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Contest with pre-contest investment☆

Qiang Fu [a,*], Jingfeng Lu [b,1]

[a] Department of Business Policy, National University of Singapore, 1 Business Link, 117592, Singapore
[b] Department of Economics, National University of Singapore, 10 Kent Ridge Crescent, 119260, Singapore

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ABSTRACT

In a standard noisy contest, more competition (more contestants) leads to lower individual equilibrium effort. We show that when contestants can make pre-contest investment to enhance their competency, neither equilibrium investment nor individual effort is monotonic in the number of contestants. Individual effort may increase with the level of participation.

1. Introduction

An enormous amount of academic effort has been devoted to studies on economic agents' competitive strategies in contests. Contestants often make costly investment to improve competency prior to the formal competition.[2] As noted by Danhof (1968) and Lichtenberg (1988), pre-contest investment is an essential dimension in analyzing competitive activities in contests. Only a handful of papers, however, have followed in this line to study contestants' investment behaviors. The latest study is by Münster (2007) who characterizes the investment equilibria in a two-person all-pay auction. He finds that only asymmetric pure-strategy equilibria exist when contestants simultaneously invest; while sequential investments could soften the subsequent competition.[3]

In this paper, we study in an imperfectly discriminating contest the strategic behaviors of contestants when they can invest on their competency. We address a classical question in contest design: does the level of participation (the number of contestants) necessarily decrease individual equilibrium effort? A usual wisdom in contest literature is that individual effort strictly decreases with the level of participation.[4] An additional trade-off arises when pre-contest investment kicks in: additional competition could affect participants' incentive of investment, which in turn affects equilibrium effort supply. The relationship between the number of contestants and resulted equilibrium effort therefore remains obscure, before we understand how the level of participation affects contestants' equilibrium investments.

Mere intuition says that more contestants necessarily crowd out equilibrium investment, as additional competition squeezes contestable rent. However, our analysis shows that its effect is a double-edged sword and the opposite could result. Positive equilibrium investment emerges when the number of contestants is small, while zero investment occurs in equilibrium when the number of contestants is large. Furthermore, the relationship between the number of contestants and equilibrium investment is non-monotonic: equilibrium investment may increase with the level of participation.

The positive effect of participation on individual effort runs in contrast to its negative effect in contests without pre-contest investment. When additional competitor participates, it disciplines contestants and may give them a stronger incentive to invest and to

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* Corresponding author. Tel.: +65 65163775; fax: +65 67795059.
E-mail addresses: bizfq@nus.edu.sg (Q. Fu), ecsljf@nus.edu.sg (J. Lu).
[1] Tel.: +65 65166026; fax: +65 67752646.
[2] For instance, an R&D company purchases laboratory equipment, which improves the efficiency of subsequent research activities.
[3] Other studies on pre-contest investments include Tan (1992), and Piccione and Tan (1996) and Kräkel (2004). Tan (1992) and Piccione and Tan (1996) study R&D investment in procurement tournaments (auctions). Kräkel (2004) study oligopolistic contests with R&D spillover and strategic delegation.
[4] Total equilibrium effort, however, increases with the number of contestants.

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preempt their rivals: they would lose more if they were "left behind at the starting line." This effect looms large when the number of contestants is small, while it diminishes when the number is large.

Furthermore, our analysis allows us to gain insights on another classical question in contest design: does it pay to limit competition by shortlisting contestants? Does lesser competition necessarily result in more competitive contestants? An affirmative answer is expected in a standard contest. However, it is called into question when pre-contest investment is available. On the one hand, the contest organizer may prefer to increase the level of participation when the number of contestants is small, because additional competition may boost the quality of contestants. The enhanced competency in turn leads to higher individual equilibrium effort. On the other hand, our result may lend support to shortlisting when the number of contestants is large. As equilibrium investment drops to zero when number of contestants gets excessively large, equilibrium effort definitely declines if participation further increases. A handful of papers discuss the optimality of "shortlisting" from other perspectives. Our result complements this literature by illuminating the additional trade-off that arises from pre-contest investment.[5]

2. The model

There are $K\geq 2$ ex ante identical contestants, indexed by $i=1, 2,\ldots, K$ who compete for an indivisible prize. Without loss of generality, we normalize the value of the prize to one. Denote by Ω_K the index set of all participants.

