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Communication and commitment in contests☆

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ABSTRACT

Players often engage in high-profile public communications to demonstrate their confidence in winning before they carry out actual competitive activities. We investigate players' incentives to engage in such pre-contest communication. Our key assumption is that a player suffers a cost when he sends a "message of confidence" but later loses the contest. Sending a message thus increases one's incentive to win. For the favorite, this has the beneficial strategic effect of decreasing the underdog's equilibrium effort. In a standard Tullock contest model, however, with no costs of entry and complete information, this strategic advantage is not strong enough to outweigh the cost of sending the message. Therefore, communication can only be beneficial if it deters the rival's entry into the contest, and under asymmetric information.

1. Introduction

Many competitive events resemble contests in that economic agents forfeit scarce resources to compete for a limited number of prizes. One example would be an R&D race in which firms attempt to develop new technology before their competitors. Other examples of such contests include the rivalry between firms who increase their marketing budgets to become market leaders, politicians who strive to win votes during political campaigns, rent-seekers who make political contributions so as to influence policy or secure the patronage of powerful politicians, and parties involved in legal disputes who incur great costs gathering evidence so that they can prevail in court. Other examples include war and international conflict, sports competitions and the market for internal labor.[3]

Whatever the context, contenders often conduct high-profile communications in public before the actual contest takes place. During a 1961 mission statement before the U.S. Congress, President John F. Kennedy expressed his goal of beating the Soviet Union in the race to reach the moon by the end of the 1960s: "I believe that this nation should commit itself to

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3 See Konrad (2009) for a survey of the literature.

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achieving the goal, before this decade is out, of landing a man on the moon and returning him safely to the Earth." The goal was ultimately achieved by the crew of the Apollo 11 in 1969. Similarly, Intel's president and chief executive, Paul Otellini, took a high-profile stance when he announced Intel's entry into the unfolding tablet market: "We are going to utilize all the assets at our disposal to win this segment" and "We will win in the tablet market." Such widely publicized statements are also made in the world of sports. Former boxing world champion Mike Tyson famously demonstrated his confidence in winning his next fight by making statements such as, "It's no doubt I am going to win this fight and I feel confident about winning this fight."

The many incidents of high-profile pre-competition communication have led to extensive discussions of this phenomenon in research on business strategy.[4] As Porter (1980) argues, a competitor sends a verbal message as "a direct or indirect indication of its intentions, motives, goals, internal situations." Such a "message" is intended to modify the structure of subsequent competition and to trigger a favorable reaction from rivals. Porter broadly defines such actions as "market signals," and regards them as a critical element of competitive strategies. Despite this, the economics literature has contributed little in the way of formal modeling to understand players' incentives to engage in such communications. We attempt to close this gap by developing a formal model to shed light on the strategic effects of public communications that precede a winner-take-all contest.

To fully understand the strategic trade-offs involved in pre-contest communications, it is worth noting that such activities can be costly. High-profile communications focus public attention on the ultimate performance of the participant who is sending the message. If the person does not achieve his stated goal (winning the contest), he risks public embarrassment and reputational damage. In the event of such a failure, the "message of confidence" or publicly stated resolve would be read by the public or press as a bluff (Kreps and Wilson, 1982; Milgrom and Roberts, 1982a,b) that amplifies the player's failure, jeopardizes his credibility, and handicaps him in future strategic maneuvers. For instance, before Germany's federal elections in 2002, Guido Westerwelle of the Free Democratic Party (FDP) initiated the ambitious so-called "project 18" and committed publicly to an objective of winning 18% of the votes. When the FDP won only 7.4%, Westerwelle was taunted by the media and public. This tarnished his reputation as an effective political leader.

While the message sender bears additional costs when he misses the stated goal, these costs enlarge the stake he places in the contest, and thus allows him to credibly commit to a tougher stance in the competition. This commitment could eventually pay off because it might discourage competitors and thus attenuate their incentive to exert effort. We label this strategic effect as the "discouragement effect."

To provide an account of such communications prior to contests, we consider a model in which two players compete for a prize and the players are allowed to send a public "message of confidence" prior to the contest. Our analysis demonstrates that the discouragement effect alone is insufficient to overcome the cost of sending the message of confidence, i.e., the cost when the stated goal is missed. Consequently, in a standard complete-information contest, in which players' "strengths" (valuations for the prize) are commonly known, there exists no equilibrium in which either player sends a message of confidence.

Our subsequent analysis, however, identifies two main contexts in which pre-contest communication arises. First, we provide an "entry deterrence" argument and demonstrate that the discouragement effect plays a more significant role when the contest involves endogenous entry. Continuing in a complete-information setting, we allow contenders to decide whether to participate in a contest, with participation involving a fixed entry cost. We show that communicative activities arise in equilibrium if the entry cost is sufficiently high. When entry costs are high enough, one player's commitment of a tough stance (through communication) could effectively prevent another's entry. Note that these arguments may explain Paul Otellini's statements when Intel decided to enter the tablet market. By stating that Intel was trying to win in the tablet market by all available means, he may have discouraged other firms from entering the market.

Second, we demonstrate that high profile pre-contest communications can arise when the contest involves incomplete and asymmetric information. This would occur even if entry was free, in which case pre-contest communication would not serve as an entry-deterrence device. To illustrate this point, we allow one player's strength (i.e., his valuation of the prize) to take either a high or a low value, which is privately known. The presence of asymmetric information breeds rich strategic trade-offs. The uninformed player, whose strength is commonly known, never conducts the communication. However, the informed player may send a message of confidence, which functions not only as a commitment device but also as a signaling device that conveys private information. Communicative activities are revived in this context through two possible avenues.

1. There could exist a separating equilibrium that demonstrates a "confirmation effect." In the equilibrium, the player with private information sends a message of confidence if and only if he has a high valuation of the prize. The separating equilibrium is made possible because the stronger player incurs a lower cost in sending the message of confidence, as he stands a lesser chance of losing. In this case, the uninformed player discerns the true strength of the other party by observing his communicative activity. The informative signal allows the stronger player to credibly convince his rival of his superior strength. The confirmed advantage of the stronger party, together with the discouragement effect of his commitment to the tough stance, successfully weakens the rival's incentive to put forth effort.

[4] See Heil and Robertson (1991) for a review of the literature.

2. There could also exist a pooling equilibrium, which embodies a “bluffing effect.” In this equilibrium, the informed party sends the message of confidence regardless of his relative strength as a contender. Such an equilibrium tends to emerge when the prior is less favorable (from the perspective of the uninformed player), i.e., when (a) the informed player is more likely to be of the high (stronger) type and/or (b) the high-type informed player possesses substantial advantages in the contest. In this case, the stronger type benefits from sending the message to maintain the pessimistic belief of his rival and thus discourages high effort from the rival. Interestingly, the weaker type may also want to send such a message, as by doing so, he hides his weakness and takes advantage of the pessimistic opponent.

