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ORIGINAL ARTICLE

Endogenous timing of contest with asymmetric information

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Abstract Simultaneous moves have been conventionally assumed in modeling rent-seeking competition. However, in reality many forms of contests involve contestants sequentially choosing their effort entries. This study allows agents to choose the timing of their moves before the contest takes place. In contrast to the previous literature, we introduce information asymmetries across agents. We find that in all sequential-move equilibria, the uninformed agent moves first. More generally we show that the order of agents' moves in a sequential contest is a regularity stemming from information asymmetries. Furthermore, under plausible assumptions, sequential moves Pareto dominate simultaneous moves (from the view point of the players) and also result in lower rent-seeking expenditures. Our result explains the timing pattern in National Presidential Conventions observed from 1948 through 2004. Our result also applies to many other formal or informal institutions, which give rise to sequential contests.

Keywords Rent-Seeking · Contests · Timing · Information asymmetry

JEL Classifications: H1, C7, D8

1. Introduction

In modeling rent-seeking competition, the conventional approach requires contestants to simultaneously determine their effort outlays. The assumption of simultaneous moves, however, runs in contrast to many contest settings that occur in reality. Numerous forms of rent-seeking activities involve contestants sequentially determining their effort outlays. For instance, since 1948, National Presidential Conventions have been held in an order in which the parties of incumbent presidents, regardless of their identities (Republican or Democrat), choose the later dates (see Morgan, 2003). In litigation, under most circumstances, the burden

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of proof is shifted to the defendant only if the plaintiff has developed plausible evidence that meets certain pleading standards. Sequential moves of rent-seekers are also required by various business rules. For example, under the protection of the Meet-Competition-Clause, incumbent firms are endowed with the privilege to make the final offer when their competitors attempt to take over their existing clientele. Similarly, as noted by Morgan (2003), in professional sports, the current employer is usually allowed to wait until other teams have made their bids to the free athletic talent.

Besides the widely observed sequential moves, asymmetric information is also ubiquitous in various contest settings. In the case that two teams bid for a professional athlete, the current employer can more precisely assess his value in terms of his physical condition, athletic capability, and personality, such as the willingness to cooperate with teammates. In the case that two firms compete for a contract, the incumbent firm presumably can more accurately value this business. In duty of care litigation, plaintiff shareholders, who allege corporate managers of reckless transactions, have to exert extra investigative effort to discover the hidden action taken by the management. Besides, since 1948, U.S. presidential candidates, who represented the parties of incumbents, have been either the sitting presidents or vice presidents, with the only exception of Stevenson in 1952 (see Table 4). Obviously, incumbents must better understand the net value of presidency, i.e. the worth of "the winner's purse", and the cost to be in office, while challengers typically only have a rough notion; thus, challengers are more likely to be subject to uncertainty. A common feature hidden behind all the aforementioned examples is that contestants who are better informed of the actual stake take their actions second in the contest. Nevertheless, the rule that governs the sequence of moves in the contest has been determined before the value of the contestable prize is realized. In other words, although the informed party obtains better notions about the value of the contestable prize, contestants are ex ante equally subject to the uncertainty when the rule is made.

The sequence of moves in rent-seeking activities can be governed by either formal or informal institutions, such as laws, customs and contracts. The intent of this paper is to investigate the endogenous formation of these institutions, and establish the commonality we observe in these examples as a regularity. We argue that the particular order of moves, where the informed party moves second, is not accidentally or arbitrarily arranged, but driven by the underlying information asymmetry. In an attempt to identify this particular order as an endogenous outcome in equilibrium, we develop a three-stage model, in which sequential contests may endogenously arise out of contestants' rational choice. In stage 0, agents determine the institution that governs the contest, i.e. the sequence of their moves in the contest, while the contest is conducted in stage 1 and/or stage 2.

We have two agents compete for an object. At the beginning of the game, i.e. the stage 0, agents simultaneously pick either stage 1 or 2 as the date they determine their effort outlays. Then they determine their effort outlays in stage 1 and/or stage 2 according to the timing they have chosen. Once the effort is made, it is publicly observable. The value of this object, which can be either high or low, is randomly drawn from a discrete distribution. The value is not realized until stage 0 ends, but its distribution is common knowledge throughout the game. Even though the object is ex post equally valued by both agents, only one of them is immediately informed of the realization of its value.

We show that choosing the later date, i.e. stage 2, at least weakly dominates choosing stage 1 in the informed agent's payoff. Given that the informed agent chooses to move on a later date, the less informed agent has the incentive to pick stage 1 and move first in the contest. We show that the institution of sequential moves, with the informed agent moving second in the contest, always emerges in equilibrium. By contrast, another institution that leads to sequential contests, under which the informed agent moves first, never arises in equilibrium. Although simultaneous-move contests may also arise in equilibrium, they emerge only if the likelihood that the low value is realized is sufficiently high. We show that the endogenous sequential contest generates lower rent-seeking expenditures and Pareto dominates a simultaneous-move contest. The intuition behind our equilibrium result is clear: If the uninformed agent is allowed to move first, she is able to lower her bid and reduce the potential overbidding due to the uncertainty, which softens the competition.

This study is mainly built upon two strands of literature. Our basic framework is close to Leininger (1993) and Morgan (2003). Leininger (1993) is among the first to challenge the conventional assumption of simultaneous moves in the rent-seeking literature. Leininger (1993) allows agents to choose the timing of their moves in the contest. He shows that sequential contests endogenously arise in equilibrium if agents are endowed with asymmetric strengths. As the first mover, the weaker agent underbids to "calm down" her stronger opponent, which makes both of them better off. Leininger (1993) assumes that agents' valuations of the object of interest are commonly known to each other when they make their timing choices. Morgan (2003), in contrast, assumes agents' valuations on the object are identically distributed but are not realized until they have committed to the timing of their moves. He finds a unique subgame perfect equilibrium, where these two ex ante identical agents sequentially make their effort outlays.

Both Leininger (1993) and Morgan (2003) examine complete-information contests. The literature that is devoted to asymmetric information contests has been rather small. Wärneryd (2003) thoroughly explores the common-value asymmetric information contests, in which one party obtains superior information about the value of the object. Fu (2005) applies this framework to the context of litigation. In contrast to Wärneryd (2003) and Fu (2005), Bernado et al. (2000) assume information asymmetry concerns one party's marginal cost of her effort. However, among papers devoted to asymmetric information contests, none of them has investigated contests with agents sequentially determining their effort outlays.

Our study addresses the endogenous sequence of moves in contests in the presence of information asymmetry. This study bears a structure close to Morgan (2003). Although Morgan (2003) establishes sequential moves as the unique equilibrium outcome, the order between contestants, such as the particular order of moves observed in the history of national presidential conventions, has yet to be identified because Morgan (2003) does not differentiate between contestants' identities (the incumbent or the challenger), but assumes they are ex ante identical. We, by contrast, distinguish between contestants by allowing them to possess asymmetric information.

The remainder of this paper is organized as follows. In Section 2, we describe the model setup. Section 3 shows the equilibrium outcome in the end game of contest when the institution has been determined. Section 4 characterizes agents' best response and the equilibrium in stage 0 and Section 5 concludes this paper.

2. The model

Two risk neutral agents, indexed by $i = I, U$, compete for an indivisible object. The object can be a piece of legislation, a business contract, a professional athlete, a court verdict, a license to operate in a regulated market, etc. The value of this object, V, is randomly drawn from a discrete distribution with a probability $q \in (0, 1)$ to be V_H, and $1 - q \in (0, 1)$ to be V_L, and $V_H > V_L > 0$. The distribution of V is a public signal. To ease the notation, we normalize V_L to be one, and V_H to be m^2, with $m \in (1, \infty)$.

To win this object, agents have to exert costly and non-refundable efforts e_I and e_U, respectively. The probability agent i wins is given by the contest success function $P_i(e_i, e_j) = \frac{e_i}{e_i+e_j}$. An agent i receives $V_i - e_i$ as her payoff if she wins, while simply bears the cost e_i if she loses.

2.1. Timeline

The timeline of the game is as follows. We consider a three-stage sequential game. Agents choose the timing of their rent-seeking activities in stage 0, while the contest takes place in stage 1 and/or stage 2. At the beginning of the game, i.e. stage 0, agents simultaneously decide whether to make their efforts in stage 1 or stage 2. An agent i picks either stage 1 or stage 2 as the date of her action, i.e. determining e_i in the contest. Then nature chooses the state according to the distribution as described above. Note that the timing choice is made prior to the realization of V, although agent I is informed of V immediately after V is realized. The realization of V then defines the "type" of agent I, which can be either "high" (H) or "low" (L). In stage 1 and stage 2, agents either simultaneously or sequentially commit to their effort outlays according to their choices in stage 0. If an effort has been made in stage 1, then it is publicly observable in stage 2.

2.2. The contest

Agents' choice in stage 0 may generate three institutions that govern the subsequent contest. To the extent that agent I and U pick the same stage, they simultaneously determine their effort outlays. We denote by SS the institution of simultaneous moves. If agent U picks stage 1, while agent I picks stage 2, agents sequentially make their effort in the subsequent contest. We denote by UI the institution with agent U as the first mover. If agent I chooses stage 1, while agent U chooses stage 2, the contest then requires agent I to move before agent U takes the action. This institution is denoted by IU.