We model the competition for the prize as a standard Tullock contest. Contestants simultaneously commit to their effort outlays x_k, $k\in\Omega_K$. A contestant k wins with a probability

$$p_k = \frac{x_k}{x_k + \sum_{k'\in\Omega_K\setminus\{k\}} x_{k'}}. \tag{1}$$

A contestant k bears a constant marginal cost c_k for each unit of effort supply. A contestant can reduce her marginal cost c_k by making technological investment prior to the contest. Specifically, the marginal cost c_k of contestant k is determined by her pre-contest investment I_k, with $c_k\equiv c(I_k)$, where $c'(\cdot)<0$ and $c''(\cdot)>0$.

The game proceeds in two stages. In the first stage, contestants simultaneously engage in technological investment I_k, $k\in\Omega_K$ in order to lower their marginal costs. The investments of contestants are publicly observable.[6] In the second stage, contestants compete in the contest.

3. Analysis

We solve the game by backward induction. For the sake of tractability, we assume that the cost function $c(I_k)$ takes a convenient form of $c(I_k)=\frac{\lambda}{\lambda+I_k}$. Clearly, $c'(\cdot)<0$ and $c''(\cdot)>0$. The parameter λ can be naturally interpreted as ones' initial stock of technological knowledge. Clearly, a larger λ implies a less significant impact of the investment.

Let the ordered set $\mathbf{I}_k=(I_1, I_2,\ldots, I_K)$ indicates any strategy profile of all contestants in the first stage. Let $x_k(I_k)$ denote a contestant k's equilibrium effort supply in the endgame (the contest). A contestant $k\in\Omega_K$ ends up with an expected payoff in the contest

$$\pi_k(I_k,\mathbf{I}_{-k}) = \frac{x_k(I_k,\mathbf{I}_{-k})}{x_k(I_k,\mathbf{I}_{-k}) + \sum_{k'\in\Omega_K\setminus\{k\}} x_{k'}(I_k,\mathbf{I}_{-k})} - c(I_k)x_k(I_k,\mathbf{I}_{-k}) - I_k, \tag{2}$$

where $\mathbf{I}_{-k}=(I_1, I_2,\ldots, I_{k-1}, I_{k+1},\ldots, I_k)$ denotes the investment strategy profile of the other $K-1$ contestants. Thus, in the first stage, each contestant k strategically chooses her investment level I_k to maximize the payoff $\pi_k(I_k,\mathbf{I}_{-k})$. For any given investment strategy profile $\mathbf{I}_{-k}$, a contestant k's best response I_k^* would be defined by the first order condition

$$\frac{\partial \pi_k}{\partial I_k^*}\begin{cases} =0 & \text{if } I_k^*>0; \\ \leq 0 & \text{if } I_k^*=0. \end{cases} \tag{3}$$

Since $\frac{\partial\pi_k}{\partial I_k}$ involves terms $\frac{\partial x_k(I_k,\mathbf{I}_{-k})}{\partial I_{k'}}$ for every k, $k'\in\Omega_K$, completely characterizing the subgame perfect equilibria of this game directly from the general best response functions characterized by Eq. (3) is not an easy task, if not impossible. Thus, in the subsequent analysis, we focus on the pure-strategy symmetric subgame equilibrium where all contestants choose the same level of investment, and therefore exert the same amount of effort in the contest.