These results shed light on why many athletes, such as Mike Tyson, often communicate confidence before taking part in a competition. While entry is typically not an issue in sports (it is unlikely that an opponent will not show up), a statement of confidence may signal one’s own strength or shape, and thus discourage the opponent.

The remainder of the paper is organized as follows. First, we briefly discuss the study’s relation to the the existing literature. In Section 2, we describe the complete-information model and present its solution. Section 3 deals with the incomplete-information case and Section 4 concludes the paper. All formal proofs are presented in the Appendix.

1.1. Relation to literature

As outlined in the previous section, the message of confidence plays the role of a commitment device in the context of this paper. The literature on strategic commitment in contests includes notable work by Dixit (1987), Baik and Shogren (1992), Morgan (2003), Yildirim (2005), Fu (2006), and Morgan and Várdy (2007). However, this strand of literature typically focuses on players’ commitment to particular timing patterns for their moves. Our players, in contrast, exert effort simultaneously and are allowed to commit to tougher stances through pre-contest communication.

Our work can be linked to the small but growing literature on communication and feedback in contests. However, again, this study differs from those papers, in that most of them analyze vertical communications between a contest organizer and contestants (e.g., Gershkov and Perry, 2009; Aoyagi, 2010; Ederer, 2010; Gürtler and Harbring, 2010; Goltsman and Mukherjee, 2011). These authors also typically focus on dynamic contest settings and investigate whether the contest designer should reveal intermediate results to the contestants. A notable exception is the paper by Sutter and Strassmair (2009), who conduct experiments on contests between teams and allow for horizontal communication between contestants. They find that within-team communication leads to higher efforts, while between-team communication leads to lower efforts. The latter finding provides indirect evidence for the discouragement effect.

The message of confidence examined in this paper can also function as a signaling device when the game involves private information. The small literature on contests with private and incomplete information includes Hurley and Shogren (1998a,b) and Malueg and Yates (2004), who assume that contestants have independent valuations of the prize. Fu (2006, 2008) and Wärneryd (2003, 2012) consider common-value contests, but allow a subset of contestants to privately know the true prize purse. Fey (2006), Münster (2009), Katsenos (2009), and Morath and Münster (2012) further allow contestants to possess private information concerning their own abilities or effort costs. However, only a few of these studies introduce an information-transmission device into contests. Fu (2006) analyzes a game in which an informed contestant moves ahead of his uninformed opponent, with his rent-seeking outlay conveying his private information. This is similar to Katsenos (2009), in which rent-seeking outlays are also used as a signaling device. In contrast to Fu (2006), Katsenos (2009) assumes that signaling activities take place in an additional stage prior to the contest. Wärneryd (2007) and Fu (2008) further allow potential contestants to negotiate for settlement before they enter conflicts; their actions in the negotiation reveal their private information. This paper introduces a novel signaling device (public communication) to the contest setting and reveals the rich strategic trade-offs that can be triggered by the signaling activities. In this regard, our paper contributes to this ongoing research agenda.

Our work is also related to the political science literature on audience costs that is based on the idea that political leaders face domestic political punishments for making public commitments and later backing down from them (Fearon, 1994). More generally, it is also linked to the broad and diverse literature on signaling and information transmission (see Spence, 1973, 1974; Crawford and Sobel, 1982).[5]

Two remarks are in order. First, it should be noted that pre-contest communication, in our context, is defined more broadly than in the information economics literature. Here, pre-contest communication is primarily a commitment device that ex post compels the sender to “act tough” without necessarily being an information-transmission device. The message of confidence is labeled as “public communication.” It directly affects the environmental factors that determine the payoff structure of the game, rather than merely communicating private information between the two active players in the game.

[5] Limit-pricing models (e.g., Milgrom and Roberts, 1982a,b) represent specific applications of signaling theory that are closely related to our analysis, in that limit-pricing models witness an entry-deterrence effect, with an incumbent choosing a price policy so as to deter others’ entry. In our model, however, the entry deterrence effect occurs even under complete information, because the communicative activity induces a player to increase his effort in the subsequent competition. Furthermore, as we show in the subsequent analysis, communication is used as a signaling device even when it can never deter a rival’s entry.

Second, in the incomplete-information setting, our model differs from conventional frameworks, as the cost of the signal (message of confidence) is endogenous, because the sender bears the cost if and only if he loses the contest.

2. Complete information

Two players, indexed by $i = 1,2$, are involved in a contest and they compete for an indivisible object. Each player i values the object at $v_i \geq 0$, which is commonly known. We assume that player 1 values the object more than the other, i.e., $v_1 > v_2$.

The game proceeds in three stages. In the first stage, the two players simultaneously choose $s_i \in \{s, n\}$, where $s_i = s$ means that player i sends a public message of confidence that states his confidence of winning the contest, while $s_i = n$ means that he does not send such a message.[6] In the second stage, players decide whether to enter the contest. One's entry entails a fixed cost $c \geq 0$. In the third stage, participating contestants choose their efforts $x_i \in \mathbb{R}_+$ to compete for the prize. When the entry cost c drops to zero, the model degenerates into a two-stage game: Their entry decision is trivial, as players always obtain non-negative expected payoffs from the contest.

If both players have entered the contest, the winner is determined through a standard lottery contest, i.e., contestant i wins with probability $p_i = x_i/(x_i + x_j)$ if $x_i + x_j > 0$. The prize is randomly assigned if both enter the contest but neither exerts positive effort. If only one of the players has entered the contest, he receives the prize automatically.

We focus on this simple setting mainly for the sake of expositional efficiency. It should be noted, however, that many of the results we derive here also hold for a more general ratio-form contest success function (which has been axiomatized by Skaperdas, 1996; Clark and Riis, 1998), and even for a completely discriminating contest, i.e., all-pay auctions (Baye et al., 1996).[7]

We assume that one's effort incurs a unity marginal cost and that a player bears the cost of his own effort regardless of whether he wins or loses. If a player sends the message $s_i = s$, but loses in the subsequent contest, he suffers an additional cost $k > 0$.[8]

The solution concept is subgame perfect Nash equilibrium (henceforth equilibrium) in the complete-information setting. For expositional clarity, we further assume that each player would enter the contest when he is indifferent between entry and exit. However, this assumption is by no means crucial for our results.

