for the sake of brevity. However, these results are available from the authors upon request.

Before proceeding, the result of Lemma 2(iii) deserves to be remarked upon more carefully. The supermodularity implies that a given prize purse contributes more to the overall output when more generous subsidies are provided. Prizes and subsidies provide *complementary* incentives, instead of being *substitutes* for each other. More subsidies amplify the effect of a winner's purse; while less subsidies weaken it. This result addresses the question raised by Lichtenberg (1988): *"Why does the government provide a subsidy for private military R&D, in addition to establishing prizes for innovation"*, if a more generous prize purse is an effective substitute for subsidies? The complementarity between the two instruments thus provides a rationale for the practice of supplementing prize incentives through subsidies in various R&D contests (e.g., DoD's design competitions). Our analysis further leads to the following.

Proposition 3. *$s_1^*(M)$, $s_2^*(M)$, $\Gamma_0^*(M)$ weakly increase with M.*

Proof. See Appendix A. ■

Proposition 3 shows that all the three instruments are "normal goods" to the sponsor: When more resources are available, all the three elements in her bundle $(s_1^*, s_2^*, \Gamma_0^*)$ are assigned more resources. This result is underpinned by the complementarity between a prize and subsidies, as well as the quasi-concavity of the optimization problem. An allocation profile that involves a corner solution (i.e., zero resources on certain instruments) could emerge as the optimum.[18] Proposition 3, nevertheless, indicates that such an equilibrium arises simply due to scarcity of resources, while a more balanced portfolio would result when the resource is more abundant.

4. Discussion

In this part, we utilize our equilibrium result to obtain further insight into this design problem. The well-behaved program allows us to explore in depth (1) *the trade-off and interaction between a prize and subsidies*, (2) *the efficient allocation of subsidies between heterogenous firms;* and (3) *alternative prize schedule*. Further, we discuss in Section 4.4. the alternative application of our analysis.

4.1. Prize vs. subsidies: the role of "r"

The trade-off between a prize and subsidies depends critically on the parameter r. As mentioned earlier, the size of this parameter is determined by the project's technological characteristics. This parameter indicates the effectiveness of a firm's research efforts in producing output (e.g., generating knowledge). Alternatively, it depicts the level of difficulty or uncertainty in the project. Recall that the quality of a firm's final product is drawn from the distribution $F(q_i)^{g(\theta_i,s_i)x_i^r}$. A larger r implies that each additional amount of effort improves product quality more reliably and deterministically. Conversely, a lower r indicates a more difficult or uncertain development process: on average, more trials will be needed to improve the quality of the product.

An increase in r amplifies the incentives provided by a prize. Firms tend to increase their R&D outlays as their effort becomes more effective. Hence, a given prize purse elicits a greater amount of effort. A larger r amplifies the return from subsidies as well. A higher overall output $(g(\theta_1, s_1)+g(\theta_2, s_2))x^{*r}$ can be expected out of a given level of subsidies, because (1) firms are more productive (a larger r), and (2) firms supply more effort (because of the aforementioned effect). To put it intuitively, laboratory equipment can contribute more when its user works harder or works more efficiently. The overall effect on the trade-off between a prize and subsidies remains obscure. Our analysis concludes the following.

Proposition 4. *When r increases, s_1^*, s_2^* decrease and Γ_0^* increases. In other words, a less challenging innovation process requires a more generous prize and lesser subsidies.*

Proof. See Appendix A. ■

Our analysis confirms that $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}$ responds more sensitively than $\frac{\partial H(s;\Gamma_0)}{\partial s}$ when r increases. Hence, a greater r results in a more generous prize. By contrast, the sponsor provides more subsidies when the innovation project involves a higher level of difficulty, i.e., when r is smaller.

The logic of this result can be interpreted intuitively. Consider a difficult project, i.e., where a small r applies. First, a prize incentive must be heavily discounted to encourage firms to exert some effort. Firms supply less effort when they anticipate less reliable returns. Second, although a more generous prize encourages firms to expend more effort, it has only a limited effect on the overall output because of the smaller r. Both forces tilt the trade-off towards a greater use of subsidies.

4.1.1. Implications

A more generous prize purse could successfully lure competitive effort on a less challenging or low-risk project, such as a standard process recipe for chemical synthesis, a rapid and accurate computer algorithm, and a diet drug, or even an effective vaccine for malaria. Our analysis recommends that greater priority should be given to providing prize incentives when such technically less demanding projects are being pursued.

By contrast, a generous prize purse would be less effective in facilitating major breakthroughs, such as a vaccine for HIV or effective cancer treatments. Our analysis can contribute to the ongoing debate on the proper level of publicly-provided incentives for critical medical research. The Center for Global Development, for instance, has suggested that prizes (e.g. government purchase commitments) be offered to incentivize private vaccine development. Michael Kremer (2001) advocates the awarding of "funds only upon the completion of a vaccine" (as an alternative to upfront funding).[19] This proposal was positively received by the Group of Seven major developed nations (G-7). However, our results recommend that the role of a (non-contingent) subsidy, as well as its trade-off against prize incentives, be assessed more carefully.

Proposition 4 espouses the possible merit of up-front subsidies that assist pharmaceutical research entities (in their pursuit of major breakthroughs). Such development requires decades of continuous input without the promise of any progress. Without adequate subsidies and assistance, this tremendous difficulty not only diminishes firms' incentives to supply effort in response to a prize, but also limits the possible output of their effort. As indicated by an IAVI (International AIDS Vaccine Initiatives) report, "many small companies who are willing to take more risks on discovery and early testing often lack the capital and expertise".[20]

4.2. The favorite vs. the underdog: handicapping or "National Champion"?

We now explore another aspect of the optimization problem: the efficient allocation of subsidies between two heterogenous firms. Without loss of generality, we assume $\theta_1 > \theta_2$. Recall that the sponsor maximizes

$$H(s_1, s_2, \Gamma_0) = \left[g(\theta_i, s_i) + g\left(\theta_j, s_j\right)\right]x^{*r}. \quad (5)$$

[18] Examples of corner solutions in parameterized settings are omitted for brevity but they are available from the authors upon request. However, Lemma A1 in Appendix in fact points towards the possibility of corner solutions.

[19] The reader is referred to "Creating New Market for Vaccines", in Adam Jaffe, Joshua Lerner, and Scott Stern, eds., *Innovation Policy and Economy*, Vol. 1, Cambridge, MA: MIT Press, 2001, 35–72.

[20] Source: *Policy Brief #2: Incentives for Private Sector Development of an AIDS Vaccine*, IAVI, 2004.

As implied by Eq. (5), a subsidy to a firm yields two effects. It improves the recipient firm's competence, which *directly* amplifies the output of x^* by increasing $[g(\theta_i, s_i)+g(\theta_j, s_j)]$. At the same time, a subsidy exercises an *indirect* effect, which moderates firms' incentives to supply effort. Evaluating equilibrium effort $x^* = \frac{rg(\theta_i,s_i)g(\theta_j,s_j)}{[g(\theta_i,s_i)+g(\theta_j,s_j)]^2}\Gamma_0$ with respect to s_i yields

$$\frac{\partial x^*}{\partial s_i} = \frac{rg_{s_i}(\theta_i,s_i)g(\theta_j,s_j)\Gamma_0}{[g(\theta_i,s_i)+g(\theta_j,s_j)]^3}\left[g(\theta_j,s_j)-g(\theta_i,s_i)\right].$$

This reveals the nature of the indirect incentive effect: an additional subsidy increases the level of effort if and only if it levels the playing field, i.e., narrowing the gap between $g(\theta_j, s_j)$ and $g(\theta_i, s_i)$. This incentive effect, as first noted by Dixit (1987), reflects the conventional wisdom in the design of contest mechanisms: A more even race leads to more competition. The contest/tournament literature thus widely espouses the notion of efficient handicapping, which suggests preferential treatment for the weaker firm.[21] However, in our context, this incentive effect could run into conflict with the direct effect. The overall effect depends on the tension between these two effects. Either the favorite or the underdog may turn out to be favored by the contest rule.

The nature of the direct effect depends critically on the properties of the function $g(\cdot,\cdot)$. It depicts how a firm's initial endowment θ_i and its subsidy s_i interact in forming its research capacity. θ_i and s_i may interact in two possible ways, which lead to differing optima.