Suppose that all contestants but k invest $I\geq 0$ in the first stage of the game, while contestant k invests $I_k\geq 0$. Denote this investment profile by (I_k, I). In the following subgame, contestant k chooses x_k to maximize his/her payoff

$$U_k = \frac{x_k}{\left(x_k + \sum_{k'\in\Omega_K\setminus\{k\}} x_{k'}\right)} - c(I_k)x_k - I_k. \tag{4}$$

Letting all contestants other than k play a symmetric strategy x, the first order condition for the optimal x_k boils down to

$$\frac{\partial U_k}{\partial x_k} = \frac{(K-1)x}{[x_k+(K-1)x]^2} - c(I_k) = 0. \tag{5}$$

Similarly, the first order condition for the best choice of any other contestant k' is

$$\frac{x_k+(K-2)x}{[x_k+(K-1)x]^2} - c(I) = 0, \forall k'\in\Omega_K\setminus\{k\}. \tag{6}$$

Solving Eqs. (5) and (6) yields the equilibrium effort for the investment profile (I_k, I):

$$x^*_{k'}(I_k,I) = x^*(I_k,I) = \frac{c(I_k)(K-1)}{[c(I)(K-1)+c(I_k)]^2}, \forall k'\in\Omega_K\setminus\{k\}, \tag{7}$$

$$\text{and } x^*_k(I_k,I) = \frac{[c(I)(K-1)-c(I_k)(K-2)](K-1)}{[c(I)(K-1)+c(I_k)]^2}. \tag{8}$$

Also note that we assume $c(I_k)=\frac{\lambda}{\lambda+I_k}$ and $c_{k'}=c(I)=\frac{\lambda}{\lambda+I}$, $\forall k'\in\Omega_K\setminus\{k\}$. Therefore, provided that all other contestants choose investment level I, a contestant k with I_k would expect to receive a payoff

$$\pi_k(I_k,I) = \left[\frac{(\lambda+I_k)(K-1)-(\lambda+I)(K-2)}{(\lambda+I_k)(K-1)+(\lambda+I)}\right]^2 - I_k. \tag{9}$$

Proposition 1. *Suppose there are $K\geq 2$ contestants. (i) A unique pure-strategy symmetric subgame perfect equilibrium with positive investment exists only if $\lambda\geq\frac{2(K-1)^2}{K^3}-\frac{1}{K^2}$. (ii) A symmetric subgame perfect equilibrium with zero investment, i.e., $I^*(K)=0$, exists if and only if*

[5] Baye et al. (1993), Fullerton and McAfee (1999) and Che and Gale (2003) emphasize the benefit of selecting the most appropriate contestants when candidates are asymmetric. The result of Fu and Lu (2008) instead replies on savings in positive entry costs. None of these papers involve pre-contest investment.

[6] We conducted analysis when investment is not observable. In this case, the contestants simultaneously choose their investment and effort. The analysis is simpler as it does not require sequential rationality. Nevertheless, the results are consistent with the findings obtained in the current setting. The details are available from the authors upon request.

$\lambda \geq \frac{2(K-1)^2}{K^3}$. In this equilibrium, each contestant exerts an equilibrium effort

$$x^*(K) = \frac{K-1}{K^2}. \tag{10}$$

(iii) If $K \geq 4$, there exists no symmetric subgame perfect equilibrium with positive investment. (iv) If $K=2,3$ and $\lambda \in \left[\frac{2(K-1)^2}{K^3} - \frac{1}{K^2}, \frac{2(K-1)^2}{K^3}\right]$, a unique symmetric equilibrium with positive equilibrium investment exists iff $x_k^*(I^*(K), I^*(K)) \geq x_k^*(0, I^*(K))$. In the equilibrium, each contestant invests

$$I^*(K) = \frac{2(K-1)^2}{K^3} - \lambda, \tag{11}$$

and exerts an equilibrium effort

$$x^*(K) = \frac{2(K-1)^3}{\lambda K^5}. \tag{12}$$

Proof. We first consider interior equilibrium where contestants make positive investment. When all other contestants choose the same investment level I, Eq. (9) leads to the best response function $I_k(I)$ of a contestant k, which is defined by the following first order condition

$$\frac{(\lambda + I_k)(K-1) - (\lambda + I)(K-2)}{(\lambda + I_k)(K-1) + (\lambda + I)} \cdot \frac{2(K-1)^2(\lambda + I)}{[(\lambda + I_k)(K-1) + (\lambda + I)]^2} = 1. \tag{13}$$