2.1. Effort stage

The game is solved by backward induction. In the third stage, positive efforts are provided only if both players have entered the contest. For notational convenience, let I_i be an indicator variable that equals k if player i has chosen $s_i = s$ and zero otherwise. Player i maximizes

$$\begin{aligned} E[u_i] &= p_i v_i - x_i - (1 - p_i) I_i - c \\ &= \frac{x_i}{x_i + x_j}(v_i + I_i) - x_i - I_i - c, \end{aligned}$$

where p_i is a player i's win likelihood. Standard technique allows us to obtain the equilibrium efforts

$$x_i = \frac{(v_i + I_i)^2 (v_j + I_j)}{(v_i + v_j + I_i + I_j)^2}, \quad i = 1, 2.$$

A player i's equilibrium payoff is given by

$$E[u_i] = \frac{(v_i + I_i)^3}{(v_i + v_j + I_i + I_j)^2} - I_i - c.$$

The strategic value of the message of confidence (i.e. choosing $I_i = k$) is demonstrated by a closer inspection of the equilibrium effort and the payoff functions. The commitment fostered by the communication ($s_i = s$) compels the sender to step up his effort, as a loss would incur additional costs. The commitment of player i, however, triggers ambiguous reactions from the rival. It lowers the equilibrium effort of his rival j if and only if the incentives of player j to win are sufficiently weak, i.e., $v_j + I_j < \sqrt{(v_i + k)v_i}$. In such a case, player j would appear to be the underdog, and a better committed player i would disincentivize player j further. We label the strategic interaction a *discouragement effect*. Its logic is similar to the conventional wisdom prevalent in the literature on strategic pre-commitment in contests (see, for example, Dixit, 1987).

[6] In Section 2.4, we also briefly explore an extension in which the public message is chosen from a continuous set.

[7] Formal proofs are available from the authors upon request.

[8] In Section 4, we discuss alternative assumptions surrounding the pre-contest communication and their effects on contest outcomes.

2.2. Entry stage

We now consider the subgame in which players decide whether or not to enter the contest. If player i does not enter, he receives a payoff of $-I_i$. If he enters, his payoff depends on various factors, including both players' communicative activities in the first stage and the entry decision of his rival.

If an excessive entry cost is involved, the outcome becomes relatively straightforward and trivial: Neither player will enter if $c > v_1 + I_1$, while only player 1 enters if $v_1 + I_1 \geq c > v_2 + I_2$. With moderate entry costs (i.e., $c \leq v_2 + I_2$), three cases are possible depending on the size of c. For the sake of brevity, we provide only a short qualitative overview of the results in the following paragraphs. More detailed analytical results are presented in Appendix A.

1. If c is sufficiently small, both players enter the contest.
2. If the entry cost remains in the medium range, only the ex post stronger player (i.e. the player i with $v_i + I_i \geq v_j + I_j$) enters. The entry cost is sufficiently small such that player i must enter regardless of player j's entry decision; meanwhile, the cost is high enough to ensure that his rival stays out given player i's entry.[9]
3. If c is sufficiently large (but still below $v_2 + I_2$), there are two pure strategy equilibria in which exactly one player enters. In this case, each player prefers to stay out whenever the other enters, and to enter whenever the other stays out. Furthermore, there also exists a mixed equilibrium in which players randomize their entry.

2.3. Communication stage

The discussion on equilibrium play in the communication stage begins with a few interesting preliminary results, which will be used frequently in the subsequent analysis.

Lemma 1. *There exists no pure strategy equilibrium in which a player i sends the message $s_i = s$ but does not enter the contest in the subsequent subgame.*

Lemma 2. *Suppose that player j would enter the contest if i has sent the message $s_i = s$. Anticipating this, player i strictly prefers not to send the message.*

Lemmas 1 and 2 directly lead to the following important result.

Proposition 1. *There is no pure strategy equilibrium in which both players send the message.*

According to Lemma 2, a player never finds it optimal to send the message of confidence if he anticipates that his rival would eventually enter the contest. The benefit accrued from the discouragement effect does not offset the potential cost of the pre-contest communication. One has no incentive to engage in such a commitment if the tough stance cannot successfully deter the rival's entry.[10] It should be noted that the result is not an artifact of the particular lottery contest model. Lemma 2 continues to hold for more generally defined ratio-form contest success functions and also for a perfectly discriminating contest (all-pay auction). The intuition for the result is that in all these situations, players' reaction functions in the competition are not sufficiently steep that the discouragement effect is important enough to overcome the concomitant cost.

Nevertheless, it is possible to construct examples based on non-canonical contest models, such that a message of confidence pays off sufficiently because of the discouragement effect alone. For player 1, this might be the case if the reaction function of player 2 is decreasing very rapidly. To illustrate this, we consider a contest that allows for draws. We further assume that for the players, a draw is as bad as losing. As above, we denote by p_i a player i's winning probability. Suppose that

$$p_1 = \frac{x_1}{x_1 + x_2},$$
$$p_2 = f(x_1)\frac{x_2}{x_1 + x_2},$$

and that there is a draw with probability

$$(1 - f(x_1))\frac{x_2}{x_1 + x_2}.$$

By introducing the possibility of a draw in this way, we can keep the reaction function of player 1 as in the standard contest, but are able to manipulate player 2's reaction function to make it sufficiently steep. Suppose, for example, that $v_1 = v_2 = k = 1$ and $c = 0$. Moreover, assume that $f(\cdot)$ is nonnegative and continuous, with $f(x_1) = 1$ for $x_1 \leq 1/4$, and $f(\cdot)$ is decreasing rapidly

[9] Since $v_1 > v_2$, player 1 typically has the higher incentive to win. The only exception is the case $v_2 + k > v_1$. In this case, player 2 has the higher incentive if only he has sent the message.

[10] This result does not depend on the assumption that both players send their messages at the same time. Even if players were communicating sequentially, none of the players would send a message of confidence if the other player could not be deterred from entering the contest.

for $x_1 > 1/4$. That is, for $x_1 > 1/4$, a further increase of x_1 enlarges the probability of a draw, and weakens the incentives of player 2 to exert effort. The reaction function of player 2 is as in a standard Tullock contest for $x_1 \leq 1/4$, but decreases rapidly for $x_1 > 1/4$. If no player sends the message, the equilibrium is as in a standard Tullock contest at $x_1 = x_2 = 1/4$. If player 1 sends the message, however, this leads to a big reduction in player 2's effort. If f is decreasing rapidly enough, the payoff of player 1 is higher if he sends the message than if he does not.[11] This counter-example to Lemma 2 clarifies the logic behind Lemma 2: In standard contest models, the reaction functions are not "steep enough" in the relevant range to give players an incentive to send the signal.

We now return to our baseline model and show that entry deterrence may be a reason for pre-contest communication.