Definition 1. *$\theta_i$and s_i are technically substitutable (complementary), if and only if $g_{\theta_i s_i}\leq(>)0$.*

The initial endowment θ_i and the subsidy s_i are *technical substitutes (complements)*, if and only if an increase in one decreases (increases) the marginal effect of the other. The substitutability and complementarity between θ_i and s_i each mirror one distinct technological environment. The economic implications of the differing model settings will be interpreted in detail after we present our analytical results.

4.2.1. Substitutable θ_i and s_i

The following result is obtained in the case where θ_i and s_i are technically substitutable.

Proposition 5. Efficient Handicapping

Whenever θ_i and s_i are technically substitutable, i.e., $\frac{\partial^2 g(\theta_i,s_i)}{\partial\theta_i\partial s_i}\leq 0$, the firm that is initially weaker always receives strictly more subsidies than the other, i.e., $s_1^*<s_2^*$, unless the entire budget is allocated to the prize.

Proof. See Appendix A. ■

Consistent with the conventional wisdom, Proposition 5 shows that the sponsor has to preferentially subsidize the firm that is initially less endowed when θ_i and s_i are technically substitutable. In this case, the direct and indirect effects both point towards a "handicapping" scheme: besides its positive indirect incentive effect, favoring the weaker firm also allows $[g(\theta_1, s_1)+g(\theta_2, s_2)]$ to increase more rapidly.

4.2.2. Complementary θ_i and s_i

A "handicapping" scheme may lose its bite when θ_i and s_i are complementary, i.e., $\frac{\partial^2 g(\theta_i,s_i)}{\partial\theta_i\partial s_i}>0$. Whenever $s_1=s_2=s$, $g_{s_1}(\theta_1, s_1)$ must be strictly greater than $g_{s_2}(\theta_2, s_2)$, because of the complementarity. Hence, favoring the stronger firm allows $[g(\theta_1, s_1)+g(\theta_2, s_2)]$ to increase more rapidly and yields a stronger direct effect. The direct effect rivals the indirect (incentive) effect in this case. The sponsor may prefer to create a "national champion" by further assisting the stronger firm.

The trade-off depends subtly on a number of factors, including the properties of $g(\cdot,\cdot)$, the effectiveness of R&D effort (the magnitude of r), and the amount of available resources. So far, little restriction has been imposed on $g(\cdot,\cdot)$, which limits our ability to come to a definitive conclusion. We focus on the class of multiplicatively separable functions $g(\theta, s)=\vartheta(\theta)\rho(s)$, with $\vartheta'(\theta)>0$, $\rho'(s)>0$, and $\vartheta''(\theta)\leq 0$, $\rho''(s)\leq 0$. The following can be concluded.

Proposition 6. "National Champion" vs. Handicapping

When $g(\theta, s)$ takes the multiplicatively separable form $g(\theta, s)=\vartheta(\theta)\rho(s)$, the stronger firm (firm 1) is preferentially subsidized if and only if $r<\frac{1}{2}$, i.e., $s_1^*\lesseqgtr s_2^*$, if $r\lesseqgtr\frac{1}{2}$.

Proof. See Appendix A. ■

Proposition 6 demonstrates the role played by the parameter r, the measure of difficulty or risk involved in the project, in moderating the trade-off. Favoring the weaker (stronger) firm is optimal if and only if R&D effort is sufficiently effective (ineffective). In contrast to previous studies, the sponsor in our contest does not necessarily prefer a more balanced competition. The optimal contest implements either a "handicapping" scheme or a "national champion" scheme (by favoring the "favorite"), depending on the specific characteristics of the technological environment.

We briefly interpret the logic of the result. Favoring firm 1 allows the overall capacity $[g(\theta_1, s_1)+g(\theta_2, s_2)]$ to increase more significantly, but sacrifices equilibrium effort. The incentive effect of handicapping is less significant when r is smaller. First, as aforementioned, a smaller r diminishes firms' incentive to exert effort. The incentive effect of handicapping would be limited, as firms anticipate lesser returns. Second, a smaller r implies less productive R&D effort. Hence, a given increase in x causes a lesser increase in overall output $H(s_1, s_2; \Gamma_0)=[g(\theta_1, s_1)+g(\theta_2, s_2)]x^{*r}$. Both of these forces diminish the possible contribution of a handicapping scheme. The direct (output-amplifying) effect thus overweighs the indirect (incentive) effect when r is sufficiently small, and leads to a "national champion" scheme.

It should be noted that the result of Proposition 6 is not limited to the setting of multiplicatively separable $g(\theta, s)$, and would continue to hold in broader contexts. As demonstrated in the proof of this proposition (see Appendix A), to determine whether firm 1 should be allocate more subsidy than firm 2, one only need to compare $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{s_1=s_2}$ with $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_1=s_2}$, the marginal products of s_1 and s_2 when subsidies are evenly distributed. Simple analysis would verify that $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big/\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_1=s_2}$ strictly decreases with r whenever θ_i and s_i are complementary, regardless of the specific functional form of $g(\theta, s)$. It implies that a smaller r would in general favor a more generous subsidy to firm 1, i.e. a "national champion" scheme, in a setting with resource complementarity.

4.2.3. Substitutability vs. Complementarity: economic interpretation

We now discuss in depth the economic implications of the relationship between θ_i and s_i, which is critical in determining the appropriate scheme for resource allocation. The subsidy s_i allows the recipient to acquire additional inputs, such as laboratory equipment, to replenish its own endowment. These assets can be either substitutes or complements to the recipient's existing capital stock (θ_i). This relationship depends on various factors, including the capability of a firm to acquire resources, the mechanisms through which various inputs interact to develop the firm's research capacity, and the condition of the firms' existing endowments.

The additional input funded through s_i can arguably be a substitute to θ_i when the subsidy allows the firm to procure assets that

[21] The reader is referred to Che and Gale (1998, 2003) and Morgan and Wang (2010) among others.

are the same as existing inputs. For example, assume that a firm's research capacity is determined mainly by its level of physical capital input (e.g. laboratory equipment), and θ_i is a measure of its current endowment. In such a scenario, providing a financial subsidy s_i to an inadequately-equipped firm can make up for its lack of physical endowments θ_i, but will be less effective if the firm is adequately equipped. An environment with $\frac{\partial^2 g(\theta_i, s_i)}{\partial\theta_i \partial s_i} \leq 0$ would plausibly emerge when the additional inputs function in a similar way to firms' existing ones.

However, the additional inputs a firm acquires may not function similarly to those in its existing capital stock. We now elaborate upon the possible scenarios where technical complementarity could plausibly emerge.

It is possible that a firm's existing mix of sunk inputs (θ_i) is sub-optimal. The firm would then prefer productive assets that are heterogenous and complementary to its own endowment.[22] The subsidy would allow a firm to optimize the composition of the inputs in its capital stock. The newly acquired resources would not be mere addition to one's endowment, as they exercise different functions instead of duplicating the roles played by the existing inputs.

Furthermore, it is not uncommon that a firm's research capacity depends critically on its intangible assets, e.g. its expertise, experience and proprietary knowledge in a certain domain. Building research capacity requires the combination of both physical and intellectual inputs. In that case, an environment with technical complementarity could plausibly occur. Imagine the following situation. Firms race towards a project on the technological frontier and its success depends heavily on firms' knowledge and research skills. Firms are endowed with differing levels of intellectual competence in R&D, so the difference in their endowment θ_i mainly reflects their differing innate abilities to conduct research. A financial subsidy is useful for building up a firm's physical capital stock (e.g. equipment), but it is less effective to narrow the gap in terms of intellectual competence, and to compensate directly for its deficiency in this aspect. When a firm acquires additional physical assets, the new resources are more likely to complement its existing intellectual resource, as they can plausibly augment each other, but would be less likely to be a substitute. For instance, a more knowledgeable scientist can utilize laboratory equipment more efficiently and more productively, i.e. $\frac{\partial^2 g(\theta_i, s_i)}{\partial\theta_i \partial s_i} > 0$.