In the stage game of contest, agent I's expected payoff is given by

$$\pi_I = \frac{e_I}{e_I + e_U} V - e_I. \tag{1}$$

We define π_I^H and π_I^L to be the expected payoffs of the high-type agent I and the low-type agent I, respectively. We further define e_I^H and e_I^L to be the efforts made by the high-agent I and low-type agent I, respectively.

Agent U chooses e_U to maximize her expected payoff

$$\pi_U = \mu \frac{e_U}{e_I^H + e_U} m^2 + (1 - \mu) \frac{e_U}{e_I^L + e_U} - e_U, \tag{2}$$

where μ is a subjective probability attached to the event that V_H is realized. Obviously, neither the institution SS nor UI involves information exchange between agent I and U. Then agent U simply maintains the prior belief throughout the game with $\mu = q$.

However, once the institution IU is in place, agent I's effort signals her type. Upon observing e_I made in stage 1, agent U forms a conjecture about the realization of V through Bayesian updating. The updated belief assigns a probability $\mu = \mu(e_I)$ to V_H, and a probability $1 - \mu$ to V_L, where the belief function $\mu(e_I)$ is a mapping $\mu\colon \Re \to [0, 1]$.

2.3. Timing choices

In stage 0, agents compare their payoffs across institutions and pick stage 1 or stage 2 as the timing of their actions. Prior to the realization of V, even agent I is unaware of her own type. Hence, agent I has to weigh her ex ante expected payoff $\widetilde{\pi}_I$ under each institution, which is given by

$$\widetilde{\pi}_I = q\pi_I^H + (1-q)\pi_I^L. \tag{3}$$

By contrast, agent U simply considers her expected payoff in contest π_U.

We further define $\widetilde{\pi}_U^{SS}$, $\widetilde{\pi}_I^{UI}$ and $\widetilde{\pi}_I^{IU}$ to be agent I's ex ante expected payoffs under the institutions SS, UI, and IU, with the superscripts to indicate the institutions that govern the contest. We also define agent U's expected payoffs under these institutions by π_U^{SS}, π_U^{UI} and π_U^{IU}.

Agents simultaneously choose whether to determine their rent-seeking efforts in stage 1 or stage 2. Agents' strategic trade-offs in stage 0 can be presented in a normal-form game as shown below, in which agent I plays the row strategy, while agent U plays the column strategy.

	Stage 1	Stage 2
Stage 1	$\widetilde{\pi}_I^{SS}$, π_U^{SS}	$\widetilde{\pi}_I^{IU}$, π_U^{IU}
Stage 2	$\widetilde{\pi}_I^{UI}$, π_U^{UI}	$\widetilde{\pi}_I^{SS}$, π_U^{SS}

In a complete information contest, regardless of the sequence of moves, each agent exerts an effort $e = \frac{V}{4}$ and receives $\frac{V}{4}$ as the expected payoff. We define Ψ to be $\Psi = q\frac{m^2}{4} + (1-\mathrm{q})\frac{1}{4} = \frac{EV}{4}$. Hence, Ψ is each agent' ex ante expected payoff if the value of V is revealed to both agents after it is realized. We use Ψ as the benchmark to compare agents' expected payoffs under different institutions.

3. Equilibrium behavior in contest

We solve the game backward. In this part, we characterize the equilibrium behavior in the contest under each institution. We first consider the institution SS, where agents move simultaneously. Then we consider the case UI, with agent U moving first. Finally, we investigate the institution IU with agent I as the first mover.

3.1. Simultaneous moves (SS)

Taking first order derivative of (1) and (2), we have

$$\frac{\partial \pi_I}{\partial e_I} = \frac{e_U}{(e_I + e_U)^2} V - 1, \tag{4}$$

$$\frac{\partial \pi_U}{\partial e_U} = q\frac{e_I^H}{\left(e_I^H + e_U\right)^2} m^2 + (1-q)\frac{\alpha e_I^L}{\left(e_I^L + e_U\right)^2} - 1. \tag{5}$$

The best response of agent I to agent U's effort e_U is given by

$$e_I = \begin{cases} \sqrt{e_U} \cdot (\sqrt{V} - \sqrt{e_U}) & \text{if} \quad e_U < V, \\ 0 & \text{if} \quad e_U \geq V. \end{cases} \tag{6}$$

(6) implies that agent I may not exert positive effort in equilibrium. Once e_U exceeds 1, no positive effort rewards the low-type agent I and her best response is to stay inactive, i.e. $e_I^L = 0$. However, the high-type agent I always exerts positive effort (because e_U never exceeds m^2), as well as agent U. Two types of equilibria may emerge in the contest under the institution SS, which differ in the behavior of the low-type agent I.

Definition 1. An active equilibrium in contest is a Bayesian Nash equilibrium where agent I makes positive effort, regardless of the realization of V.

Definition 2. An inactive equilibrium in contest is a Bayesian Nash equilibrium where the low-type agent I stays inactive, i.e. $e_I^L = 0$.

Agent U's best response to e_I is given by

$$e_U = \begin{cases} qe_I^H + (1-q)e_I^L & \text{if } e_I^L > 0, \\ qe_I^H & \text{if } e_I^L = 0. \end{cases} \tag{7}$$

Proposition 1. *Under the institution SS, a unique active Bayesian Nash equilibrium exists if and only if $\mu < \frac{1}{m-1}$; while a unique inactive Bayesian Nash equilibrium exists if and only if $\mu \geq \frac{1}{m-1}$.*

Agents' equilibrium effort outlays and expected payoffs are summarized in Table 1.

Corollary 1. *Under the institution SS, $\tilde{\pi}_I^{SS} > \Psi$.*

Corollary 1 states that under the institution SS, agent I expects to receive more than Ψ, the ex ante expected payoff if information is complete in the contest. This implies that agent I benefits from her private information and extracts positive information rent.

Table 1 Equilibrium under SS

	$q < \frac{1}{m-1}$	$q \geq \frac{1}{m-1}$
e_I^H	$\frac{E\sqrt{V}}{4}[2m - E\sqrt{V}]$	$\frac{qm^2}{(1+q)^2}$
e_I^L	$\frac{E\sqrt{V}}{4}[2 - E\sqrt{V}]$	0
π_I^H	$[m - \frac{E\sqrt{V}}{2}]^2$	$\frac{1}{(1+q)^2}m^2$
π_I^L	$[1 - \frac{E\sqrt{V}}{2}]^2$	0
$\tilde{\pi}_I^{SS}$	$EV - \frac{3(E\sqrt{V})^2}{4}$	$\frac{q}{(1+q)^2}m^2$
e_U	$\frac{(E\sqrt{V})^2}{4}$	$\frac{q^2m^2}{(1+q)^2}$
π_U (π_U^{SS})	$\frac{(E\sqrt{V})^2}{4}$	$\frac{q^3}{(1+q)^2}m^2 + (1-q)$

3.2. Agent U moves first (UI)

As the first mover in the contest, agent U takes into account agent I's response to e_U as given by (6). Agent U therefore chooses e_U to maximize her expected payoff

$$\pi_U = \begin{cases} \sqrt{e_U} \cdot (E\sqrt{V} - \sqrt{e_U}) & \text{if } e_U < 1, \\ \sqrt{e_U} \cdot (qm - \sqrt{e_U}) + (1-q) & \text{if } e_U \geq 1. \end{cases} \tag{8}$$

Lemma 1. *π_U is continuous on $\sqrt{e_U}$.*

We illustrate agent U's expected payoff as a function of $\sqrt{e_U}$ in Figure 1. π_U is a kinked curve, with $\sqrt{e_U} = 1$ as the kinked point. $\sqrt{e_U} \cdot (E\sqrt{V} - \sqrt{e_U})$ and $\sqrt{e_U} \cdot (qm - \sqrt{e_U}) + (1-q)$ are both bell-shaped functions of $\sqrt{e_U}$, with the unique maximizers $\frac{E\sqrt{V}}{2}$ and $\frac{qm}{2}$, respectively. Agent U's equilibrium behavior then depends on where the kinked point lies. Three possibilities exist. The top panel of Figure 1 shows the first possibility, where $\frac{qm}{2} < \frac{E\sqrt{V}}{2} < 1$ and π_U is maximized at $\frac{E\sqrt{V}}{2}$. The panel in the middle instead shows the case that $\frac{qm}{2} \leq 1 \leq \frac{E\sqrt{V}}{2}$. Hence, π_U rises before $\sqrt{e_U}$ reaches 1, but starts to fall when $\sqrt{e_U}$ exceeds 1. In the bottom panel, $1 < \frac{qm}{2} < \frac{E\sqrt{V}}{2}$, then π_U keeps rising until $\sqrt{e_U} = \frac{qm}{2}$.

Similar to the circumstance with the institution SS taking place, both active and inactive equilibria, which differ in the behavior of the low-type agent, may emerge in the equilibrium, depending on the size of q.

Proposition 2. *Under the institution UI, a unique active equilibrium exists if and only if $q < \frac{1}{m-1}$; an inactive equilibrium exists if $\frac{1}{m-1} \leq q \leq \frac{2}{m}$; and an inactive equilibrium exists if $q > \frac{2}{m}$.*

Agents' equilibrium effort outlays and expected payoffs aresummarized inTable 2.