Lemma 3. *Suppose $v_i > c$. Player i strictly prefers to send $s_i = s$ if the message deters player j from entering the contest.*

In what follows, we study equilibria in which players actively communicate. By Proposition 1, there is no pure strategy equilibrium, in which both players send the message of confidence. In Section 2.3.1, we focus on pure strategy equilibria in which exactly one player communicates. In Section 2.3.2, we discuss mixed equilibria in which both players engage in pre-contest communication with positive probabilities.

2.3.1. Equilibria in which one player communicates

We now consider equilibria where exactly one player communicates. By Lemmas 1 and 2, the player who did communicate enters, whereas the other player stays out. We first consider the equilibrium in which only player 1 communicates and enters.

Proposition 2. *An equilibrium in which only player 1 communicates and only player 1 enters exists if and only if*

$$c \in \left(\frac{v_2^3}{(v_1 + v_2 + k)^2}, v_1 \right].$$

Proposition 2 demonstrates the possible benefit of the message of confidence in that it serves as a deterrence to the rival's entry. As a player commits to a tougher stance in the subsequent contest, the rival player may find it no longer worthwhile to participate in the competition. This "entry-deterrence effect" compels a player to engage in high-profile communication. Naturally, player 2 can be prevented from entering the contest only if the contest requires substantially costly entry.

The equilibrium is in general not unique when c falls in the interval $(v_2^3/(v_1 + v_2 + k)^2, v_1]$. There can be an equilibrium in which only player 2 sends the message and enters. The next result demonstrates this possibility and establishes the conditions under which such an equilibrium exists.

Proposition 3. *An equilibrium where only player 2 communicates and only player 2 enters exists if and only if either (i)*

$$\frac{v_1^3}{(v_1 + v_2 + k)^2} < c \leq \min \left\{ v_2, \frac{(v_2 + k)^3}{(v_1 + v_2 + 2k)^2} \right\},$$

or (ii)

$$\frac{(v_1 + k)^3}{(v_1 + v_2 + 2k)^2} < c \leq v_2.$$

[11] For a fully-worked out example, suppose that for $x_1 > 1/4$,

$$f(x_1) = \max \left\{ 1 - 100 \left(x_1 - \frac{1}{4} \right), 0 \right\}.$$

As mentioned before, a draw is assumed to be as bad as losing the contest. Hence, winning gives player i a utility $v_i = 1$, while losing or a draw gives $-I_i$. Suppose $I_1 = k = 1$. Then the reaction functions are (in the relevant range)

$$x_1 = \sqrt{2x_2} - x_2,$$
$$x_2 = \sqrt{\left(1 - 100 \left(x_1 - \frac{1}{4} \right) \right) x_1} - x_1.$$

The equilibrium is approximately at $x_1 = 0.256$ and $x_2 = 4.56 \times 10^{-2}$. Player 1's payoff is approximately 0.4415. This payoff is bigger than 1/4 (the payoff when choosing $I_1 = 0$).

Two facts deserve to be noted. First, the two sets defined by conditions (i) and (ii) are disjoint.[12] Second, the union of the two sets is included in $\left(\frac{v_2^3}{(v_1+v_2+k)^2}, v_1\right]$.[13] This implies that the ex ante favorite (player 1) is more likely to engage in pre-contest communication. Whenever there exists an equilibrium with only player 2 sending the message and entering the contest, there must exist an equilibrium with only player 1 doing so. The converse, however, is not true.

These results yield interesting efficiency implications. First, pre-contest communication may allow one player to successfully prevent his rival's entry, thereby decreasing rent dissipation in the contest. In this way, such communication improves social welfare in contexts in which efforts are unproductive, e.g., in political campaigns, as it helps avoid the waste of resources. However, the communication may backfire under circumstances in which competition leads to productive efforts, e.g., in R&D races, architectural design competitions, and promotion tournaments within firms. In these situations, pre-contest communication instead facilitates coordination and dilutes the competition.

Second, pre-contest communication does not exert a definitive impact on allocative efficiency. Allocative efficiency, in the current context, requires that the player who values the prize more wins it. However, as demonstrated by Propositions 2 and 3, multiple equilibria exist when the entry cost is sufficiently high. Even player 2 may successfully deter the other's entry and win the prize, thereby jeopardizing the allocative efficiency of the contest.

2.3.2. (Mixed) equilibria in which both communicate with positive probabilities

As implied by Proposition 1, equilibria where both players communicate must involve mixed strategies in the communication stage. We briefly discuss the possibility of such mixed-strategy equilibria. They may emerge when sufficiently high entry costs lead to multiple pure-strategy equilibria, as shown in Propositions 2 and 3. Under these circumstances, the strategic interaction in the current context (in terms of communication and entry) resembles that of a standard coordination game.

Suppose, for instance, that entry cost c falls in the interval $\left(\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{v_2^3}{(v_1+v_2)^2}\right)$. In this case, there exists an equilibrium in which both players send $s_i = s$ with positive probabilities. The players' strategy plays in the entry subgames are as follows.

1. If only player i has sent the message, i enters and j stays out.
2. If neither player has sent the message, both enter the contest subsequently.
3. If both have sent the message, then both enter.[14]

Anticipating such strategic play in the entry stage, $s_i = s$ is strictly optimal for player i if $s_j = n$ by Lemma 3, whereas $s_i = n$ is strictly optimal for i if $s_j = s$ by Lemma 2. Hence, there must exist a $p_j \in (0, 1)$ such that, if j sends $s_j = s$ with probability p_j, then i is indifferent between sending and not sending the message. Therefore, a mixed-strategy equilibrium exists in which both players send the signal with positive probabilities and both may enter the contest.

Such equilibria may explain why, in many real world cases, both competing parties send messages of confidence and then engage in an actual competition for the prize—a phenomenon that is not literally captured by the pure strategy equilibrium of our model.

2.4. A robustness check: commitment as a continuous variable

In this subsection, we briefly explore a direct extension of our baseline: A player can choose the strength of his commitment, which is measured by the continuous variable I_i. In the first stage, a player i chooses $I_i \in [0, k]$, where $k \in \mathbb{R}_+$ is an exogenous parameter. He suffers an additional cost equal to I_i if he fails to win the subsequent contest.

For players' given choices of commitment strength (I_1, I_2), the solutions of equilibrium efforts and payoffs in Section 2.1 remain valid in this extension. Lemmas 1 and 2 can similarly be generalized: (1) there is no pure strategy equilibrium in which a player i sends a message $I_i > 0$ but does not enter the contest and (2) player i strictly prefers $I_i = 0$ over any message $I_i' > 0$ when j will enter the contest if i has sent the message I_i'. Consequently, there is no pure strategy equilibrium with $I_1, I_2 > 0$, analogous to Proposition 1. In any pure strategy equilibrium, at most one player sends a message with a positive strength. Moreover, Lemma 3 can be generalized as well: Given $v_i > c$, player i strictly prefers to send a message $I_i' > 0$ over $I_i = 0$ if the message I_i' deters player j from entering the contest.