There are a few reasons to believe that a firm will find it easier to acquire physical assets than intellectual resources. First, firms can be subject to administrative constraints, and can be intensively monitored on the use of the subsidies. For instance, private contractors in DoD's design competition can only claim the expenditure that is verifiably dedicated to winning the contest.[23] Such requirement may limit their ability to acquire intangible assets that have broader applications.

Second, acquiring intellectual assets (such as technological know-how) is usually a more difficult task, even when the subsidies are allowed to be used to purchase human capital resources. Although a firm can solicit knowledge in the market by technology licensing or outsourcing, its ability in this aspect is limited. The economics and management literature points out that knowledge is often tacit in nature, which prevents efficient contracting in market transactions and leads parties to seek alternative means for facilitating informations sharing (e.g. partial ownership).[24] Moreover, critical intellectual assets, which determine a firm's strategic competence, are often heavily protected by owners from leakage. As documented by Cohen et al. (2000) and Arundel (2001), firms reap more benefits from their unpublicized "trade secrets" than their publicized patents of innovative activities. They usually restrict trade secrets for their own use and go to great lengths to prevent them leaking out. The response of Nicole Wong, Google's Associate General Counsel, to the DoJ's 2006 motion provides some evidence of the exclusivity of such knowledge. As she stated, "Google avidly protects every aspect of its search technology from disclosure, even including the total number of searches conducted on any given day." Such knowledge is usually accessible to only a very limited subset of personnel in a firm. Non-disclosure clauses are popularly included in employment contracts as a protective measure, which prevents the disclosure of critical knowledge even when critical employees are hired by competitors.

Moreover, a financial subsidy would not allow a firm to acquire direct substitutes to θ_i even if it can effectively build up its knowledge stock. In fact, newly-acquired intellectual resources may often complement a firm's existing intellectual resources. As Png (2011) finds, the knowledge a firm obtains from other sources complements the knowledge it has developed itself, instead of substituting for it.

In summary, our analysis demonstrates that the optimal structure relies critically on the relationship between θ_i and s_i. The results yield rich implications for both theory and practice. Our analysis suggests that a sponsor should identify firms' endowment conditions when allocating subsidies and should be aware of the critical factors that determine the gap in the firms' competence. For instance, if a firm leads its rivals mainly because it has more hardware, this gap can be easily narrowed by subsidizing the weaker firms. However, the implications can be reversed and a handicapping mechanism may not be very effective if the leader is the exclusive owner of a critical intellectual asset. For instance, a more knowledgeable or experienced firm may "deserve" more subsidies, as it could make better use of any given physical resource (e.g. laboratory equipment).

In the latter case, the efficient allocation of subsidies depends not only on firms' endowment conditions, but also the nature of the targeted project. Recall that r reflects the level of difficulty or uncertainty involved in the development process. When θ_i and s_i are complementary, a National Champion scheme gains more appeal especially when the innovation being pursued requires more trials and errors. While a handicapping rule can be more effective when firms chase a regular industrial solution by exercising conventional techniques, it will be less effective when firms compete for a less foreseeable major breakthrough, such as the next generation stealth jet fighter or an effective HIV vaccine. In these cases, as we have shown, a smaller r decreases the attractiveness of a handicapping scheme. Its positive incentive effect comes at an excessive cost: a handicapping scheme has to spread resources on the weaker firm that is less likely to deliver the superior product ex post.

4.3. Heterogenous prizes

We have assumed so far that the sponsor promises a uniform prize schedule to the firms, even though they can be provided heterogenous prizes. This modeling nuance conforms to our observations of various public R&D contests, such as NASA's Centennial Challenges (NASA), GE's "Open Innovation Challenge" and the various design competitions sponsored by the DoD's Defense Advanced Research Projects Agency (DARPA). In this context, the sponsor may have more flexibility in setting discriminatory subsidy schemes than providing identity-dependent reward schemes. The prize is usually announced publicly to solicit firms' participation. By contrast, as pointed out by Lichtenberg (1988, 1990), among others, the amount of subsidies the DoD provides to each competing firm is decided through individual negotiation after the firm has demonstrated a serious interest in the

[22] We thank an anonymous referee to alert us of this possibility.

[23] As suggested by an anonymous referee, the use of the subsidy is usually monitored heavily. For example, it could only be allowed to be used for purchasing equipment.

[24] See Mowery et al. (1996) and Ghosh and Morita (2010), among many others, for discussions on the tacit nature of knowledge and the difficulty involved in market transactions involving the transfer of knowledge.

design competition. The DoD takes into account various specific characteristics of the firm in making its decision.

Although the setting conveniently reflects the convention in practice, the ramifications of the "uniform prize rule" have yet to be explored more seriously. It is unclear whether this practice is adopted simply for the sake of convenience, or whether it makes the contest more efficient. We now relax this assumption and explore the contest design in a more generalized setting. We allow the sponsor to choose a portfolio $(s_1, s_2, \Gamma_1, \Gamma_2)$, where Γ_i, $i \in \{1,2\}$, denotes the prize she promises to firm i if it succeeds. The identity-dependent prizes do not have to be uniform. The budget constraints are thus written as

$$s_1 + s_2 + \Gamma_i \leq M, \forall i = 1, 2. \tag{6}$$

We now consider an arbitrary contest $(s_1, s_2, \Gamma_1, \Gamma_2)$ with $\Gamma_1 \neq \Gamma_2$. By standard techniques, one can verify that each firm i sinks equilibrium R&D effort

$$x_i^* = \frac{rg(\theta_1, s_1)g(\theta_2, s_2)\Gamma_1^r\Gamma_2^r}{\left[g(\theta_1, s_1)\Gamma_1^r + g(\theta_2, s_2)\Gamma_2^r\right]^2}\Gamma_i, \forall i = 1, 2. \tag{7}$$

Hence, the overall output of the contest is given by

$$\begin{aligned} H(s_1, s_2, \Gamma_1, \Gamma_2) &= g(\theta_1, s_1)x_1^{*r} + g(\theta_2, s_2)x_2^{*r} \\ &= \left[rg(\theta_1, s_1)g(\theta_2, s_2)\Gamma_1^r\Gamma_2^r\right]^r \times \left[g(\theta_1, s_1)\Gamma_1^r + g(\theta_2, s_2)\Gamma_2^r\right]^{1-2r}. \end{aligned} \tag{8}$$

While the sponsor has plenty of flexibility in modifying the portfolio, we restrict our attention to the effect of varying the prize structure marginally. Let $\Gamma_i > \Gamma_j$. The sponsor can always feasibly increase Γ_j without breaking its budget constraint. On one hand, a non-uniform prize rule leaves the budget non-binding if firm j turns out to win, which leaves unused resources ex post. On the other hand, it moderates the balance of the competition. Firms of differing characteristics respond to prize incentives asymmetrically. Heterogenous prizes, which are customized to firms' specific characteristics, may further incentivize the firms (see Kaplan and Wettstein, 2010). Indeed, when Γ_j increases, firms respond asymmetrically and a firm's equilibrium R&D effort may either decrease or increase, depending on the specific structure of the contest.[25] As a result, the overall effect on innovation quality remains ambiguous. Our formal analysis, however, illustrates that a contest with heterogenous prizes is suboptimal.

Proposition 7. *(i) Consider an arbitrary feasible allocation plan that satisfies the constraint (6). Let $\Gamma_j < \Gamma_i$, with $i, j \in \{1, 2\}$ and $i \neq j$. The output of the contest $H(s_1, s_2, \Gamma_1, \Gamma_2)$ strictly increases with Γ_j. (ii) In the optimum, prize does not depend on the identity of the winner, i.e. $\Gamma_1^* = \Gamma_2^*$.*

Proof. See Appendix A. ■

Proposition 7 establishes that overall output increases when the sponsor narrows the gap between the prizes it promises the two firms. This immediately implies that the optimal contest requires the sponsor to adopt a uniform prize rule, such that the reward for success does not depend on the identity of the winning firm. The optimal contest provides an ex post equal prize which rewards successful firms equally, irrespective of their identity. In our setting, the sponsor does not lose its flexibility in designing an efficient contest when committing to a uniform prize. Two remarks are provided as follows.