If $q < \frac{1}{m-1}$, the equilibrium outcome in this setting is equivalent to the case of simultaneous moves. However, they diverge once q exceeds $\frac{1}{m-1}$. As the first mover, agent U does not attempt to preempt the follower like a Stackelberg leader. Compared to the simultaneous-move contest, both agent U and the high-type agent I behave less aggressively. Under both UI and SS, agent U "overbids" if she turns out to encounter a low-type agent I, because any effort $e_U > 1$ influences only the response of the high-type agent I, while the low type stays inactive anyway regardless of the level of e_U. Hence, if agent U is allowed to move first, she is willing to lower her effort in order to avoid potential overbidding, which softens the competition.

Corollary 2. *Under the institution UI, $\tilde{\pi}_I^{UI} > \Psi$.*

The institution UI requires agent U to move first, so it does not involve information transmission between agents. Hence, we see that agent I still enjoys the information rent and receives more than Ψ under the institution UI.

Table 2 Equilibrium under UI

	$q < \frac{1}{m-1}$	$\frac{1}{m-1} \leq q \leq \frac{2}{m}$	$q > \frac{2}{m}$
e_I^H	$\frac{E\sqrt{V}}{4}[2m - E\sqrt{V}]$	$m - 1$	$\frac{qm}{2} \cdot (m - \frac{qm}{2})$
e_I^L	$\frac{E\sqrt{V}}{4}[2 - E\sqrt{V}]$	0	0
π_I^H	$[m - \frac{E\sqrt{V}}{2}]^2$	$(m-1)^2$	$(1 - \frac{q}{2})^2 m^2$
π_I^L	$[1 - \frac{E\sqrt{V}}{2}]^2$	0	0
$\widetilde{\pi}_I^{UI}$	$EV - \frac{(E\sqrt{V})^2}{4}$	$q(m-1)^2$	$q(1 - \frac{q}{2})^2 m^2$
e_U	$\frac{(E\sqrt{V})^2}{4}$	1	$\frac{q^2 m^2}{4}$
π_U (π_U^{UI})	$\frac{(E\sqrt{V})^2}{4}$	$q(m-1)$	$\frac{q^2 m^2}{4} + (1-q)$

3.3. Agent I as the first mover (IU)

Under the institution IU, agent U does not determine her effort entry until she observes agent I's action. With agent I as the first mover, the sequential contest bears the structure of a signaling game. We solve the equilibrium under this institution with the concept of sequential equilibrium, which follows in line with Kreps and Wilson (1982).

We denote agent I's strategy in stage 1 by $\sigma_I^H(e_I)$ or $\sigma_I^L(e_I)$, where the superscripts denote agent I's types. $\sigma_I^H(e_I)$ or $\sigma_I^L(e_I)$ are the probability distributions of their efforts e_I over the supports Δ_I^H or Δ_I^L. Upon observing e_I, agent U infers agent I's type through the belief system $\mu = \mu(e_I)$, which is obtained through Bayesian updating. The expected payoff of agent U is given by

$$\begin{aligned}\pi_U &= \mu \frac{e_U}{e_I + e_U} m^2 + (1-\mu) \frac{e_U}{e_I + e_U} - e_U \\ &= \frac{e_U}{e_I + e_U} E_\mu V - e_U, \end{aligned} \tag{9}$$

where $E_\mu V$ denotes the expectation of V with a belief that attaches the probability μ to the event that V_H is realized, i.e. $E_\mu V = \mu m^2 + (1-\mu)$.

The best response of agent U to e_I is then given by

$$e_U = \sqrt{e_I} \cdot (\sqrt{E_\mu V} - \sqrt{e_I}). \tag{10}$$

As the first mover, agent I must take into account agent U's response to e_U. The expected payoff of agent I is given by

$$\pi_I = \sqrt{e_I} \cdot \left(\frac{V}{\sqrt{E_\mu V}} - \sqrt{e_I} \right). \tag{11}$$

It is obvious to see that π_I is strictly decreasing in μ. The more optimistic agent U's belief is, the more aggressively agent U acts in the contest, and the less agent I expects to receive. However, in contrast to simultaneous-move contests and the sequential contest under UI, the institution IU does not give rise to the inactive equilibrium where the low-type agent I exerts zero effort. (11) instead implies that the high-type agent I receives no less than $\frac{m^2}{4}$ in any equilibrium, which is her payoff in a complete information contest; while the low type receives at least $\frac{1}{4m^2}$ in any equilibrium.[1]

As a signaling game, the contest under the institution IU may yield multiple equilibria. These equilibria differ in the degree of information revelation. We obtain the separating equilibrium if the private information is completely revealed to agent U. To put it formally, the separating equilibrium emerges if $\Delta_I^H \cap \Delta_I^L = \varnothing$. Agent U believes $\mu\,(e_I|\,e_I \in \Delta_I^H) = 1$, and $\mu\,(e_I|\,e_I \in \Delta_I^L) = 0$. We obtain the pooling equilibrium if $\sigma_I^H(e_I) = \sigma_I^L(e_I)$. In this case, e_I does not contain any new information, and agent U has to maintain her prior belief. In addition, we may also obtain semi-separating equilibrium, in which $\sigma_I^H(e_I) \neq \sigma_I^L(e_I)$, but $\Delta_I^H \cap \Delta_I^L$ is nonempty.

Lemma 2. *In any equilibrium under the institution IU,*
(a) $\Delta_I^H \cap \Delta_I^L$ consists of at most one element;
(b) $\Delta_I^H \backslash \{\Delta_I^H \cap \Delta_I^L\}$ consists of a single element $e_I = \frac{m^2}{4}$, if it is nonempty.

The first part of Lemma 2 implies that agent I plays a uniform pure strategy in a pooling equilibrium regardless of her type. In addition, this implies that in a semi-separating equilibrium, Δ_I^H and Δ_I^L do not completely overlap with each other. Lemma 2 (b) shows that $\frac{m^2}{4}$ is the only effort level that may be exerted by the high type agent I to separate herself from the low type.

Proposition 3. *There exist a continuum of pure strategy separating equilibria. In the equilibrium, the high-type agent I chooses an effort level $e_I^H = \frac{m^2}{4}$, while the low-type agent I's effort $e_I^L \in [\frac{(1-\sqrt{1-\frac{1}{m^2}})^2}{4}, \frac{[m^2(1-\sqrt{1-\frac{1}{m^2}})]^2}{4}]$.*

The separating equilibrium always exists. As implied by Lemma 2, in the separating equilibrium, the high-type agent I behaves as if she were in a complete information contest. However, the low type chooses an effort e_I^L, which is less than $\frac{1}{4}$, her equilibrium effort level in a complete information contest. The low type underbids to separate herself from her high-type counterpart, and solely bears the cost of the signaling.

Proposition 4. *When $q \leq \frac{3m^2+1}{(m^2+1)^2}$, there exist a continuum of pooling equilibria, where agent I chooses a uniform effort level e_I regardless of her type, with $e_I \in [[\frac{m^2}{2\sqrt{EV}} \cdot (1 - \sqrt{1-\frac{EV}{m^2}})]^2, [\frac{1}{2\sqrt{EV}} \cdot (1+\sqrt{1-\frac{EV}{m^2}})]^2]$.*

The pooling equilibrium exists only if q is sufficiently small. A greater q raises the marginal return of e_U, and induces agent U to act more aggressively in the contest. As a consequence, the low-type agent I tends to reduce her effort, which forces the high-type agent

[1] To see why this is true, consider the low-type agent I's expected payoff with the belief $\mu = 1$, which is given by $\pi_I = \sqrt{e}(\frac{1}{m} - \sqrt{e})$. she maximizes her expected payoff by choose $e = \frac{1}{4m^2}$ and receives $\frac{1}{4m^2}$. Because $\mu = 1$ is the most hostile belief to agent I, then the low type does not receives less than $\frac{1}{4m^2}$ in any equilibrium.

I to underbid in the pooling equilibrium, which raises the high type's cost to play a pooling strategy.

Besides the separating equilibrium and the pooling equilibrium, there may also exist the semi-separating equilibrium, in which the private information held by agent I is only partially revealed to agent U through e_I. The contest may potentially yield three types of such equilibria. In the first type of such equilibria, the high-type agent I plays $e_I^H = \frac{m^2}{4}$ with positive probability, which separates her from the low type. She also replicates the low-type agent I's strategy with a positive probability. Another type of semi-separating equilibrium has the low type underbid to separate herself from the high type. The low type also replicates the high-type agent I's strategy with a positive probability. The third type of semi-separating equilibrium requires both types of agent I to play separating strategies with positive probabilities, i.e. $\Delta_I^H \backslash \{\Delta_I^H \cap \Delta_I^L\} \neq \varnothing$ and $\Delta_I^L \backslash \{\Delta_I^H \cap \Delta_I^L\} \neq \varnothing$.

The focus of this paper is on agents' timing choice in stage 0. It is not instructive to characterize all these equilibria in details. However, the partial characterization has been sufficient for us to derive the general property of the equilibrium payoff structure under this institution.

Proposition 5. *Under the institution IU, $\widetilde{\pi}_I^{IU} \leq \Psi$.*

Proposition 5 states that under the institution IU, agent I ex ante expects to receive no more than Ψ, her ex ante expected payoff if the information is complete in the contest. The inequality is binding only in a pooling equilibrium with $e_I = \frac{EV}{4}$, while it is non-binding under all other circumstances. Agent I does not enjoy the information rent, but may suffer as the first mover.