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[12] To appreciate this, note that $\frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$ is greater than both $\frac{(v_2+k)^3}{(v_1+v_2+2k)^2}$ and $\frac{v_1^3}{(v_1+v_2+k)^2}$. The latter is true because $\frac{\partial (v_1+u)^3/(v_1+v_2+k+u)^2}{\partial u} = \frac{3(v_1+u)^2(v_1+v_2+k+u)-2(v_1+u)^3}{(v_1+v_2+k+u)^3} = \frac{(v_1+u)^2[v_1+3v_2+3k+u]}{(v_1+v_2+k+u)^3} > 0$.

[13] To see that, note $\frac{(v_1+k)^3}{(v_1+v_2+2k)^2} > \frac{v_1^3}{(v_1+v_2+k)^2} > \frac{v_2^3}{(v_1+v_2+k)^2}$ and $v_2 < v_1$.

[14] In this subgame, it is optimal for player 2 to enter. Note that he receives an expected payoff $\frac{(v_2+k)^3}{(v_1+v_2+2k)^2} > \frac{v_2^3}{(v_1+v_2)^2} > c$. The first inequality follows from the fact that the left hand side is strictly increasing in k: $\frac{\partial}{\partial k}\left(\frac{(v_2+k)^3}{(v_1+v_2+2k)^2}\right) = \frac{(v_2+k)^2(3v_1+2k-v_2)}{(v_1+v_2+2k)^3} > 0$.

Now suppose that there exists a pure strategy equilibrium in which a player i chooses some $I_i' \in (0, k)$. In such an equilibrium, the opponent j must be deterred from entry. Since j's payoff from participating in the contest is decreasing in I_i, any message $I_i'' > I_i'$ will also deter j from entry. Moreover, given that j does not enter if player i chooses I_i' or I_i'', player i is indifferent between sending I_i' and I_i''. In other words, the two messages, I_i' and I_i'', would lead to the same outcome. As a result, it is without loss of generality to limit our attention to a binary strategy space, as we have assumed in the baseline setting, when identifying the conditions for the existence of an equilibrium in which only player i communicates and enters. It follows that in the extension, Propositions 2 and 3 are valid as they stand. Consequently, the efficiency implications pointed out above do not lose their bite, either.

It is interesting to note what would happen if there were no upper bound on the strength of the commitment a player can choose. Suppose that player i can choose any $I_i \in \mathbb{R}_+$. Then an equilibrium in which only player $i \in \{1, 2\}$ communicates and enters the contest exists if and only if $c \in (0, v_i]$. That is, such an equilibrium exists as long as the benefit of the prize outweighs his entry cost c, so he is willing to enter (alone): Given the unbounded strategy space, he can always choose some sufficiently large I_i to deter his opponent. Note that the conditions in Propositions 2 or 3 simply converge to $c \in (0, v_i]$ in the limiting case of $k \to \infty$.

3. Incomplete information

In this section, we provide an alternative rationale for pre-contest communication. We demonstrate that communication could emerge when the game involves incomplete and asymmetric information, even if entry costs are trivial and entry deterrence is impossible.

For the sake of expositional efficiency, we focus on a simple and direct variation of the basic model to ascertain the role played by information asymmetry. Player 1's valuation of the prize takes either a high (h) or a low (l) value, which determines his type (t). For simplicity, we normalize the valuation of a type-l player 1 to one, which occurs with probability $\lambda \in (0, 1)$. The type-h player 1 has a valuation of $v > 1$, which occurs with the complementary probability. Player 1's valuation is privately known, while its distribution is common knowledge. The valuation of player 2 for the prize is low, with $v_2 = 1$, and it is commonly known. We make this admittedly special assumption to illustrate the possibility of pre-contest communication in a setting that is as simple as possible. It should be noted, however, that qualitatively our results are robust to small variations of these assumptions; see the conclusion for a further discussion.

We focus on the limiting case of $c = 0$. The game then essentially reduces to two stages, because the entry decision is trivial. Players simultaneously choose their message s_i in the first stage. They then expend their efforts vying for the prize in the second stage of the game. This simplification allows us to eliminate the confounding effects of the entry-deterrence mechanism, but identify and highlight the role played by asymmetric information. We adopt the solution concept of perfect Bayesian equilibrium (PBE) in our analysis to obtain predictions.

Several useful facts deserve to be highlighted before we present our main analysis. First, there exists no equilibrium in which the uninformed player (player 2) actively engages in communication. The trade-off faced by the player is analogous to that in the complete-information setting. Player 2 is notably an "underdog" in the competition, as player 1 has a higher valuation on average. A tougher stance comes at a cost, as it would only trigger a more aggressive response from the opponent.[15] As a result, the message of confidence can only be sent by player 1. The interaction thus alludes to a signaling game with the message being a signaling device, as only the informed player may actively engage in such activity.

Second, the high-type player has a stronger incentive to communicate. This is formally stated by the following proposition, which further narrows the set of possible equilibria.

Proposition 4. *There exists no equilibrium in which only the low-type player 1 sends the message with a positive probability.*

The proof of Proposition 4 verifies that the model satisfies a "single-crossing property." It arises out of two effects. First, the high-type player 1 always benefits more from the communication. The commitment discourages player 2 from exerting competitive effort and increases the winning odds of player 1. Any given effort reduction from player 2—or, equivalently, any given increase in player 1's winning odds—must yield a larger gain to the high type, because he values the prize more. Second, recall that the message of confidence backfires only when the sender loses. The high type chooses a higher effort than the low type, and hence loses less often. Communication thus has a lower expected cost for the high type. In summary, the high type bears a lesser cost when sending the message of confidence, while he also reaps a larger gain from it.

Communication can appear in equilibrium through two mechanisms: (1) the "confirmation effect" and (2) the "bluffing effect." The former is exercised through a separating equilibrium in which only the high-type player 1 sends a message of confidence, while the latter emerges in a pooling equilibrium in which both types of player 1 participate in the communication.

[15] The rationale is intuitive, but the proof is lengthy and tedious. We omit it in the paper for brevity, but it is available from the authors upon request.