First, Kaplan and Wettstein (2010) demonstrate that it is optimal to set different rewards for asymmetric firms. The different results are driven by the differences in the settings. In our setting, the sponsor has a fixed budget. She internalizes and accommodates the inter-firm differentials through ex ante discriminatory subsidy schemes. By contrast, the sponsor in Kaplan and Wettstein's (2010) setting does not provide subsidies to firms, but has more financial flexibility in setting contingent reward schemes, which help to accommodate the asymmetry between firms. The differences in the constraints and available instruments in contest design lead to diverging predictions. The two papers thus complement each other by providing insights on contest design in different contexts.

Second, we assume that the budget constraint has to hold ex post. We focus on this setting to maintain consistency with the benchmark setup. One line of further extension can be enlightened immediately by considering ex ante budget constraint, which means that the sponsor only needs to ensure that the expected expenditure (on prizes and subsidies) to be bounded by the budget. The sponsor obviously has more flexibility in financial planning under an ex ante budget constraint, which allows for heterogenous winner prizes and ex post budgetary slack without losing efficiency ex ante. Ex post binding budget constraint would instead appear only as a special case. The ex ante budget constraint triggers interesting trade-off. On the one hand, when the sponsor attempts to further incentivize one firm with a more generous prize that is above the expected available budget, she has to disincentivize the other by reducing his prize below the expected level. On the other hand, the flexibility to "over-incentivize" one contestant (i.e. a winner prize higher than the ex ante expected level) may compensate for the loss from "under-incentivizing" the other one. The additional flexibility in budget alludes to the possibility that non-uniform prizes could be optimal, while the overall incentive effect of non-uniform prizes remains less than explicit. Further, the discussion so far does not exhaust the complexity involved in allowing for ex ante budget constraint. Note that for given subsidy allocation (s_1, s_2), an ex ante budget constraint is written as $p_1\Gamma_1 + p_2\Gamma_2 \leq M - (s_1 + s_2)$. The winning probabilities of the contestants depend on the prize structure. The budget constraint is therefore endogenously determined in nature. A complete characterization of the optimum under ex ante budget constraint goes beyond the scope of our paper. However, the optimum deserves to be explored more formally in future studies.

4.4. Alternative interpretation: "first past the post" contests

Our analysis explicitly focuses on the setting of a "best of simultaneous submission" contest. However, the results from the current framework would not lose their bites in alternative contexts. For instance, the results shed light on the efficient design of the R&D contest in another popular form, i.e. "first past the post" contests.

Consider the following scenario. Two R&D firms race towards an innovation of a given nature. The firm that achieves the sought after discovery sooner than the other wins the race. A sponsor benefits from the innovation. For example, it can be a public agency like the World Health Organization (WHO) that requires a particular technology, such as an effective vaccine, to fulfill a particular goal, such as to effective reining in a deadly epidemic. The sponsor can also be a firm that tries to outsource a technical solution to facilitate the obtainment of a greater market share. Alternatively, the sponsor can be a non-profit science foundation dedicated to inspiring scientific breakthroughs, such as the solving of a major mathematical puzzle. The sponsor attempts to maximally speed up the discovery by optimally utilizing her fixed budget, by properly choosing a portfolio (s_1, s_2, Γ_0). It includes a prize for the winner, as well as subsidies to firms.

The framework of Dasgupta and Stiglitz (1980) can be adopted to model this R&D race. Upon observing (s_1, s_2, Γ_0), each firm i invests an R&D effort x_i in order to achieve a quicker discovery. The actual amount of time t_i for a firm i to accomplish this task is a random variable that follows a Weibull (minimum) distribution. To put it

[25] For brevity, we do not provide the details of the comparative statics in the paper, but they are available from the authors upon request.

formally, given x_i, the probability that firm i successfully innovates before time t is given by

$$F_i(t|x_i) = 1-e^{-f_i(x_i)t},\ t\geq 0,\ i = 1,2, \tag{9}$$

where $f_i(x_i)$ is firm i's hazard rate of success, i.e., $f_i(x_i)\Delta t$ measures firm i's conditional probability of making the discovery between time t and time $t+\Delta t$, provided that the discovery has not been achieved before time t. It is assumed that $f_i(x_i)$ is strictly increasing with effort x_i, and is concave in its argument. The function $f_i(x_i)$ arguably measures the technical output of firm i's innovation activities. Specifically, we assume that the hazard rate $f_i(x_i)$ takes the functional form

$$f_i(x_i) = g(\theta_i, s_i)x_i^r, \tag{10}$$

with $g(\theta_i, s_i)>0$ and $r\in(0, 1]$. We follow our basic setup in interpreting the role of $g(\theta_i, s_i)$ as a measure of a firm i's research capacity. We also continue to use θ_i to label firm i's initial endowment.

It has been well known that Tullock contest model, patent race model and R&D tournaments are strategically equivalent (see Baye and Hoppe, 2003). We now briefly demonstrate the isomorphism between the alternative setting and our main setup. From the viewpoint of the sponsor, for a given effort profile (x_1, x_2), the innovation time (i.e. $\min\{t_1, t_2\}$) follows a cumulative distribution function

$$\begin{aligned} F(t|x_1,x_2) &= 1-(1-F_1(t|x_1))\ (1-F_2(t|x_2)) \\ &= 1-e^{-[f_1(x_1)\ +\ f_2(x_2)]t},\quad t\geq 0. \end{aligned} \tag{11}$$

The expected innovation time is then given by

$$E(t|x_1,x_2) = \frac{1}{f_1(x_1) + f_2(x_2)}. \tag{12}$$

First, the strategic investment game between the two firms is strategically equivalent to that in the benchmark setup. A firm wins if it realizes the desired discovery sooner than the other. The winning probability of a firm i is given by

$$\begin{aligned} Pr\left(t_i<t_j|x_i,x_j\right) &= \int_0^\infty \left(\int_0^{t_j} F_i'(t_i|x_i)dt_i\right)F_j'\left(t_j|x_j\right)dt_j \\ &= \frac{f_i(x_i)}{f_i(x_i) + f_j\left(x_j\right)}, \end{aligned} \tag{13}$$

which is no different from (1) in the benchmark setup.

Second, given $E(t|x_1,x_2) = \frac{1}{f_1(x_1) + f_2(x_2)}$, the sponsor who prefers earlier innovation would attempt to maximize $[f_1(x_1)+f_2(x_2)]$, which also the objective function of the benchmark setup.

The analysis in our benchmark setup thus automatically extends to this alternative context of patent race. Our results are thus not limited to the design of "best of simultaneous submission" contest that maximizes expected quality of the winning product. The same principles, i.e. those embodied through Propositions 2–7, as well as the supplementary discussion on the results, would continue to apply in designing contests to shorten the cycle of R&D process.

5. Concluding remark

This paper studies the optimal design of research contests. The sponsor designs the contest using three strategic vehicles: subsidies to two competing firms and an inducement prize. The sponsor faces a budget constraint and her objective is to maximize the expected quality of the winning submission. Our main results are the following. First, prizes and subsidies provide complementary incentives. Second, more subsidies are provided when the innovation process involves a higher level of difficulty. Finally, the optimal contest may either preferentially subsidize the ex ante weaker firm to balance the competition, or create a "national champion" by further assisting the firm that is stronger ex ante. The optimum depends not only on the distribution of firms' initial endowments, but also on various characteristics of the target project.

Analytical tractability and expositional efficiency have limited our analysis to a stylized two-firm R&D tournament model. However, our main results would continue to hold in a more extensive setting, e.g., where more than two firms compete. However, the analysis for such a scenario is beyond the scope of the current paper. When more than two heterogenous firms compete, and a subset of firms are excessively disadvantaged against the favorites, they may stay inactive (i.e., exert zero effort) in the equilibrium. This possibility would greatly complicate the analysis, because the sponsor's budget allocation problem becomes discontinuous. A subsidy allocation rule could serve as an incentive-compatible mechanism that allows the sponsor to select participating (active) firms so as to optimize the structure of the subsequent competition. The sponsor may either exclude an otherwise active firm by heavily subsidizing other firms, or revive an otherwise inactive firm by preferentially subsidizing it. A number of forces could interact to determine the optimal subset of competing firms in the race. The problem of optimal shortlisting through the strategic allocation of subsidies among competing parties remains unsolved in the literature. However, this will be pursued in future research efforts despite its technical challenge.