To obtain equilibrium strategy I^* in a symmetric equilibrium, we replace I_k and I by I^* in Eq. (13) and solve for $I^*(K) = \frac{2(K-1)^2}{K^3} - \lambda$. By doing so, we therefore obtain the unique candidate for symmetric interior investment equilibrium strategy. Each contestant's payoff in this subgame is

$$\pi^*(K) = \frac{1}{K^2}\left[1 - \frac{2(K-1)^2}{K}\right] + \lambda. \tag{14}$$

Apparently, such a symmetric equilibrium with positive investment exists only if $I^*(K)>0$ and $\pi^*(K) \geq 0$, i.e. $\lambda \in \left[\frac{2(K-1)^2}{K^3} - \frac{1}{K^2}, \frac{2(K-1)^2}{K^3}\right]$. Thus part (i) is shown.

Next we show part (ii). When $\lambda \geq \frac{2(K-1)^2}{K^3}$ there exists a unique pure-strategy symmetric equilibrium, where all contestants invests nothing, i.e. $I^*=0$. Obviously, in this case a symmetric equilibrium that involves positive investment does not exist. We then claim that zero investment is the best response of a contestant given that all others do not invest. We thus have to establish that the *LHS* of Eq. (13) ≤ 1 for any positive I_k when $I=0$. Rewrite LHS of Eq. (13) with $I=0$ as $\Phi(I_k) = \frac{\lambda + I_k(K-1)}{[\lambda K + I_k(K-1)]^3} \cdot 2(K-1)^2\lambda$. We have $\frac{d\Phi}{dI_k} = \frac{(K-1)}{[\lambda K + I_k(K-1)]^4} \cdot \lambda(3-4K)$, which is negative for any $K \geq 2$. Thus, it suffices to show $\Phi(0) \leq 1$. When $I_k=0$, we have $\Phi(0) = \frac{2(K-1)^2}{\lambda K^3}$, which is less than one by the assumption on λ.

Note that $\lambda \geq \frac{2(K-1)^2}{K^3}$ is also necessary for $\Phi(0) \leq 1$. This means when $\lambda < \frac{2(K-1)^2}{K^3} - \frac{1}{K^2}$, zero investment cannot be a symmetric equilibrium. Note that when $\lambda < \frac{2(K-1)^2}{K^3} - \frac{1}{K^2}$, there is no symmetric pure-strategy equilibrium with positive investment. Thus, when $\lambda < \frac{2(K-1)^2}{K^3} - \frac{1}{K^2}$, there is no symmetric pure-strategy equilibrium.

Part (iii) is true as the second order condition does not hold at $I_k = I^*(K)$ as long as $K \geq 4$. Simple algebra verifies that $\frac{d^2\pi_k(I_k, I^*(K))}{dI_k^2}$ shares the same sign with $(I^*(K)+\lambda)(3K-5)-(I_k+\lambda)(2K-2)$. Second order condition $\frac{d^2\pi_k(I_k, I^*(K))}{(dI_k)^2}\Big|_{I_k=I^*(K)} \leq 0$ if and only if $\frac{3K-5}{2K-2} \leq 1$, i.e. $K \leq 3$.

We now turn to Part (iv). As $\frac{d^2\pi_k(I_k, I^*(K))}{(dI_k)^2}$ shares the same sign with $(I^*(K)+\lambda)(3K-5)-(I_k+\lambda)(2K-2)$, there exists a threshold I_K^c for I_k such that $\frac{d^2\pi_k(I_k, I^*(K))}{dI_k^2} \leq 0$ if and only if $I_k \geq I_K^c$. When $K=2,3$, we have $I^*(K) > I_K^c$ as $\frac{d^2\pi_k(I_k, I^*(K))}{(dI_k)^2}\Big|_{I_k=I^*(K)} \leq 0$. Thus $\frac{d\pi_k(I_k, I^*(K))}{dI_k}$ is bell shaped and a positive maximum is achieved at $I_k = I_K^c$. For $I_k = I^*(K)$ to be a global maximizer, the necessary and sufficient condition is thus $x_k^*(I^*(K), I^*(K)) \geq x_k^*(0, I^*(K))$.