The intuition is as follows. In the separating equilibrium, the high type receives exactly $\frac{m^2}{4}$, because she has completely revealed the private information, and surrendered the potential information rent. On the other hand, the low type has to underbid in order to credibly separate herself from the high type. Hence, a separating equilibrium never benefits agent I. In the pooling equilibrium, because agent U is unaware of the type of the competitor, she turns out to overbid when she is confronted with the low type agent I, which makes the low type worse off. On the other hand, a pooling equilibrium requires either/both the low type to overbid, or/and the high type to underbid, in order to pool the two types of agent I together. Although one type of agent I may benefit, the gain always comes at the cost of the other type. The potential benefit that accrues to one type, however, does not offset the cost to the other type. Hence, prior to the realization of V, agent I can not ex ante expect to receive more than Ψ.

4. Endogenous sequence of moves in contests

Now we look at agents' timing choices in stage 0. Prior to the realization of V, agents simultaneously commit whether to determine their effort outlays in stage 1 or 2. To solve the equilibrium, we need find out each agent's best response given the other player's timing choice. In other words, we compare each agent's ex ante expected payoffs across possible institutions, given the other's choice.

4.1. Agent I's best response

4.1.1. Agent U moves in stage 1

Suppose agent U chooses stage 1. The institution SS results if agent I also picks stage 1, while UI emerges if she picks stage 2. It pays agent I to pick stage 1 if $\tilde{\pi}_I^{SS} > \tilde{\pi}_I^{UI}$, while it pays her to pick stage 2 if $\tilde{\pi}_I^{SS} < \tilde{\pi}_I^{UI}$.

Lemma 3. *If agent U picks stage 1, then,*
(a) when $q \leq \frac{1}{m-1}$, agent I is indifferent between stage 1 and stage 2, i.e. $\tilde{\pi}_I^{SS} = \tilde{\pi}_I^{UI}$;
(b) when $q > \frac{1}{m-1}$, agent I prefers to pick stage 2, i.e. $\tilde{\pi}_I^{SS} < \tilde{\pi}_I^{UI}$.

In the case that $q \leq \frac{1}{m-1}$, UI and SS yield equivalent outcomes. In contrast, in the case that $q > \frac{1}{m-1}$, if agent U moves first, she lowers her effort to avoid potential overbidding, which softens the competition and makes the high-type agent I strictly better off.

4.1.2. Agent U moves in stage 2

Given that agent U moves in stage 2, agent I has to weigh her ex ante expected payoff $\tilde{\pi}_I^{IU}$ against $\tilde{\pi}_I^{SS}$, to decide whether or not to make her effort in stage 1. We may compare $\tilde{\pi}_I^{IU}$ with $\tilde{\pi}_I^{SS}$ through the medium Ψ.

Lemma 4. *If agent U picks stage 2, then agent I strictly prefers to pick stage 2, i.e. $\tilde{\pi}_I^{IU} < \tilde{\pi}_I^{SS}$.*

By Corollary 1, agent I extracts information rent under the institution SS and her ex ante payoff outweighs Ψ. However, by Proposition 5, she receives no more than Ψ if she moves first. The intuition is straightforward: Agent I takes advantage of her superior information under the institution SS.

Lemma 3 and Lemma 4 give rise to the following.

Proposition 6. *(a) When $q \leq \frac{1}{m-1}$, choosing stage* 2 *is a weakly dominant strategy of agent I;*
(b) when $q > \frac{1}{m-1}$, choosing stage 2 *is a strictly dominant strategy of agent I.*

When q is below $\frac{1}{m-1}$, agent I strictly prefers stage 2 if agent U chooses stage 2, but is indifferent between stage 1 and stage 2 if agent U chooses stage 1. However, once q exceeds $\frac{1}{m-1}$, agent I strictly prefers to move in the second stage regardless of the choice of agent U.

4.2. Agent U's best response

4.2.1. Agent I moves in stage 2

Given that agent I picks stage 2, agent U obtains π_U^{UI} if she picks stage 1, or π_U^{SS} if she picks stage 2. Compare π_U^{SS} and π_U^{UI}, and we have the following result.

Lemma 5. *If agent I picks stage 2, then,*
(a) when $q \leq \frac{1}{m-1}$*, agent U is indifferent between stage 1 and stage 2, i.e.* $\pi_U^{SS} = \pi_U^{UI}$*;*
(b) when $q > \frac{1}{m-1}$*, agent U strictly prefers to pick stage 1, i.e.* $\pi_U^{SS} < \pi_U^{UI}$*.*

Again, when q is below $\frac{1}{m-1}$, the institutions UI and SS yield equivalent outcomes. Hence, agent U is indifferent between these two institutions. However, in the case that q exceeds $\frac{1}{m-1}$, agent U benefits as the first mover, because she is able to lower her bid and avoid potential overbidding.

4.2.2. Agent I moves in stage 1

Suppose agent I picks stage 1. If agent U chooses stage 1, she expects to receive π_U^{SS}. If she chooses stage 2, the contest is conducted sequentially and agent U receives π_U^{IU}. However, the latter institution induces multiple equilibria. Agent U's preference then depends on the equilibrium being played in the contest. Agent U's preference between SS and IU is therefore ambiguous.

In the case that a separating equilibrium is to be played in the contest under the institution IU, agent U must prefer to pick stage 2 and receive π_U^{IU}. In the separating equilibrium, agent I completely surrenders the information rent. The high type behaves in the same way as she does in a complete information contest, with $e_I^H = \frac{m^2}{4}$; while the low type agent I has to bear the cost of signaling and underbid with $e_I^L < \frac{1}{4}$. As a consequence, agent U earns strictly more than Ψ, her expected payoff if the value of V is revealed to both agents in contest.

However, agent U may prefer institution SS to IU, if a pooling equilibrium is to be played in the sequential contest. As shown by Proposition 4, the feasible pooling strategy e_I exists in an interval $[[\frac{m^2}{2\sqrt{EV}} \cdot (1 - \sqrt{1 - \frac{EV}{m^2}})]^2, [\frac{1}{2\sqrt{EV}} \cdot (1 + \sqrt{1 - \frac{EV}{m^2}})]^2]$. Agent U's preference can go either way depending on the equilibrium strategy being played. Agent U may suffer if e_I is high, while may benefit if e_I is low.[2]

4.3. Equilibria in stage 0

By the results shown above, we may examine the equilibrium in stage 0, where agents simultaneously decide whether to determine their efforts in stage 1 or stage 2. Agents' strategic trade-offs are demonstrated as a normal-form game in Figure 2, with agent I playing row strategy and agent U playing column strategy.

The top panel of Figure 2 shows that when $q \leq \frac{1}{m-1}$, there may exist three Nash equilibria: (1) agent I picks stage 2 and agent U picks stage 1; (2) both of them pick stage 2; and (3) both

[2] Consider a pooling equilibrium, $\pi_U^{IU} = q \cdot \frac{(\sqrt{EV} - \sqrt{e_I})}{\sqrt{EV}} m^2 + (1-q) \cdot \frac{(\sqrt{EV} - \sqrt{e_I})}{\sqrt{EV}} - \sqrt{EV} \cdot (\sqrt{EV} - \sqrt{e_I}) = (\sqrt{EV} - \sqrt{e_I})^2$. Hence, $\pi_U^{IU} < \pi_U^{SS}$ if $\sqrt{EV} - \sqrt{e_I} < \frac{E\sqrt{V}}{2}$. Agent U may or may not prefer to move second. Recall Proposition 4, a pooling equilibrium exists if $\sqrt{e_I} \in [\frac{m^2}{2\sqrt{EV}} \cdot (1 - \sqrt{1 - \frac{EV}{m^2}}), \frac{1}{2\sqrt{EV}} \cdot (1 + \sqrt{1 - \frac{EV}{m^2}})]$, and $q \leq \frac{3m^2+1}{(m^2+1)^2}$. Now consider a numeric example: $m = 1.2, q = 0.1 < \frac{3m^2+1}{(m^2+1)^2} \approx 0.894$. Hence, $EV = 1.044$, $\sqrt{EV} \approx 1.022$, $E\sqrt{V} = 1.02$, $\frac{m^2}{2\sqrt{EV}} \cdot (1 - \sqrt{1 - \frac{EV}{m^2}}) \approx 0.328$, and $\frac{1}{2\sqrt{EV}} \cdot (1 + \sqrt{1 - \frac{EV}{m^2}}) \approx 0.730$. It is easy to verify if agents play a pooling equilibrium with $\sqrt{e_I} \in (0.512, 0.730]$, $\pi_U^{IU} < \pi_U^{SS}$, and agent U prefers to pick stage 1.

of them pick stage 1.[3] Hence, both UI and SS may emerge as the equilibrium outcomes. However, the bottom panel shows that UI is the only equilibrium as long as q exceeds $\frac{1}{m-1}$.

Theorem 1. *(a) When $q \leq \frac{1}{m-1}$, both the institution UI and the institution SS may emerge in the equilibrium;*

(b) when $q > \frac{1}{m-1}$, the institution UI is the unique Nash equilibrium;

(c) the institution IU is not a Nash equilibrium.