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Appendix A

Proof of Proposition 1

Proof. Clearly, $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial \Gamma_0}>0$. $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_i}$ is obviously positive because $r\leq 1$ and $\frac{g(\theta_j,s_j)}{g(\theta_i,s_i)}>0$. ■

Proof of Lemma 1

Proof. By Proposition 1, the resource s must be used up. Then it suffices to show when s_i increases while $s_1+s_2=s$, $y(s_i) = \frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_i} - \frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_j}$ can intersect with $y=0$ at most once. We have $y(s_1) = \frac{H(s_1,s_2,\Gamma_0)}{g(\theta_1,s_1) + g(\theta_2,s_2)}\Phi$, where $\Phi = g_{s_1}(\theta_1,s_1)\,[(1-r) + r\frac{g(\theta_2,s-s_1)}{g(\theta_1,s_1)}] - g_{s_2}(\theta_2,s-s_1)\left[(1-r) + r\frac{g(\theta_1,s_1)}{g(\theta_2,s-s_1)}\right]$. Because $g(\theta_i, s_i)$ is increasing with but concave in s_i, Φ must decrease when s_1 increases and $s_2 = s-s_1$ decreases. ■

Lemma A1 and its Proof

With an abuse of notation, we denote by i_1 the firm that has a larger $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_i}\Big|_{s_1=s_2=0}$, and denote by i_2 the other. For convenience, let

$i_1=1$ in the case of $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{s_1=s_2=0}=\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_1=s_2=0}$. The following Lemma A1 characterizes important properties of the optimal allocation profile $(s_{i_1}^*(s,\Gamma_0),s_{i_2}^*(s,\Gamma_0))$. The proof of this lemma (see below) verifies that there exists a unique cutoff $\bar{s}\geq 0$, such that $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_{i_1}}=\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_{i_2}}$ when $s_{i_1}=\bar{s}$, $s_{i_2}=0$. The cutoff degenerates to zero if and only if $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{s_1=s_2=0}=\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_1=s_2=0}$.

Lemma A1. *Under Condition 1, (i) both $s_{i_1}^*(s,\Gamma_0)$ and $s_{i_2}^*(s,\Gamma_0)$ are increasing and continuous with s; (ii) $s_{i_1}^*(s,\Gamma_0)=s$, and $s_{i_2}^*(s,\Gamma_0)=0$ for $s\leq\bar{s}$; and (iii) both $s_{i_1}^*(s,\Gamma_0)$ and $s_{i_2}^*(s,\Gamma_0)$ strictly increase with s for $s\geq\bar{s}$.*

Proof. (i) Recall $\Phi=g_{s_1}(\theta_1,s_1)\left[(1-r)+r\frac{g(\theta_2,s-s_1)}{g(\theta_1,s_1)}\right]-g_{s_2}(\theta_2,s-s_1)\left[(1-r)+r\frac{g(\theta_1,s_1)}{g(\theta_2,s-s_1)}\right]$. We look at the sign of

$$\Phi|_{s_{i_2}=0}=\left\{g_{s_{i_1}}\left(\theta_{i_1},s_{i_1}\right)\left[(1-r)+r\frac{g\left(\theta_{i_2},s_{i_2}\right)}{g\left(\theta_{i_1},s_{i_1}\right)}\right]-g_{s_{i_2}}\left(\theta_{i_2},s_{i_2}\right)[(1-r)]+r\frac{g\left(\theta_{i_1},s_{i_1}\right)}{g\left(\theta_{i_2},s_{i_2}\right)}\right]\Bigg\}\Bigg|_{s_{i_2}=0}.$$

When s_{i_2} is fixed at zero, but s_{i_1} increases, $g_{s_{i_1}}(\theta_{i_1},s_{i_1})$ weakly decreases, and $\frac{g(\theta_{i_2},s_{i_2})}{g(\theta_{i_1},s_{i_1})}$ strictly decreases. Hence, $g_{s_{i_1}}(\theta_{i_1},s_{i_1})[(1-r)]+r\frac{g(\theta_{i_2},s_{i_2})}{g(\theta_{i_1},s_{i_1})}\Big]-g_{s_{i_2}}(\theta_{i_2},s_{i_2})[(1-r)]+r\frac{g(\theta_{i_1},s_{i_1})}{g(\theta_{i_2},s_{i_2})}\Big]$ must strictly decrease. By the definition of i_1, there must exist a unique $\bar{s}\geq 0$, which leads to $\Phi|_{s_{i_2}=0}=0$. $\bar{s}=0$ holds if and only if $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{s_1=s_2=0}=\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_1=s_2=0}$, which is obvious.

We now claim both $s_1^*(s,\Gamma_0)$ and $s_2^*(s,\Gamma_0)$ weakly increase with s. We prove it by contradiction. Suppose the total amount of subsidy increases from s to $\tilde{s}$. When total subsidy is s, the optimal allocation is (s_1^*,s_2^*); when total subsidy is $\tilde{s}$, the optimal allocation is $\left(\tilde{s}_1^*,\tilde{s}_2^*\right)$. Suppose (s_1^*,s_2^*) is an interior solution and without loss of generality $\tilde{s}_1^*<s_1^*$. In this case, $\tilde{s}_2^*>s_2^*$. As (s_1^*,s_2^*) is an interior solution, we have $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{\left(s_1^*,s_2^*\right)}=\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{\left(s_1^*,s_2^*\right)}$, i.e.,

$$\left\{g_{s_1}(\theta_1,s_1)\cdot\left[(1-r)+r\frac{g(\theta_2,s_2)}{g(\theta_1,s_1)}\right]\right\}\Bigg|_{\left(s_1^*,s_2^*\right)}=\left\{g_{s_2}(\theta_2,s_2)\cdot\left[(1-r)+r\frac{g(\theta_1,s_1)}{g(\theta_2,s_2)}\right]\right\}\Bigg|_{\left(s_1^*,s_2^*\right)}.$$

Since $\tilde{s}_1^*<s_1^*$ and $\tilde{s}_2^*>s_2^*$, by Condition 1, we have $g_{s_1}(\theta_1,s_1)\Big|_{\left(\tilde{s}_1^*,\tilde{s}_2^*\right)}>g_{s_1}(\theta_1,s_1)\Big|_{\left(s_1^*,s_2^*\right)}$, $g(\theta_2,s_2)\Big|_{\left(\tilde{s}_1^*,\tilde{s}_2^*\right)}>g(\theta_2,s_2)\Big|_{\left(s_1^*,s_2^*\right)}$, $g(\theta_1,s_1)\Big|_{\left(\tilde{s}_1^*,\tilde{s}_2^*\right)}<g(\theta_1,s_1)\Big|_{\left(s_1^*,s_2^*\right)}$. It follows that

$$\Phi\Big|_{\left(\tilde{s}_1^*,\tilde{s}_2^*\right)}=\left\{g_{s_1}(\theta_1,s_1)\cdot\left[(1-r)+r\frac{g(\theta_2,s_2)}{g(\theta_1,s_1)}\right]-g_{s_2}(\theta_2,s_2)\cdot\left[(1-r)+r\frac{g(\theta_1,s_1)}{g(\theta_2,s_2)}\right]\right\}\Bigg|_{\left(\tilde{s}_1^*,\tilde{s}_2^*\right)}>0.$$

It implies that $H(s_1,s_2,\Gamma_0)$ can be further increased if the sponsor redirects resource from s_2 to s_1, which apparently violates the assumption that $\left(\tilde{s}_1^*,\tilde{s}_2^*\right)$ is optimal with total subsidy $\tilde{s}$.

Suppose (s_1^*,s_2^*) is a corner solution and without loss of generality $s_1^*=0$. In this case, we cannot have $\tilde{s}_1^*<s_1^*$. Suppose $\tilde{s}_2^*<s_2^*$. As (s_1^*,s_2^*) is a corner solution, we have $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{\left(s_1^*,s_2^*\right)}<\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{\left(s_1^*,s_2^*\right)}$. Since $\tilde{s}_1^*>s_1^*$ and $\tilde{s}_2^*<s_2^*$, we have $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{\left(\tilde{s}_1^*,\tilde{s}_2^*\right)}>\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{\left(\tilde{s}_1^*,\tilde{s}_2^*\right)}$ from Condition 1. Similar to the previous case, it violates the assumption that $\left(\tilde{s}_1^*,\tilde{s}_2^*\right)$ is optimal with total subsidy $\tilde{s}$.