3.1. Confirmation effect: separating equilibrium

In a separating equilibrium, only the high-type player 1 chooses $s_1 = s$, while the low type chooses $s_1 = n$. Player 2 perfectly infers the true type of player 1, which gives the posterior $\Pr(t = l|n) = 1$ and $\Pr(t = l|s) = 0$. The following result establishes the conditions for the existence of a separating equilibrium.[16]

Proposition 5.

(a) *Suppose that* $v \geq 81/16$. *There is a critical value* $\tilde{k} \in (0, \infty)$ *such that a separating equilibrium, with the high-type player* 1 *choosing* $s_1 = s$ *and the low-type player* 1 *choosing* $s_1 = n$, *exists if and only if* $k \geq \tilde{k}$.
(b) *Suppose that* $v < 81/16$. *There are two cutoff values* $\hat{k}$ *and* $\tilde{k}$, *with* $0 < \tilde{k} < \hat{k} < \infty$, *such that the separating equilibrium exists if and only if* $k \in [\tilde{k}, \hat{k}]$.

We briefly interpret the conditions that ensure the existence of the separating equilibrium. The equilibrium requires (1) that the high type be incentivized to send the message of confidence and (2) that the low type be prevented from mimicry. The costly commitment allows the high type to credibly reveal his type, which yields two types of strategic benefits. First, the aforementioned discouragement effect, which is exercised through the commitment of k, continues to exist. Second, the message credibly verifies the high-type player's competitive advantage, thereby further discouraging player 2 from exerting competitive effort. The latter effect is referred to as the *confirmation effect*. The combination of the two effects compels the high type to send the message.

A larger v amplifies the gains to the high type from these effects. It implies a more significant advantage in the competition once he credibly verifies his type, which allows him to disincentivize player 2 further. Moreover, he reaps a larger marginal gain from player 2's concession when he values winning more. In addition, a larger v also suppresses the cost of communication, because a more significant advantage reduces the likelihood of losing. As a result, he prefers to send the message whenever v is sufficiently large, i.e., $v \geq 81/16$. When v is relatively small, i.e. $v < 81/16$, the player can still engage in communication so long as the cost of communication is sufficiently small, i.e., k falling below the cutoff $\hat{k}$.

The low type is always tempted to bluff by misrepresenting his type. By doing so, he fools his opponent into believing that he is a high type, and this allows him to disincentivize his rival, thereby reducing the competition. A separating equilibrium requires that such an incentive be checked by a nontrivial k (i.e., $k \geq \tilde{k}$): The low type would refrain from misrepresentation only if he would be punished severely if he loses. A larger v encourages the low type's mimicry: It disincentivizes player 2 further, if he believes that he will encounter an extremely strong high-type player 1.

Hence, an increase in v may either facilitate or preclude the separating equilibrium. It incentivizes the high type more to engage in the communication on the one hand, while it further entices the low type to mimic on the other. The competing effects can be witnessed by the following observations.

Corollary 1. *Both the lower bound* $\tilde{k}$ *and the upper bound* $\hat{k}$ *increase with* v.

3.2. Bluffing effect: pooling equilibrium

The low type's incentive to bluff spawns pooling equilibria in which both types of player 1 send the message. In such an equilibrium, no additional information is transmitted through communication, which leads to the posterior $\Pr(t = l|s) = \lambda$ in equilibrium. We first characterize the general property of a pooling equilibrium.

Lemma 4. *Suppose that a pooling equilibrium exists, in which both types of player* 1 *send the message. Player 2 exerts an equilibrium effort*

$$x_2 = \left(\frac{\lambda(v+k)\sqrt{1+k} + (1-\lambda)(1+k)\sqrt{v+k}}{(1+k)(v+k) + \lambda(v+k) + (1-\lambda)(1+k)} \right)^2.$$

It decreases in v *and* k, *while it increases in* λ.

We then provide conditions under which such a pooling equilibrium exists.

Proposition 6. *For every given* k, *there exist a unique cutoff* $\bar{v} > 1$ *and a cutoff probability* $\bar{\lambda}(v)$ *associated with every* $v > \bar{v}$, *such that a pooling equilibrium, in which player 1 sends the message regardless of his type, exists if and only if* $v > \bar{v}$, *and* $\lambda \leq \bar{\lambda}(v)$.

The existence of a pooling equilibrium with communication is underpinned by the low type's incentive to hide his type by mimicking his high-type counterpart. Player 2 is uncertain about the true competence of his opponent, which discourages him from putting forth effort. A bluff pays off more significantly if and only if (1) the prior is sufficiently unfavorable to player 2 ($\lambda \leq \bar{\lambda}(v)$) and (2) the high-type player 1 possesses a significant advantage ($v > \bar{v}$). The former lets player 2 believe that he is more likely to meet the high-type rival, while the latter lets him believe that he is handicapped more severely

[16] The equilibrium in Proposition 5 is unique in the class of all separating equilibria. As we show in Section 3.2, other types of equilibria may exist.

in the competition if he indeed meets the high-type rival. As Lemma 4 shows, he behaves less competitively under these circumstances. The benefit of bluffing thus outweighs the possible cost of the message when these conditions are met.

Two remarks are in order. First, player 1's incentive to communicate crucially depends on player 2's posterior when observing no communication. In the proof of Proposition 6 we adopt the most favorable off-equilibrum belief to identify the (sufficient and necessary) conditions for the existence of pooling equilibria with communication, i.e., a belief under which player 1 is punished most severely when deviating.[17] It thus allows us to find the widest range of parameters that allows for a pooling equilibrium: No pooling equilibria would exist if v fell below the cutoff $\bar{v}$ identified in Proposition 6, regardless of the prevailing out-of-equilibrium belief. When a less harsh out-of-equilibrium belief is considered, a pooling equilibrium demands larger v and/or smaller λ.

Second, in general the equilibrium is not unique. The conditions for existence of a separating equilibrium (in Proposition 5) and a pooling equilibrium with communication (in Proposition 6) do partially overlap. Moreover, there can be a hybrid equilibrium: The high-type player 1 sends the message with probability one, while his low-type counterpart randomizes. Both the confirmation effect and the bluffing effect loom large in such an equilibrium. For brevity, we do not provide detailed analysis, as its main logic has been explicated in the current discussion.

3.3. *Efficiency implications of pre-contest communication*

In what follows, we discuss the efficiency implications of pre-contest communication in the incomplete-information setting. We explore two main issues: (1) Does the communication increase or decrease equilibrium efforts? (2) Does the communication improve the allocative efficiency of the contest?