Theorem 1 establishes that the sequential contest where agent U moves first always arises in the equilibrium. Proposition 6 shows that choosing stage 2 is at least a weakly dominant strategy of agent I. When $q \leq \frac{1}{m-1}$, both agent U and agent I are indifferent between UI and SS. Both UI and SS may therefore arise. However, when $q > \frac{1}{m-1}$, the equilibrium is unique. Agent I strictly prefers stage 2, regardless of agent U's choice. On the other hand, given that agent I picks stage 2, agent U must pick stage 1, because $\pi_U^{SS} < \pi_U^{UI}$, which gives rise to a sequential contest where agent I moves second.

In addition, we show that the order of moves in a sequential contest is not arbitrarily chosen. Although the institution *UI* always arises in the equilibrium, the institution IU, with agent I as the first mover, never takes place in the equilibrium.

4.4. Welfare

We now compare the welfare resulting from the equilibrium sequential contest (*UI*) and the simultaneous-move contest. By Lemma 3 and Lemma 5, it is straightforward to have the following result.

Theorem 2. *(a) When $q \leq \frac{1}{m-1}$, both agents are ex ante indifferent between the simultaneous-move contest and the equilibrium sequential contest (UI);*

(b) when $q > \frac{1}{m-1}$, the equilibrium sequential contest (UI) strictly Pareto dominates the simultaneous-move contest.

Theorem 2 states that the endogenous sequential contest (*UI*) creates Pareto improvement over the simultaneous-move contest. We suggest that the endogenous institution design process at the beginning of the game can be viewed as a natural coordination mechanism that facilitates Pareto efficient outcome.

Obviously, an institution that yields higher social surplus must induce lower total expected rent-seeking effort outlays.

Corollary 3. *(a) When $q \leq \frac{1}{m-1}$, the sequential contest (UI) and simultaneous-move contest generate the same level of total expected rent-seeking outlays;*

(b) when $q > \frac{1}{m-1}$, the sequential contest (UI) generates lower total expected rent-seeking outlays than the simultaneous-move contest (SS).

[3] This occurs if certain pooling equilibrium is played in the contest under the institution IU.

When $q > \frac{1}{m-1}$, the sequential contest allows agent U to lower bids, which softens the competition. We then establish that the endogenous sequential contest (UI) reduces rent dissipation.

4.5. Examples

The timing pattern we observe in the history of National Presidential Conventions provides a natural experiment to test our model. In U.S., the timing of national presidential conventions has demonstrated a persistent pattern since 1948: The parties of incumbent presidents always choose the later dates. Morgan (2003) shows that sequential contest is the unique subgame perfect equilibrium if contestants are allowed to choose the timing before the contest. However, Morgan (2003) does not establish the particular order as a regularity, because he assumes contestants are ex ante identical.

However, we provide an alternative rationale for these observations. We argue that this particular order is not arbitrarily chosen, but stems from the underlying information asymmetry between incumbents and challengers.Table 4 shows that all president candidates who represented the parties of incumbent presidents were either sitting presidents or vice-presidents, with the only exception of Stevenson in 1952. The information asymmetry over the value of the contestable prize may arise from various sources. First of all, it is natural to assume that incumbent candidates better understand the worth of the presidency, i.e. "the winner's purse". Secondly, the incumbents know more about the exact cost one has to bear in order to be in office. By contrast, challengers, who have only rough notions about the benefit and cost associated with the presidency, are more likely to be subject to uncertainty. This timing pattern can be interpreted in light of our model. We show that a sequential contest is always a Nash equilibrium in stage 0. In addition, the model shows that the ordering in the sequential contest is not arbitrary. The institution with the incumbent as the first mover does not emerge in the equilibrium, while a sequential contest must have the challenger move first.

Besides National Presidential Conventions, many other examples may fit in our framework. For example, the Meet-Competition-Clause awards incumbent firms the privilege to make the final bid when their competitors attempt to take over their existing clients. Clearly, incumbent firms can more accurately assess the value of the business than their competitors. As shown by our model, such a rule may endogenously arise in equilibrium, which softens the competition and yields lower rent-seeking expenditures. It benefits not only incumbent firms but also their competitors.

5. Concluding remark

This paper endogenizes the sequence of moves in rent-seeking contests in the presence of one-sided information asymmetry. We allow agents to simultaneously choose the timing of their actions in the contest. We find that the equilibrium that leads to sequential contests always exists; but nevertheless the informed agent does not move first in equilibrium. We establish the particular order in which the uninformed party moves first as a regularity stemming from the underlying information asymmetry. This institution allows agents to soften competition, and generates higher social surplus under certain conditions.

Our prediction explains many contest settings in reality that involve agents sequentially determining rent-seeking efforts. The result sheds light on the formation of customs and the design of institutions that give rise to rules of sequential contests.

Appendix A: Tables

Table 3 The dates of national presidential conventions

Year	Start date democrat	Republican	Incumbent	Last mover
1948	July 12	June 21	Democrat	Democrat
1952	July 21	July 7	Democrat	Democrat
1956	August 13	August 20	Republican	Republican
1960	July 11	July 25	Republican	Republican
1964	August 24	July 13	Democrat	Democrat
1968	August 26	August 5	Democrat	Democrat
1972	July 10	August 21	Republican	Republican
1976	July 12	August 16	Republican	Republican
1980	August 11	July 14	Democrat	Democrat
1984	July 16	August 20	Republican	Republican
1988	July 18	August 15	Republican	Republican
1992	July 13	August 17	Republican	Republican
1996	August 26	August 12	Democrat	Democrat
2000	August 13	July 30	Democrat	Democrat
2004	July 26	August 30	Republican	Republican

(Source: See Morgan (2003))

Table 4 Democrat and republican presidential and vice-presidential candidates

	Democrat		Republican	
Year	Presidential	Vice presidential	Presidential	Vice-presidential
1948	Harry S. Truman	Alben W. Barkley	Thomas E. Deway	Earl Warren
1952	Adlai E. Stevenson	John J. Sparkman	Dwight D. Eisenhower	Richard M. Nixon
1956	Adlai E. Stevenson	Estes Kefauver	Dwight D. Eisenhower	Richard M. Nixon
1960	John F. Kennedy	Lyndon Johnson	Richard Nixon	Henry Cabot Lodge
1964	Lyndon B. Johnson	Hubert H. Humphrey	Barry M. Goldwater	William E. Miller
1968	Hubert H. Humphrey	Edmund S. Muskie	Richard Nixon	Spiro T. Agnew
1972	George McGovern	Sargent Shriver	Richard Nixon	Spiro T. Agnew
1976	Jimmy Carter	Walter F. Mondale	Gerald R. Ford	Robert J. Dole
1980	Jimmy Carter	Walter F. Mondale	Ronald Reagan	George Bush
1984	Walter F. Mondale	Geraldine A. Ferraro	Ronald Reagan	George Bush
1988	Michael S. Dukakis	Lloyd Bentsen	George Bush	J. Danforth Quayle
1992	William J. Clinton	Albert A. Gore, Jr.	George Bush	J. Danforth Quayle
1996	William J. Clinton	Albert A. Gore, Jr.	Robert Dole	Jack F. Kemp
2000	Albert A. Gore. Jr.	Joseph I. Lieberman	George W. Bush	Richard B. Cheney
2004	John F. Kerry	John Edwards	George W. Bush	Richard B. Cheney

Appendix B: Proof

Proof of Corollary 1

Proof: First consider the case $q < \frac{1}{m-1}$. $EV = (\sqrt{EV})^2$. By Jensen's inequality, we then have $EV = (\sqrt{EV})^2 > (E\sqrt{V})^2$. Hence, $\widetilde{\pi}_I^{SS} = EV - \frac{3(E\sqrt{V})^2}{4} > EV - \frac{3EV}{4} = \Psi$.

Then we consider the case $q \geq \frac{1}{m-1}$. $\pi_I^{SS} - \Psi = \frac{q}{(1+q)^2}m^2 - \frac{qm^2+(1-q)}{4} = \frac{4q-(1+q)^2q}{4(1+q)^2}m^2 - \frac{1-q}{4} = \frac{(3+q)(1-q)}{4(1+q)^2}qm^2 - \frac{1-q}{4} = \frac{1-q}{4} \cdot [\frac{(3+q)qm^2}{(1+q)^2} - 1]$.

Because $q \geq \frac{1}{m-1}$, we have $m \geq \frac{1+q}{q}$. Hence, $\frac{(3+q)qm^2}{(1+q)^2} - 1 \geq \frac{3+q}{q} - 1 > 0$. □

Proof of Lemma 1

Proof: $\lim_{e_U \uparrow 1} \pi_U = E\sqrt{V} - 1 = \pi_U(1) = \lim_{e_U \downarrow 1} \pi_U = qm - 1 + 1 - q = q(m-1) = E\sqrt{V} - 1$. □

Proof of Proposition 2

Proof: Look at Figure 1.