(ii) Let us proceed by conducting the following thought experiment. Fix Γ_0, and allocate s between s_1 and s_2 optimally. By Lemma 1, the optimality only requires any additional s be added to s_i with a higher marginal impact $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_{i_1}}$. Whenever $s<\bar{s}$, any additional s must be given to i_1, by the definition of i_1 and $\bar{s}$. Hence, for $s\leq\bar{s}$, the optimum must involve $s_{i_1}^*(s,\Gamma_0)=s$, and $s_{i_2}^*(s,\Gamma_0)=0$.

(iii) We then claim for any $s\geq\bar{s}$, an increase in s must lead both $s_1^*(s,\Gamma_0)$ and $s_2^*(s,\Gamma_0)$ to strictly increase. By the previous argument, we cannot have either of them strictly decrease when s increases. Hence, we only need to verify that it is impossible that all the extra subsidy goes to one firm. We again verify the claim by contradiction. Assume that $\Delta s>0$ is available to be given to these firms. Further assume $i_1=1$ without loss of generality.

First, assume initially $s=\bar{s}$ and $\Phi|_{s_1=\bar{s},s_2=0}=0$. Suppose that all Δs goes to one firm, then we must have $\Phi<0$ if $s_1=\bar{s}+\Delta s$, which requires some subsidy money be redirected to s_2, and $\Phi>0$ if $s_2=\Delta s$, which requires some subsidy money be redirected to s_1. Contradiction results. By Lemma 1, there must exist a unique $\Delta s'\in(0,\Delta s)$, which gives $\Phi|_{s_1=\bar{s}}+\Delta s'$, $s_2=\Delta s-\Delta s'=0$. This exercise implies that whenever $s>\bar{s}$, both firms receive positive subsidies, with $s_{i_1}^*(s,\Gamma_0)>\bar{s}$, and $s_{i_2}^*(s,\Gamma_0)>0$.

Second, assume initially $s>\bar{s}$, which then leads to an interior solution. Again assume all Δs goes to one firm. By the same logic as before, we must have $\Phi<0$ if $s_1=s_1^*(s,\Gamma_0)+\Delta s$, which requires some subsidy money be redirected to s_2, and $\Phi>0$ if $s_2=s_2^*(s,\Gamma_0)+\Delta s$, which requires some subsidy money be redirected to s_1. Contradiction. There must exist a unique $\Delta s'\in(0,\Delta s)$, which gives $\Phi|_{s_1=s_1^*(s,\Gamma_0)+\Delta s',s_2=s_2^*(s,\Gamma_0)+\Delta s-\Delta s'}=0$. ■

Proof of Lemma 2

Proof. (i) The unique optimal allocation profile $(s_1^*(s,\Gamma_0),s_2^*(s,\Gamma_0))$ for the sub-maximization problem is independent of the value of winner's purse Γ_0. By Lemma 1, to efficiently split s, one only needs to focus on the local property of $H(s_1,s_2,\Gamma_0)$ with respect to s_1 and s_2. The solution can be obtained by comparing $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{s_2=s-s_1}$ with $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_2=s-s_1}$ and satisfying Kuhn–Tucker conditions. By (3), the comparison between $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{s_2=s-s_1}$ with $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_2=s-s_1}$ does not depend on Γ_0, because it appears only in the common term $\frac{H(s_1,s_2,\Gamma_0)}{g(\theta_1,s_1)+g(\theta_2,s_2)}$.

We have $H(s;\Gamma_0)=\left[g\left(\theta_1,s_1^*(s,\Gamma_0)\right)+g\left(\theta_2,s_2^*((s,\Gamma_0))\right)\right](x^*(\Gamma_0,s_1^*(s,\Gamma_0),s_2^*(s,\Gamma_0)))^r=\frac{r^rg\left(\theta_1,s_1^*(s,\Gamma_0)\right)^rg\left(\theta_2,s_2^*(s,\Gamma_0)\right)^r}{\left[g\left(\theta_1,s_1^*(s,\Gamma_0)\right)+g\left(\theta_2,s_2^*(s,\Gamma_0)\right)\right]^{2r-1}}(\Gamma_0)^r$. Because $s_1^*(s,\Gamma_0)$ and $s_2^*(s,\Gamma_0)$ are both continuous, $H(s;\Gamma_0)$ must be continuous with s. Because of the independence between $(s_1^*(s,\Gamma_0),s_2^*(s,\Gamma_0))$ and Γ_0, the continuity and differentiability of Γ_0 are obvious.

However, $H(s;\Gamma_0)$ may not be differentiable with s, because $s_1^*(s,\Gamma_0)$ and $s_2^*(s,\Gamma_0)$ are both kinked at $\bar{s}$. However, both of them are differentiable with s when $s<\bar{s}$, and $s\geq\bar{s}$. When $s<\bar{s}$, $\frac{ds_{i_1}^*(s,\Gamma_0)}{ds}=1$, and $\frac{ds_{i_2}^*(s,\Gamma_0)}{ds}=0$. When $s\geq\bar{s}$, $s_1^*(s,\Gamma_0)$ and $s_2^*(s,\Gamma_0)$ are obtained as interior solutions, and both of them must be differentiable with s. When $s<\bar{s}$, we have $\frac{\partial H(s;\Gamma_0)}{\partial s}=\frac{H(s_1,s_2,\Gamma_0)}{\partial s_{i_1}}\Big|_{s_{i_1}=s,\ s_{i_2}=0}$. When $s\geq\bar{s}$, we have $\frac{\partial H(s;\Gamma_0)}{\partial s}=\frac{H(s_1,s_2,\Gamma_0)}{\partial s_{i_1}}\Big|_{s_{i_1}=s_{i_1}^*(s,\Gamma_0),s_{i_2}=s_{i_2}^*(s,\Gamma_0)}\cdot\frac{ds_{i_1}^*(s,\Gamma_0)}{ds}+\frac{H(s_1,s_2,\Gamma_0)}{\partial s_{i_2}}\Big|_{s_{i_1}=s_{i_1}^*(s,\Gamma_0),s_{i_2}=s_{i_2}^*(s,\Gamma_0)}\cdot\frac{ds_{i_2}^*(s,\Gamma_0)}{ds}$. Because $s_{i_1}=s_{i_1}^*(s,\Gamma_0)$,

$s_{i_2} = s_{i_2}^*(s, \Gamma_0)$ for $s \ge \bar{s}$, and $\frac{ds_{i_1}^*(s,\Gamma_0)}{ds} + \frac{ds_{i_2}^*(s,\Gamma_0)}{ds} = 1$, we must have $\frac{\partial H(s;\Gamma_0)}{\partial s} = \frac{H(s_1,s_2,\Gamma_0)}{\partial s_{i_1}}\Big|_{s_{i_1}=s_{i_1}^*(s,\Gamma_0), s_{i_2}=s_{i_2}^*(s,\Gamma_0)}$ for $s \ge \bar{s}$. We then have $lim^-_{s\to\bar{s}} \frac{\partial H(s;\Gamma_0)}{\partial s} = lim^+_{s\to\bar{s}} \frac{\partial H(s;\Gamma_0)}{\partial s}$, which then guarantees the differentiability of $H(s;\Gamma_0)$ with respect to s.

Apparently, $H(s;\Gamma_0)$ is strictly increasing with both s and Γ_0.