The equilibrium individual effort in the contest is obtained by applying Eq. (7) or (8) with the equilibrium symmetric investment. When equilibrium investment is zero, the model reduces to a standard Tullock contest. The well-known result directly applies. □

The implications of Proposition 1 are two-folds. Firstly, a pure-strategy symmetric equilibrium with positive investment, requires the size of λ remain in an intermediate range, i.e., $\lambda \in \left[\frac{2(K-1)^2}{K^3} - \frac{1}{K^2}, \frac{2(K-1)^2}{K^3}\right]$. The size of this parameter is inversely related to the marginal benefit of pre-contest investment. Only a sufficiently large λ can discipline one's incentive to invest, and guarantees a sensible interior solution; while excessively large λ diminishes the benefit of the positive equilibrium investment as indicated by Eq. (13). Note that the upper bound $\frac{2(K-1)^2}{K^3}$ converges to zero when K increases. Secondly, a model that allows for pre-contest investment bites only if the size of the competition is relative small. When the number of contestants is sufficiently large, they simply invest nothing in equilibrium even if contestants are allowed to make pre-contest investment, and a standard model is restored.

Corollary 1. *For $K=2,3$, when λ belongs to a small neighborhood of $\lambda_0 = 0.23$, a unique symmetric equilibrium with positive investment exists. Both equilibrium investment $I^*(K)$ and equilibrium individual effort $x^*(K)$ increases when the number of contestants increases from 2 to 3, i.e., $I^*(3)>I^*(2)>0$, and $x^*(3)>x^*(2)$.*

Proof. By part (iii) of Proposition 1, we only need to check whether $x_k^*(I^*(K), I^*(K)) \geq x_k^*(0, I^*(K))$ hold for $K=2,3$ when $\lambda=0.23$. This can be verified by simple algebra. We then apply Proposition 1 and Corollary 1 to obtain $I^*(3)>I^*(2)$ and verify $x^*(3)>x^*(2)$ by simple algebra. □

Corollary 1 shows that additional competition could incentivize contestants to compete more aggressively. Although additional competion squeezes the contestable rent, it urges contestants not to slack off, as they would lose more if they were left behind in the investment stage. The latter effect dominates the former when the number of contestants is small. It further leads to higher individual effort, which runs in contrast to the prediction of a standard contest model without pre-investment. As we can see from Proposition 1(ii), a standard contest yields equilibrium individual effort $x = \frac{K-1}{K^2}$, which unambiguously decreases with K. However, this positive relationship cannot be sustained for a large number of contestants.

Corollary 2. *When λ belongs to a small neighborhood of $\lambda_0 = 0.23$, equilibrium investment is non-monotonic in competition, as well as equilibrium individual effort.*

Proof. Corollary 1 says that when K increases from 2 to 3, the individual investment is positive and increases with K. However, it drops to zero when K reaches a large number, say, $K \geq 7$ as implied by Proposition 1(ii) and 1(iii).[7] The similar can be said about individuasl effort. When K increases from 2 to 3, it increases with K. However, it drops with K when $K \geq 7$. Note that when $K=7$, the individual effort level must be lower than that of $K=3$. □

The negative effect eventually looms large, while the positive effect diminishes. Intuitively, a marginal increase in competency may make a substantial difference when one confronts few, while it would be less significant when he competes against many. As a result, a non-monotonic relationship emerges.

[7] Proposition 1(*ii*) and 1(*iii*) imply that for $4 \leq K \leq 6$, there exists no symmetric pure-strategy equilibrium with zero or positive investment.

Our result therefore provides insights to the literature on "shortlisting." Let there be N potential contestants. The contest organizer needs to shortlist K of them to participate. To promote the quality of competition (to boost individual equilibrium effort), limiting the number of contestants may not be desirable. The equilibrium effort does not necessarily decrease with the number of contestants when N is small. However, when N is large, it is always optimal to shortlist as contestants would not invest and lower individual effort always results with more contestants.

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