3.3.1. *Effort comparison*

In both the separating equilibrium and the pooling equilibrium, pre-contest communication exercises countervailing effects on the expected overall effort exerted in the contest. First, whenever a player sends a message of confidence, he ends up with a larger stake in the contest because of his commitment k, which compels him to step up his effort to avoid a loss. Second, player 2 is disincentivized, because he faces a more committed rival and—in the case of the separating equilibrium—is convinced of his rival's superior strength, which reduces his effort. Because of these countervailing effects, the expected overall effort in the contest can be either higher or lower than the corresponding effort in a benchmark case when pre-contest communication is absent. We omit the detail for brevity, but more extensive discussion is available in an older version of this paper.[18]

3.3.2. *Allocative efficiency*

While no definitive conclusion can be reached for effort comparison, our results demonstrate that allocative efficiency would improve if pre-contest communication were possible.

Allocative efficiency in this contest would improve if the high-type player 1, who has a higher valuation of the prize, becomes more likely to win. By contrast, the outcome of the contest does not make a difference in terms of allocative efficiency if player 1 turns out to be of the low type, because the competing parties equally value the prize.

We first consider a benchmark case in which players are not allowed to communicate before the contest. We then compare the high-type player 1's winning likelihood in the separating equilibrium to that in the benchmark case. Simple analysis leads to the following.

Proposition 7. *The winning-probability of the high-type player* 1 *is higher in the separating equilibrium than in the benchmark case.*

In the separating equilibrium, a high-type player 1 can successfully convince the rival of his advantage. The communication discourages player 2 as he learns that he has to compete against a stronger and better committed opponent, which leads the high-type player 1 to win more often.

A similar result applies to the pooling equilibrium examined in Section 3.2.

Proposition 8. *The winning-probability of the high-type player* 1 *is higher in a pooling equilibrium with communication than in the benchmark case.*

In a pooling equilibrium with communication, player 1's commitment compels player 2 to reduce his effort, which allows both types of player 1 to win the contest more often. A similar logic will also apply to any hybrid equilibrium involving communication. Propositions 7 and 8 therefore show that pre-contest communication improves allocative efficiency.

[17] The effort of player 2 increases in the probability with which he believes that player 1 has a low type. Therefore, the belief under which player 1 is punished most severely is as follows: If no message is sent, player 2 believes that player 1 is of low type with probability one. (This belief is also quite plausible since it reflects the fact that a low-type player 1 always benefits less from communication.)

[18] See the working paper version (Fu et al., 2011) for more detailed results on the effects of pre-contest communication on the expected overall effort.

Table 1
Equilibrium payoffs for different message combinations conditional on entry.

	$s_2 = s$	$s_2 = n$
$s_1 = s$	$\frac{(v_1+k)^3}{(v_1+v_2+2k)^2} - k - c$, $\frac{(v_2+k)^3}{(v_1+v_2+2k)^2} - k - c$	$\frac{(v_1+k)^3}{(v_1+v_2+k)^2} - k - c$, $\frac{v_2^3}{(v_1+v_2+k)^2} - c$
$s_1 = n$	$\frac{v_1^3}{(v_1+v_2+k)^2} - c$, $\frac{(v_2+k)^3}{(v_1+v_2+k)^2} - k - c$	$\frac{v_1^3}{(v_1+v_2)^2} - c$, $\frac{v_2^3}{(v_1+v_2)^2} - c$

4. Concluding remarks

The current paper offers an economic model that helps us to understand why contest participants often make statements of confidence even before they carry out any competitive activities. While it is generally shown that pre-contest communication increases one's incentives to win and helps to disincentivize the opponent, we demonstrate that the commitment value of such communication never offsets its cost in a standard contest setting. Nevertheless, the statement of confidence may be valuable for several reasons. First, it may deter the rival's entry into the contest. Second, it can function as a signaling device to strategically manipulate the belief of the opponent when the contest involves incomplete and asymmetric information. Pre-contest communications also yield interesting efficiency implications. We show, for instance, that such communications may improve the allocative efficiency and reduce wasteful rent dissipation under certain circumstances.

Although we have assumed in our model that a player who has sent a message of confidence prior to the contest suffers a fixed cost if he does not succeed eventually, our setup can be extended in a number of ways. First, players could make pessimistic statements such as "I am going to lose the contest!." A player may receive additional gains if his performance exceeds expectations and he ultimately wins. Thus, a pessimistic statement may also increase one's incentive to win the contest. A strategic analysis of players' incentive to make pessimistic statements would complement the current work.

Second, the cost of the message can be assumed not to depend on the outcome of the contest at all. Obviously, the players would never find it in their interest to send the message in the complete-information case. However, communication may still occur in the presence of incomplete information. For instance, a separating equilibrium, in which only the high type of player 1 sends the signal, still exists for certain parameter constellations. While both types of player 1 suffer the same cost of sending the signal, the high type gains relatively more from confirming his strength and discouraging the opponent, because he has a higher value for the prize. Therefore, for certain parameterizations, only the high type would find it beneficial to "burn the money" by sending the message. Similarly, with a sufficiently pessimistic prior belief and sufficiently low cost of sending the message, a pooling equilibrium exists, in which both types of player 1 communicate confidence.

Third, in our analysis of the incomplete-information setting, the low type's valuation for the prize is set to one. The simple setting not only reduces analytical complexity, but also substantially improves the expositional efficiency. It should be noted that the main logic laid out in Section 3 would not lose its bite in a broader setting. Consider, for instance, an extended setting in which a type-t player 1 has a valuation v_t, with $v_h > 1$ and $v_l \in [1/v_h, 1]$. Players face essentially the same trade-offs as in the current context. The same effects loom large, and similar equilibria emerge. No predictions would vary qualitatively. For the sake of brevity, we do not include this analysis, but it is available from the authors upon request. It should be noted, however, that additional qualifications would result when v_l falls below $1/v_h$. An excessively small v_l may entice the high type to misrepresent his type, as player 2 would substantially reduce his effort if he believes that he encounters a (weak) low-type player 1. This additional strategic concern conflicts with the confirmation effect, which attenuates player 1's incentive to communicate and makes pre-contest communication less likely.

Analytical complexity and expositional efficiency have limited our analysis to a stylized setting. The setup, however, can be generalized in various other ways, and the key insights would extend to wider contexts. For example, we have indicated that some findings are robust with respect to the form of the contest-success function. In addition, varying other assumptions (e.g., the number of contestants) might affect the magnitude of the strategic effects highlighted in this paper, but it would not qualitatively alter the main predictions. These extensions will be attempted by the authors in future. We believe, however, that our model captures the most important forces at work that lead to pre-competition communication.

Appendix A. The entry stage

Here we give details of players' behavior at the entry stage. We focus on play in pure strategies. If player i does not enter, he receives a payoff of $-I_i$. If a player enters, his payoff depends on the message combinations chosen in the first stage. The equilibrium payoffs for the different message combinations conditional on entry are displayed in Table 1.