(ii) Since $H(s;\Gamma_0)$ is strictly increasing with both s and Γ_0, the indifference curve of $H(s;\Gamma_0)$ must be negatively sloped. Hence, to verify the quasi-concavity, we only need to verify that the indifference curve of $H(s;\Gamma_0)$ gets flatter when Γ_0 increases and s decreases. The independence between $s_i^*(s, \Gamma_0)$ and Γ_0 gives $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0} = H\left(s_1^*(s,\Gamma_0), s_2^*(s,\Gamma_0), \Gamma_0\right)\frac{r}{\Gamma_0}$. Further,

$$\frac{\partial H(s;\Gamma_0)}{\partial s} = \frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_{i_1}}\Big|_{s_1=s_1^*(s,\Gamma_0), s_2=s_2^*(s,\Gamma_0)}$$
$$= H\left(s_1^*(s,\Gamma_0), s_2^*(s,\Gamma_0)\right)\frac{g_{s_{i_1}}\left(\theta_i, s_{i_1}\right)}{g\left(\theta_{i_1}, s_{i_1}\right) + g\left(\theta_{i_2}, s_{i_2}\right)}$$
$$\times\left[(1-r) + r\frac{g\left(\theta_{i_2}, s_{i_2}\right)}{g\left(\theta_{i_1}, s_{i_1}\right)}\right].$$

Both $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}$ and $\frac{\partial H(s;\Gamma_0)}{\partial s}$ contain the common term $H(s_1^*(s, \Gamma_0), s_2^*(s, \Gamma_0))$. Consider a decrease in Γ_0, and an associated increase in s such that $\Gamma_0 + s$ fixed at M. $\frac{r}{\Gamma_0}$ strictly increases. We now claim $\frac{g_{s_{i_1}}(\theta_i, s_{i_1})}{g(\theta_{i_1}, s_{i_1}) + g(\theta_{i_2}, s_{i_2})}\left[(1-r) + r\frac{g(\theta_{i_2}, s_{i_2})}{g(\theta_{i_1}, s_{i_1})}\right]$ strictly decreases.

Suppose the total amount of resource for subsidy increases from s' to s''. Denote the initial optimal subsidy allocation plan by (s_1', s_2'), and the new optimum by (s_1'', s_2''). We consider three cases.

Case 1: Suppose $s' < s'' \le \bar{s}$. In this case, all additional subsidy $s'' - s'$ goes to firm i_1. $\frac{g_{s_{i_1}}(\theta_i, s_{i_1})}{g(\theta_{i_1}, s_{i_1}) + g(\theta_{i_2}, s_{i_2})}\left[(1-r) + r\frac{g(\theta_{i_2}, s_{i_2})}{g(\theta_{i_1}, s_{i_1})}\right]$ must strictly decrease.

Case 2: $\bar{s} \le s' < s''$. There is an initial interior solution to the subsidy allocation problem, which yields $\frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{s=s'} = \frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{s_1=s_1', s_2=s_2'} = \frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_1=s_1', s_2=s_2'}$. By Lemma A1, we must have $\frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{s=s''} = \frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1}\Big|_{s_1=s_1'', s_2=s_2''} = \frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2}\Big|_{s_1=s_1'', s_2=s_2''}$ as well, and we must have $s_i'' > s_i'$ for $i \in \{1, 2\}$.
Suppose that the increase in s causes $\frac{g_{s_{i_1}}(\theta_i, s_{i_1})}{g(\theta_{i_1}, s_{i_1}) + g(\theta_{i_2}, s_{i_2})}\left[(1-r) + r\frac{g(\theta_{i_2}, s_{i_2})}{g(\theta_{i_1}, s_{i_1})}\right]$ to weakly increase. Then we must have $\frac{g_{s_i}(\theta_i, s_i)}{g(\theta_i, s_i) + g(\theta_j, s_j)}\left[(1-r) + r\frac{g(\theta_j, s_j)}{g(\theta_i, s_i)}\right]$ weakly increases for both i_1 and i_2. When s_1 and s_2 both strictly increase, $\frac{g_{s_i}(\theta_i, s_i)}{g(\theta_i, s_i) + g(\theta_j, s_j)}$ must strictly decrease for both i_1 and i_2. It implies that $r\frac{g(\theta_j, s_j)}{g(\theta_i, s_i)}$ strictly increases for both i_1 and i_2, which is impossible. Contradiction results.

Case 3: $s' < \bar{s} < s''$. The arguments laid out above directly lead to

$$\frac{g_{s_{i_1}}\left(\theta_i, s_{i_1}^*(s,\Gamma_0)\right)}{g\left(\theta_{i_1}, s_{i_1}^*(s,\Gamma_0)\right) + g\left(\theta_{i_2}, s_{i_2}^*(s,\Gamma_0)\right)}\left[(1-r) + r\frac{g\left(\theta_{i_2}, s_{i_2}^*(s,\Gamma_0)\right)}{g\left(\theta_{i_1}, s_{i_1}^*(s,\Gamma_0)\right)}\right]\Bigg|_{s=s'}$$
$$> \frac{g_{s_{i_1}}\left(\theta_i, s_{i_1}^*(s,\Gamma_0)\right)}{g\left(\theta_{i_1}, s_{i_1}^*(s,\Gamma_0)\right) + g\left(\theta_{i_2}, s_{i_2}^*(s,\Gamma_0)\right)}\left[(1-r) + r\frac{g\left(\theta_{i_2}, s_{i_2}^*(s,\Gamma_0)\right)}{g\left(\theta_{i_1}, s_{i_1}^*(s,\Gamma_0)\right)}\right]\Bigg|_{s=\bar{s}}$$
$$> \frac{g_{s_{i_1}}\left(\theta_i, s_{i_1}^*(s,\Gamma_0)\right)}{g\left(\theta_{i_1}, s_{i_1}^*(s,\Gamma_0)\right) + g\left(\theta_{i_2}, s_{i_2}^*(s,\Gamma_0)\right)}\left[(1-r) + r\frac{g\left(\theta_{i_2}, s_{i_2}^*(s,\Gamma_0)\right)}{g\left(\theta_{i_1}, s_{i_1}^*(s,\Gamma_0)\right)}\right]\Bigg|_{s=s''}.$$

We conclude that $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big/\frac{\partial H(s;\Gamma_0)}{\partial s}$ must strictly decrease when there is a marginal increase in Γ_0 and a simultaneous marginal decrease in s. Hence, the indifference curve of $H(s;\Gamma_0)$ in the (Γ_0, s) space must gets flatter when Γ_0 increases. Hence, the upper contour set must be concave, which then verifies the quasi-concavity of $H(s;\Gamma_0)$.

(iii) It is straightforward to verify the supermodularity since $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0} = H\left(s_1^*(s,\Gamma_0), s_2^*(s,\Gamma_0), \Gamma_0\right)\frac{r}{\Gamma_0}$ and $H(s_1^*(s, \Gamma_0), s_2^*(s, \Gamma_0), \Gamma_0)$ strictly increase with s. ■

Proof of Proposition 3

Proof. We first show that when M increases, both total subsidy s^* and Γ_0^* increase, using contradiction. Suppose that when budget is M, the optimal solutions of s^* and Γ_0^* are interior. Then $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{\left(s^*, \Gamma_0^*\right)} = \frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{\left(s^*, \Gamma_0^*\right)}$. Suppose that when budget is $\tilde{M}$ $(>M)$, the optimum $\tilde{s}^*$ and $\tilde{\Gamma}_0^*$ are such that (without loss of generality) $\tilde{s}^* < s^*$ and thus $\tilde{\Gamma}_0^* > \Gamma_0^*$. Lemma 2(iii) implies that $\frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{\left(\tilde{s}^*, \tilde{\Gamma}_0^*\right)} > \frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{\left(s^*, \Gamma_0^*\right)}$, and $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{\left(\tilde{s}^*, \tilde{\Gamma}_0^*\right)} < \frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{\left(s^*, \Gamma_0^*\right)}$, which violates the fact that $\left(\tilde{s}^*, \tilde{\Gamma}_0^*\right)$ is optimal.

Suppose (s^*, Γ_0^*) is a corner solution. Without loss of generality, $s^* = 0$, $\Gamma_0^* = M$. Then $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{\left(s^*, \Gamma_0^*\right)} < \frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{\left(s^*, \Gamma_0^*\right)}$. Suppose $s^* < \tilde{s}^*$ and $\Gamma_0^* > \tilde{\Gamma}_0^*$. Lemma 2(iii) implies that $\frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{\left(\tilde{s}^*, \tilde{\Gamma}_0^*\right)} < \frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{\left(s^*, \Gamma_0^*\right)}$, and $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{\left(\tilde{s}^*, \tilde{\Gamma}_0^*\right)} > \frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{\left(s^*, \Gamma_0^*\right)}$, which leads to $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{\left(\tilde{s}^*, \tilde{\Gamma}_0^*\right)} > \frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{\left(\tilde{s}^*, \tilde{\Gamma}_0^*\right)}$. This violates the fact that $\left(\tilde{s}^*, \tilde{\Gamma}_0^*\right)$ is optimal.