To have an active equilibrium where agent I exerts positive effort regardless of the realization of V, we must have the payoff-maximizing effort lower than V_L. Look at Figure 1. Agent U chooses $e_U < V_L$ if and only if $\frac{E\sqrt{V}}{2} < \sqrt{V_L}$, which implies $\frac{qm+(1-q)}{2} < 1 \Leftrightarrow q < \frac{1}{m-1}$. In response to $e_U = \frac{(E\sqrt{V})^2}{4}$, the high-type agent I chooses $e_I^H = \sqrt{e_U} \cdot (m - \sqrt{e_U}) = \frac{E\sqrt{V}}{2} \cdot (m - \frac{E\sqrt{V}}{2})$, while the low type chooses $e_I^L = \sqrt{e_U} \cdot (1 - \sqrt{e_U}) = \frac{E\sqrt{V}}{2} \cdot (1 - \frac{E\sqrt{V}}{2})$.

When $e_U \geq 1$, Agent I remains inactive. Agent U chooses $e_U = 1$ if and only if 1 falls between $\frac{qm}{2}$ and $\frac{E\sqrt{V}}{2}$, which implies $\frac{qm}{2} \leq 1 \leq \frac{qm+(1-q)}{2} \Leftrightarrow \frac{1}{m-1} \leq q \leq \frac{2}{m}$. In response to $e_U = 1$, the high-type agent I chooses $e_I^H = \sqrt{e_U} \cdot (m - \sqrt{e_U}) = m - 1$, while the low type stays inactive with $e_I^L = 0$.

Agent U chooses $e_U = \frac{qm}{2}$ if and only if $\frac{qm}{2}$ falls between 1 and $\frac{E\sqrt{V}}{2}$, which implies $\frac{qm}{2} > 1 \Leftrightarrow q > \frac{2}{m} > \frac{1}{m-1}$. In response to $e_U = \frac{qm}{2}$, the high-type agent I chooses $e_I^H = \sqrt{e_U} \cdot (m - \sqrt{e_U}) = \frac{qm}{2} \cdot (m - \frac{qm}{2})$, while the low type stays inactive with $e_I^L = 0$. □

Proof of Corollary 2

Proof: In the case of UI, the equilibrium outcome is equivalent to the case of SS. Then we consider the case $\frac{1}{m-1} \leq q \leq \frac{2}{m}$. $\widetilde{\pi}_I^{UI} - \frac{EV}{4} = q(m-1)^2 - \frac{q(m^2-1)+1}{4} = qm[(m-1) - \frac{m+1}{4}] - \frac{1}{4} = qm(\frac{3m-5}{4}) - \frac{1}{4}$. Because $q \geq \frac{1}{m-1}$, we have $m > 2$, and $q(m-1) \geq 1$. Hence, $qm(\frac{3m-5}{4}) - \frac{1}{4} > \frac{1}{4} - \frac{1}{4} = 0$.

Now we consider the case $q > \frac{2}{m}$. $\widetilde{\pi}_I^{UI} - \frac{EV}{4} = q(1 - \frac{q}{2})^2 m^2 - \frac{q(m^2-1)+1}{4} = \frac{qm^2}{4}[(2-q)^2 - 1] - \frac{1-q}{4} = \frac{qm^2}{4}[(3-q)(1-q)] - \frac{1-q}{4} = \frac{1-q}{4}[qm^2(3-q) - 1] > \frac{1-q}{4}(2-q) > 0$. □

Proof of Lemma 2

Proof: (a) Suppose the contrary. Assume two effort entries, $e_1, e_2 \in \Delta_I^H \cap \Delta_I^L$, and $e_1 \neq e_2$. Define μ_1 and μ_2 to be $\mu_1 = \mu(s_1)$ and $\mu_2 = \mu(S_2)$ as agent U's equilibrium belief upon observing e_1 and e_2, respectively. A mixed-strategy equilibrium requires agents to be

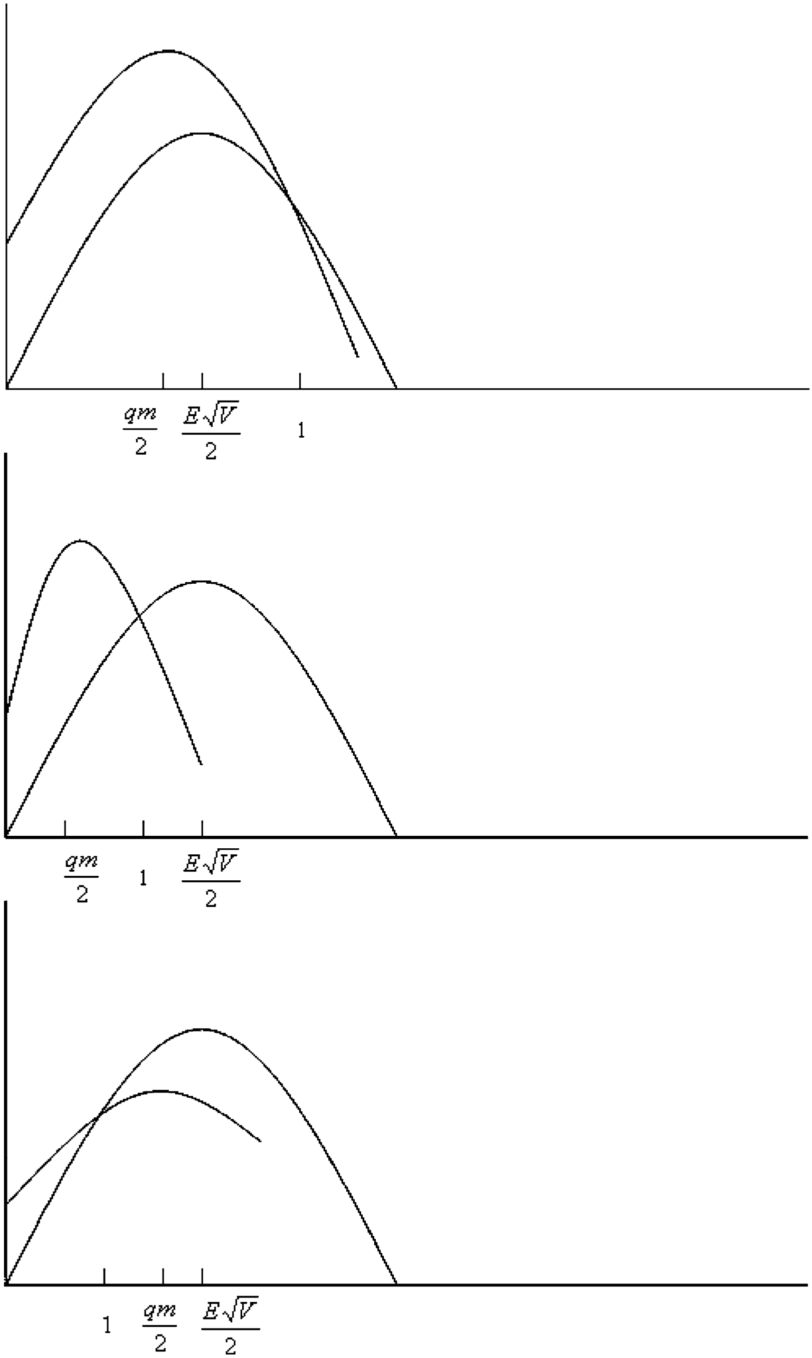

Fig. 1 π_U under the institution *UI*

indifferent between these two effort levels. The following conditions must hold:

$$\sqrt{e_1}\left(\frac{m^2}{\sqrt{E_{\mu_1}V}} - \sqrt{e_1}\right) = \sqrt{e_2}\left(\frac{m^2}{\sqrt{E_{\mu_2}V}} - \sqrt{e_2}\right), \tag{12}$$

$$\sqrt{e_1}\left(\frac{1}{\sqrt{E_{\mu_1}V}} - \sqrt{e_1}\right) = \sqrt{e_2}\left(\frac{1}{\sqrt{E_{\mu_2}V}} - \sqrt{e_2}\right). \tag{13}$$

Divide Equation (12) by (13) on both sides, and we have

$$\frac{m^2 - \sqrt{E_{\mu_1}V}\cdot\sqrt{e_1}}{1 - \sqrt{E_{\mu_1}V}\cdot\sqrt{e_1}} = \frac{m^2 - \sqrt{E_{\mu_2}V}\cdot\sqrt{e2}}{1 - \sqrt{E_{\mu 2}V}\cdot\sqrt{e2}} \Leftrightarrow$$

$$\sqrt{E_{\mu_1}V}\cdot\sqrt{e_1} = \sqrt{E_{\mu 2}V}\cdot\sqrt{e2}. \tag{14}$$

Rearrange (12) on both sides, we have

$$\frac{\sqrt{e_1}}{\sqrt{E_{\mu_1}V}}(m^2 - \sqrt{e_1}\cdot\sqrt{E_{\mu_1}V}) = \frac{\sqrt{e_2}}{\sqrt{E_{\mu_2}V}}(m^2 - \sqrt{e_2}\cdot\sqrt{E_{\mu_2}V}). \tag{15}$$

By Equation (14), (15) implies

$$\frac{\sqrt{e_1}}{\sqrt{E_{\mu_1}V}} = \frac{\sqrt{e_2}}{\sqrt{E_{\mu_2}V}}. \tag{16}$$

Combining (14) and (16) yields $\sqrt{E_{\mu_1}V}\cdot\sqrt{e_1} = \sqrt{E_{\mu 2}V}\cdot\sqrt{e2}. = \sqrt{E_{\mu_2}V}\cdot\sqrt{e_1} = \sqrt{E_{\mu_1}V}\cdot\sqrt{e_2}$, which implies $e_1 = e_2$. Contradiction. Hence, we conclude that $\Delta_I^H \cap \Delta_I^L$ contains at most one element.

(b) Suppose $\Delta_I^H \backslash \{\Delta_I^H \cap \Delta_I^L\}$ is nonempty. Agent U believes $\mu(e_I^H) = 1$ for all $e_I^H \in \Delta_I^H \backslash \{\Delta_I^H \cap \Delta_I^L\}$. Given the belief, π_I^H is given by$\sqrt{e_I^H}\cdot(m - \sqrt{e_I^H})$, which has a unique maximizer $e_I^H = \frac{m^2}{4}$, which gives the reservation payoff $\frac{m^2}{4}$. □

Proof of Proposition 3

Proof: By Lemma 2, in a separating equilibrium, the high type agent I plays a pure strategy $e_I^H = \frac{m^2}{4}$. Let $e \in \Delta_I^L$ denote an effort made by the low type in equilibrium. The individual rationality condition requires

$$\sqrt{e}(1 - \sqrt{e}) \geq \frac{1}{4m^2}. \tag{17}$$

The incentive compatibility constraints are given by

$$\sqrt{e}\cdot\left(\frac{m^2}{1} - \sqrt{e}\right) \leq \frac{m^2}{4}, \tag{18}$$

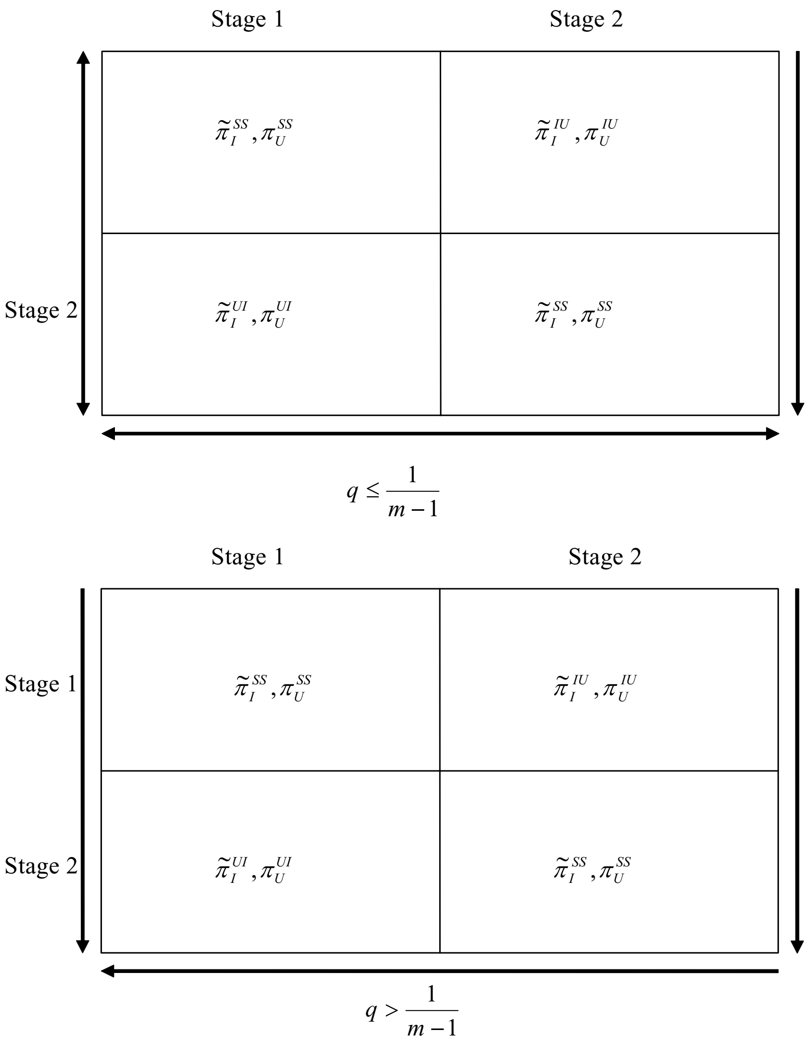

Fig. 2 Equilibrium in Stage 0

$$\frac{m}{2} \cdot \left(\frac{1}{m} - \frac{m}{2}\right) \leq \sqrt{e} \cdot (1 - \sqrt{e}) \tag{19}$$

(17) is satisfied if and only if $\frac{1-\sqrt{1-\frac{1}{m^2}}}{2} \leq \sqrt{e} \leq \frac{1+\sqrt{1-\frac{1}{m^2}}}{2}$. (18) is satisfied if $\sqrt{e} \geq \frac{1+\sqrt{1-\frac{1}{m^2}}}{2} \cdot m^2$ or $\sqrt{e} \leq \frac{1-\sqrt{1-\frac{1}{m^2}}}{2} \cdot m^2$. It is obvious to see $\frac{1+\sqrt{1-\frac{1}{m^2}}}{2} \cdot m^2 > \frac{1+\sqrt{1-\frac{1}{m^2}}}{2}$, and $\frac{1-\sqrt{1-\frac{1}{m^2}}}{2} \cdot m^2 > \frac{1-\sqrt{1-\frac{1}{m^2}}}{2}$. The low-type agent I's equilibrium strategy is $e_I^L = \frac{1}{4}$. It is straightforward to verify that $\frac{1-\sqrt{1-\frac{1}{m^2}}}{2} \cdot m^2$ is smaller than $\frac{1}{2}$, since $\frac{1-\sqrt{1-\frac{1}{m^2}}}{2} \cdot m^2 - \frac{1}{2} =$

$\frac{m^2}{2} \cdot [(1 - \frac{1}{m^2}) - \sqrt{1 - \frac{1}{m^2}}] < 0$. Some $\sqrt{e} \in [\frac{1-\sqrt{1-\frac{1}{m^2}}}{2}, \frac{1-\sqrt{1-\frac{1}{m^2}}}{2} \cdot m^2]$ always exists to sustain a separating equilibrium.

(19) always holds. Consider (19). LHS of (19)$-\frac{1}{4m^2} = \frac{1}{2} - \frac{m^2}{4} - \frac{1}{4m^2} = \frac{1}{4}(2 - m^2 - \frac{1}{m^2}) = -\frac{1}{4}(m^2 - 2 + \frac{1}{m^2}) = -\frac{1}{4}(m - \frac{1}{m})^2 < 0$. Hence, the low-type gent I has no incentive to pretend to be the high type.

These separating equilibria can be supported by an out-of-equilibrium belief where $\mu = 1$ for $e > e_I^L$, and $\mu = 0$ for $e \leq e_I^L$. By (18), given agent U's belief, the agent I has no incentive to pretend to be the low type and she simply chooses her first best strategy $e = \frac{m^2}{4}$. By (19), the low type has no incentive to pretend to be the high type. By (17), since $e_I^L < \frac{1}{4}$, the low type has no incentive to choose $e < e_I^L$. In addition, by (18), she has no incentive to choose any effort $e > e_I^L$, since she can expect to obtain no more than her reservation payoff given by Lemma 1.

In addition, in any separating equilibrium, the low type chooses a unique e_I^L. As we show, e_I^L is less $\frac{1}{4}$. However, $\pi_I^L = \sqrt{e_I^L}(1 - \sqrt{e_I^L})$ monotonically increases with e_I^L until $\frac{1}{4}$. Thus, the low-type agent I does not play mixed strategy. □

Proof of Proposition 4

Proof: By Lemma 2, the pooling equilibrium is a pure strategy equilibrium, with $e_I^H = e_I^L = e_I$ to be the uniform equilibrium effort made by both types. An equilibrium requires

$$\sqrt{e_I} \cdot \left(\frac{1}{\sqrt{EV}} - \sqrt{e_I}\right) \geq \frac{1}{4m^2} \tag{20}$$

$$\sqrt{e_I} \cdot \left(\frac{m^2}{\sqrt{EV}} - \sqrt{e_I}\right) \geq \frac{m^2}{4} \tag{21}$$

(20) and (21) are both satisfied if and only if $\frac{1}{2\sqrt{EV}} \cdot (1 - \sqrt{1 - \frac{EV}{m^2}}) \leq \sqrt{e_I} \leq \frac{1}{2\sqrt{EV}} \cdot (1 + \sqrt{1 - \frac{EV}{m^2}})$, and $\frac{m^2}{2\sqrt{EV}} \cdot (1 - \sqrt{1 - \frac{EV}{m^2}}) \leq \sqrt{e_I} \leq \frac{m^2}{2\sqrt{EV}} \cdot (1 - \sqrt{1 - \frac{EV}{m^2}})$. To have a pooling equilibrium, it is necessary to have $\frac{m^2}{2\sqrt{EV}} \cdot (1 - \sqrt{1 - \frac{EV}{m^2}}) \leq \frac{1}{2\sqrt{EV}} \cdot (1 + \sqrt{1 - \frac{EV}{m^2}})$, which is satisfied if and only if $q \leq \frac{3m^2+1}{(m^2+1)^2} (< 1)$. Hence, the existence of a pooling equilibrium requires $e_I \in [(\frac{m^2}{2\sqrt{EV}} \cdot (1 - \sqrt{1 - \frac{EV}{m^2}})^2, (\frac{1}{2\sqrt{EV}} \cdot (1 + \sqrt{1 - \frac{EV}{m^2}})^2]$. These pooling equilibria can be supported by an out-of-equilibrium belief where $\mu = 1$ for any $e \neq e_I$.