We thus have shown that when M increases, both total subsidy s^* and Γ_0^* increase. Lemma A1 further predicts that both s_1^* and s_2^* increase. ■

Proof of Proposition 4

Proof. We first show that when r increases, total subsidy s^* decreases and Γ_0^* increases. Note that $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}$ is given by (4) and $\frac{\partial H(s;\Gamma_0)}{\partial s}$ is determined by the maximum of $\frac{\partial H(s_1, s_2;\Gamma_0)}{\partial s_i}$ that are given by (3). The comparison between $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}$ and $\frac{\partial H(s;\Gamma_0)}{\partial s}$ is determined by $\Phi_{\Gamma_0} = \frac{r}{\Gamma_0}$ and $\Phi_{s_i} = \frac{g_{s_i}(\theta_i, s_i)}{[g(\theta_i, s_i) + g(\theta_j, s_j)]}\cdot\left[(1-r) + r\frac{g(\theta_j, s_j)}{g(\theta_i, s_i)}\right]$. Equivalently, we can also compare between $\frac{1}{r}\Phi_{\Gamma_0} = \frac{1}{\Gamma_0}$ and $\frac{1}{r}\Phi_{s_i} = \frac{g_{s_i}(\theta_i, s_i)}{[g(\theta_i, s_i) + g(\theta_j, s_j)]}\cdot\left[\frac{1}{r} - 1 + \frac{g(\theta_j, s_j)}{g(\theta_i, s_i)}\right]$.

If (s^*, Γ_0^*) is an interior optimum, then $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{(s^*,\Gamma_0^*)} = \frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{(s^*,\Gamma_0^*)}$, which leads to that $\frac{1}{r}\Phi_{\Gamma_0}$ equals the maximum of $\frac{1}{r}\Phi_{s_i}$.When r increases, $\frac{1}{r}\Phi_{\Gamma_0}$ remains unchanged while $\frac{1}{r}\Phi_{s_i}$ decreases. An optimum requires that s^* decreases and Γ_0^* increases. Lemma A1 further means that both s_1^* and s_2^* decrease.

Suppose that the optimum s^* and Γ_0^* are corner solutions. If $s^*=M$ and $\Gamma_0^*=0$, then $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{(s^*,\Gamma_0^*)} \le \frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{(s^*,\Gamma_0^*)}$. Regardless of the prevailing optimum, when r increases, it must follow that s^* decreases and Γ_0^* increases. If $s^*=0$ and $\Gamma_0^*=M$, then $\frac{\partial H(s;\Gamma_0)}{\partial \Gamma_0}\Big|_{(s^*,\Gamma_0^*)} \ge \frac{\partial H(s;\Gamma_0)}{\partial s}\Big|_{(s^*,\Gamma_0^*)}$, which means that $\Phi_{\Gamma_0} \ge max_i\{\Phi_{s_i}\}$. An increase in r further reinforces this inequality. Thus the optimum still entails $s^*=0$ and $\Gamma_0^*=M$. ■

Proof of Proposition 5

Proof. We conduct the following thought experiment to examine the optimal division rule. Consider an arbitrary allocation profile (s_1, s_2, Γ_0) with $s_1=s_2=s/2>0$.

We claim that when $s>0$ and $\theta_i>\theta_j$, only an allocation profile with $s_j^*>s/2>s_i^*$ can be optimal. Lemma 1 means that we only need to look at the local property for the position of global optimum. To allocate subsidies between firms, we only need to compare $\frac{\partial H(s_1,s_2;\Gamma_0)}{\partial s_i}$ with $\frac{\partial H(s_1,s_2;\Gamma_0)}{\partial s_j}$. Hence, we only need to compare $g_{s_i}(\theta_i,s_i)\left[(1-r)+r\frac{g(\theta_j,s_j)}{g(\theta_i,s_i)}\right]$ with $g_{s_j}(\theta_j,s_j)\left[(1-r)+r\frac{g(\theta_i,s_i)}{g(\theta_j,s_j)}\right]$ when $s_i=s_j=s/2$ to find out which firm should receive more subsidy in the optimum. When $s_i=s_j=s/2$, we must have $\frac{\partial H(s_1,s_2;\Gamma_0)}{\partial s_j} > \frac{\partial H(s_1,s_2;\Gamma_0)}{\partial s_i}$ if and only if $\theta_i>\theta_j$, because if and only if $\theta_i>\theta_j$, we have (1) $\frac{g(\theta_i,s/2)}{g(\theta_j,s/2)} > 1 > \frac{g(\theta_j,s/2)}{g(\theta_i,s/2)} > 0$, and (2) $g_{s_i}(\theta_i, s/2) \le g_{s_j}(\theta_j, s/2)$ by the definition of substitutability. Hence, an allocation with $s_i^*=s_j^*=s/2$ can be optimal only if $s=0$. When $s>0$, more subsidies should be allocated to the initially weaker firm.

Proof of Proposition 6

Proof. We continue to conduct the previous thought experiment. $\frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_1} - \frac{\partial H(s_1,s_2,\Gamma_0)}{\partial s_2} = \frac{H(s_1,s_2,\Gamma_0)}{g(\theta_1,s_1)+g(\theta_2,s_2)}\Phi$. Recall

$$\Phi = \frac{g_{s_1}(\theta_1,s_1)}{g(\theta_1,s_1)}[(1-r)g(\theta_1,s_1)+rg(\theta_2,s_2)] - \frac{g_{s_2}(\theta_2,s_2)}{g(\theta_2,s_2)} \times [(1-r)g(\theta_2,s_2)+rg(\theta_1,s_1)].$$

When $g(\theta, s)=\vartheta(\theta)\rho(s)$, we must have $\frac{g_{s_1}(\theta_1,s/2)}{g(\theta_1,s/2)} = \frac{g_{s_2}(\theta_2,s/2)}{g(\theta_2,s/2)}$. Hence, Lemma A1 means that we only need to compare $g(\theta_1, s/2)+r[g(\theta_2, s/2)-g(\theta_1, s/2)]$, and $g(\theta_2, s/2)+r[g(\theta_1, s/2)-g(\theta_2, s/2)]$ to determine how to allocate the total subsidy. The difference of the two terms is $\Delta|_{s_1=s_2=s/2} \propto [g(\theta_1, s/2)-g(\theta_2, s/2)]+2r[g(\theta_2, s/2)-g(\theta_1, s/2)]=(2r-1)[g(\theta_2, s/2)-g(\theta_1, s/2)]$. Since $g(\theta_2, s/2)-g(\theta_1, s/2)<0$ if $\theta_1>\theta_2$, we obtain $\Delta|_{s_1=s_2=s/2} \le 0$, iff $r \ge \frac{1}{2}$. Applying Lemma A1, we have iff $r \ge \frac{1}{2}$, $s_2^* \ge s_1^*$. ■

Proof of Proposition 7

Proof. (i): Without loss of generality, we assume $\Gamma_2<\Gamma_1$. Apparently, the sponsor can promise a more generous reward for firm 2 as the budget is not binding when firm 2 wins under the current allocation scheme. We then evaluate H with respect to Γ_2, we can directly verify that

$$\begin{aligned}\frac{\partial H}{\partial \Gamma_2} &\propto r\left(g(\theta_1,s_1)\Gamma_1^r + g(\theta_2,s_2)\Gamma_2^r\right) + g(\theta_2,s_2)\Gamma_2^r(1-2r) \\ &= rg(\theta_1,s_1)\Gamma_1^r + (1-r)g(\theta_2,s_2)\Gamma_2^r.\end{aligned} \tag{14}$$

Because $r\le 1$, (14) must be strictly positive. It implies an increase in Γ_2 would improve the output of the contest without spending more than the budget. Hence, an allocation scheme with heterogenous prizes must be suboptimal.

(ii): With the result of (i), it is clear that at the optimum, the prize to winner cannot be identity contingent. ■

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