Given agent U's belief, agent I has no incentive to to choose any $e \neq e_I$, since she is unable to earn more than $\frac{m^2}{4}$ or $\frac{1}{4m^2}$. □

Proof of Proposition 5

Proof: Assume $e_1 \in \Delta_I^H \backslash \{\Delta_I^H \cap \Delta_I^L\}$, $e_2 \in \Delta_I^H \cap \Delta_I^L$ and $e_3 \in \Delta_I^L \backslash \{\Delta_I^H \cap \Delta_I^L\}$. In equilibrium, the high-type agent I plays e_2 with probability P_1, while the low-type plays e_2 with probability P_2. By Bayesian updating, agent U obtains a belief $\mu = \mu(e_2) = \frac{qP_1}{qP_1 + (1-q)P_2}$ upon observing e_2.

The ex ante expected payoff of agent I is given by

$$\begin{aligned}\tilde{\pi}_I^{IU} &= q\left[(1-P_1)\frac{m^2}{4} + P_1\sqrt{e_2}\left(\frac{m^2}{\sqrt{E_\mu V}} - \sqrt{e_2}\right)\right] \\ &\quad +(1-q)\left[(1-P_2)\sqrt{e_3}(1-\sqrt{e_3}) + P_2\sqrt{e_2}\left(\frac{1}{\sqrt{E_\mu V}} - \sqrt{e_2}\right)\right] \qquad (22)\\ &= \left[q(1-P_1)\frac{m^2}{4} + (1-q)(1-P_2)\sqrt{e_3}(1-\sqrt{e_3})\right] \\ &\quad +\left[qP_1\sqrt{e_2}\left(\frac{m^2}{\sqrt{E_\mu V}} - \sqrt{e_2}\right) + (1-q)P_2\sqrt{e_2}\left(\frac{1}{\sqrt{E_\mu V}} - \sqrt{e_2}\right)\right]. \qquad (23)\end{aligned}$$

Define Γ_1 to be the term in the first bracket. Because e_3, which separates the low type from the high type, requires underbidding, we have$\sqrt{e_3}(1-\sqrt{e_3})$. Thus, if follows

$$\begin{aligned}\Gamma_1 &= q(1-P_1)\frac{m^2}{4} + (1-q)(1-P_2)\sqrt{e_3}(1-\sqrt{e_3}) \\ &< q(1-P_1)\frac{m^2}{4} + (1-q)(1-P_2)\frac{1}{4}. \qquad (24)\end{aligned}$$

Define Γ_2 to be the term in the second bracket.

$$\begin{aligned}\Gamma_2 &= \sqrt{e_2}\left[qP_1\frac{m^2}{\sqrt{E_\mu V}} + (1-q)P_2\frac{1}{\sqrt{E_\mu V}} - \sqrt{e_2}\right] \\ &= \sqrt{e_2}\left\{qP_1\frac{m^2}{\sqrt{\frac{qP_1}{qP_1+(1-q)P_2}m^2 + \frac{(1-q)P_2}{qP_1+(1-q)P_2}}}\right. \\ &\quad \left. + (1-q)P_2\frac{1}{\sqrt{\frac{qP_1}{qP_1+(1-q)P_2}m^2 + \frac{(1-q)P_2}{qP_1+(1-q)P_2}}} - [qP_1+(1-q)P_2]\sqrt{e_2}\right\} \\ &= \sqrt{e_2}\left\{\frac{qP_1m^2+(1-q)P_2}{\sqrt{\frac{qP_1m^2+(1-q)P_2}{qP_1+(1-q)P_2}}} - [qP_1+(1-q)P_2]\sqrt{e_2}\right\} \\ &= \sqrt{e_2}\left\{\sqrt{qP_1m^2+(1-q)P_2}\cdot\sqrt{qP_1+(1-q)P_2} - [qP_1+(1-q)P_2]\sqrt{e_2}\right\} \\ &= [qP_1+(1-q)P_2]\sqrt{e_2}\left[\sqrt{\frac{qP_1m^2+(1-q)P_2}{qP_1+(1-q)P_2}} - \sqrt{e_2}\right] \\ &= [qP_1+(1-q)P_2]\sqrt{e_2}(\sqrt{E_\mu V} - \sqrt{e_2}) \\ &\le [qP_1+(1-q)P_2]\frac{E_\mu V}{4}. \qquad (25)\end{aligned}$$

Combine (24) and (25), and we have

$$
\begin{aligned}
\tilde{\pi}_I^{IU} &= \Gamma_1 + \Gamma_2 \\
&< q(1-P_1)\frac{m^2}{4} + (1-q)(1-P_2)\frac{1}{4} + [qp_1 + (1-q)]\frac{E_\mu V}{4} \\
&= q(1-P_1)\frac{m^2}{4} + [qP_1 + (1-q)P_2]\frac{qP_1 m^2}{4[qP_1+(1-q)P_2]} \\
&\quad + (1-q)(1-P_2)\frac{1}{4} + [qP_1 + (1-q)P_2]\frac{(1-q)P_2}{4[qP_1+(1-q)P_2]} \\
&= \frac{qm^2}{4} + \frac{1-q}{4} = \Psi.
\end{aligned}
$$

□

Proof of Lemma 3

Proof: When $q < \frac{1}{m-1}$, $\tilde{\pi}_I^{UI} = \tilde{\pi}_I^{SS} = EV - \frac{3(E\sqrt{V})^2}{4}$.

When $\frac{1}{m-1} \le q \le \frac{2}{m}$, $\tilde{\pi}_I^{UI} - \tilde{\pi}_I^{SS} = q(m-1)^2 - \frac{q}{(1+q)^2}m^2$. Because $q \ge \frac{1}{m-1}$, we must have $\frac{1}{m-1} < 1$, which implies $m \ge \frac{1+q}{q}$. Hence,

$$
\begin{aligned}
q(m-1)^2 - \frac{q}{(1+q)^2}m^2 &= \frac{q}{(1+q)^2}[(m-1)^2(1+q) - m^2] \\
&= \frac{q}{(1+q)^2}[(m-1)(1+q)+m][(m-1)(1+q)-m] \\
&= \frac{q}{(1+q)^2}[(m-1)(1+q)+m][(m-1)(1+q)-m] \quad (26)
\end{aligned}
$$

$(m-1)(1+q) - m = mq - (1+q) = q[m - \frac{1-q}{q}] \ge 0$. The inequality is binding if $q = \frac{1}{m-1}$.

When $q > \frac{2}{m}$,

$$
\begin{aligned}
\tilde{\pi}_I^{UI} - \tilde{\pi}_I^{SS} &= q\left(1-\frac{q}{2}\right)^2 m^2 - \frac{q}{(1+q)^2}m^2 \\
&= qm^2\left[\left(1-\frac{q}{2}\right)^2 - \frac{1}{(1+q)^2}\right] \\
&= qm^2\left(1-\frac{q}{2}+\frac{1}{1+q}\right)\left(1-\frac{q}{2}-\frac{1}{1+q}\right) \quad (27)
\end{aligned}
$$

$$
1 - \frac{q}{2} - \frac{1}{1+q} = \frac{(2-q)(1+q)-2}{2(1+q)} = \frac{q-q^2}{2(1+q)} > 0
$$

□

Proof of Lemma 5

Proof: When $q < \frac{1}{m-1}$, $\pi_U^{UI} = \pi_U^{SS} = \frac{(E\sqrt{V})^2}{4}$.

When $\frac{1}{m-1} \leq q \leq \frac{2}{m}$, $qm \geq 1 + q$. We have

$$q(m-1) - \frac{q^3}{(1+q)^2}m^2 - (1-q) = \frac{1}{(1+q)^2}[qm(1+q)^2 - (1+q)^2 - q^3m^2] \quad (28)$$

Consider the term in the bracket.

$$\begin{aligned} qm(1+q)^2 - (1+q)^2 - q^3m^2 &= qm[(1+q)^2 - q^2m] - (1+q)^2 \\ &= qm[(1+q) + q + q^2 - q^2m] - (1+q)^2 \\ &= [qm - (1+q)](1+q) + qm[q(1+q) - q^2m] \\ &= [qm - (1+q)](1+q) - q^2m[qm - (1+q)] \\ &= [qm - (1+q)][(1+q) - q^2m] \end{aligned} \quad (29)$$

When $q \leq \frac{2}{m}$, $q^2m \leq 2q$. Hence, we have $(1+q) - q^2m \geq 1 - q > 0$. We therefore shows that that $\pi_U^{UI} \geq \pi_U^{SS}$ if $\frac{1}{m-1} \leq q \leq \frac{2}{m}$. The inequality is binding if and only if $q = \frac{1}{m-1}$.

When $q > \frac{2}{m}$,

$$\begin{aligned} \pi_U^{UI} - \pi_U^{SS} &= \frac{q^2m^2}{4} - \frac{q^3m^2}{(1+q)^2} \\ &= q^2m^2\frac{(1+q)^2 - 4q}{4(1+q)^2} \\ &= q^2m^2\frac{(1-q)^2}{4(1+q)^2} > 0 \end{aligned} \quad (30)$$

We see $\pi_U^{UI} > \pi_U^{SS}$ if $q > \frac{2}{m}$. □

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