As noted in the main text, the case where $c > v_2 + I_2$ is straightforward: only player 1 enters when $c \le v_1 + I_1$, and no player enters when $c > v_1 + I_1$. The following lemmas formally characterize behavior at the entry stage in case $c \le v_2 + I_2$. We provide a formal proof of Lemma A.1. The proofs of the other lemmas are analogous and therefore omitted.

Lemma A.1. *Suppose $s_1 = s_2 = s$ and $c \le v_2 + k$.*

(1) *If $c \le \frac{(v_2+k)^3}{(v_1+v_2+2k)^2}$, then both players enter the contest. Payoffs are $E[u_1] = \frac{(v_1+k)^3}{(v_1+v_2+2k)^2} - k - c$ and $E[u_2] = \frac{(v_2+k)^3}{(v_1+v_2+2k)^2} - k - c$.*

(ii) *If* $c \in \left(\frac{(v_2+k)^3}{(v_1+v_2+2k)^2}, \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}\right]$, *only player 1 enters the contest and payoffs are* $E[u_1] = v_1 - c$ *and* $E[u_2] = -k$.

(iii) *If* $c > \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$, *only one of the players enters and receives* $E[u_i] = v_i - c$, *while the other player receives* $E[u_j] = -k$ $(j = 1, i \neq j)$.

Proof *(Proof of Lemma A.1).* It is straightforward to see that entry is the dominating choice for player 1 if $c \le \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$. Parts (i) and (ii) then follow from a comparison of the entry payoff to the non-entry payoff of player 2. If $c > \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$, no player would enter if the opponent player enters the contest, while each player would enter if the other player stays out. □

Lemma A.2. *Suppose* $s_1 = s_2 = n$ *and* $c \le v_2$.

(i) *If* $c \le \frac{v_2^3}{(v_1+v_2)^2}$, *then both players enter the contest. Payoffs are* $E[u_1] = \frac{v_1^3}{(v_1+v_2)^2} - c$ *and* $E[u_2] = \frac{v_2^3}{(v_1+v_2)^2} - c$.

(ii) *If* $c \in \left(\frac{v_2^3}{(v_1+v_2)^2}, \frac{v_1^3}{(v_1+v_2)^2}\right]$, *only player 1 enters the contest and payoffs are* $E[u_1] = v_1 - c$ *and* $E[u_2] = 0$.

(iii) *If* $c > \frac{v_1^3}{(v_1+v_2)^2}$, *only one of the players enters and receives* $E[u_i] = v_i - c$, *while the other player receives* $E[u_j] = 0$.

Lemma A.3. *Suppose* $s_1 = s$, $s_2 = n$, *and* $c \le v_2$.

(i) *If* $c \le \frac{v_2^3}{(v_1+v_2+k)^2}$, *then both players enter the contest. Payoffs are* $E[u_1] = \frac{(v_1+k)^3}{(v_1+v_2+k)^2} - k - c$ *and* $E[u_2] = \frac{v_2^3}{(v_1+v_2+k)^2} - c$.

(ii) *If* $c \in \left(\frac{v_2^3}{(v_1+v_2+k)^2}, \frac{(v_1+k)^3}{(v_1+v_2+k)^2}\right]$, *only player 1 enters the contest and payoffs are* $E[u_1] = v_1 - c$ *and* $E[u_2] = 0$.

(iii) *If* $c > \frac{(v_1+k)^3}{(v_1+v_2+k)^2}$, *only one of the players enters and receives* $E[u_i] = v_i - c$, *while the other player receives* $E[u_j] = -I_j$.

Lemma A.4. *Suppose* $s_1 = n$, $s_2 = s$, *and* $c \le v_2 + k$.

(i) *If* $c \le \min\left\{\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{(v_2+k)^3}{(v_1+v_2+k)^2}\right\}$, *then both players enter the contest. Payoffs are* $E[u_1] = \frac{v_1^3}{(v_1+v_2+k)^2} - c$ *and* $E[u_2] = \frac{(v_2+k)^3}{(v_1+v_2+k)^2} - k - c$.

(ii) *If* $c \in \left(\min\left\{\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{(v_2+k)^3}{(v_1+v_2+k)^2}\right\}, \max\left\{\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{(v_2+k)^3}{(v_1+v_2+k)^2}\right\}\right]$, *only the player with the higher incentive enters the contest and receives* $E[u_i] = v_i - c$, *while the other player receives* $E[u_j] = -I_j$.

(iii) *If* $c > \max\left\{\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{(v_2+k)^3}{(v_1+v_2+k)^2}\right\}$, *only one of the players enters and receives* $E[u_i] = v_i - c$, *while the other player receives* $E[u_j] = -I_j$.

Appendix B. Proofs of main results

B.1. Complete information

Proof *(Proof of Lemma 1).* If i chooses $s_i = s$ and stays out, he gets $-k$. But i can guarantee himself a payoff of zero by choosing $s_i = n$ and staying out. □

Proof *(Proof of Lemma 2).* Suppose that j enters if $s_i = s$. There are two cases to consider (they differ in whether or not j enters if $s_i = n$).

First, suppose that j enters regardless of the choice of s_i. If i chooses $s_i = s$, he gets $\max\left\{\frac{(v_i+k)^3}{(v_i+k+v_j+I_j)^2} - k - c, -k\right\}$. If i chooses $s_i = n$, he gets $\max\left\{\frac{v_i^3}{(v_i+v_j+I_j)^2} - c, 0\right\}$.

Define $h(u) := \frac{(v_i+u)^3}{(v_i+u+v_j+I_j)^2} - u - c$. Since for all $u \ge 0$,

$$h'(u) = -(I_j + v_j)^2 \frac{3u + I_j + 3v_i + v_j}{(v_i + u + v_j + I_j)^3} < 0,$$

we have $h(0) > h(k)$. It follows that i 's payoff is strictly higher if he chooses $s_i = n$.

Second, suppose that j enters if and only if i communicates. If this is part of j's strategy, the payoff of i upon not communicating is $\max\{v_i - c, 0\}$, and upon communicating it is $\max\{h(k), -k\}$. Since $v_i - c > h(0) > h(k)$, not communicating is strictly better. □

Proof *(Proof of Proposition 1).* Suppose $s_1 = s_2 = s$. Then i enters the contest in the resulting subgame by Lemma 1. But then j should choose $s_j = n$ by Lemma 2. □

Proof *(Proof of Lemma 3).* If i chooses $s_i = s$ and j does not enter, i gets $v_i - c$. If i chooses $s_i = n$ and j enters, i either stays out and gets zero; or i enters, spends a positive effort in equilibrium, and wins with probability less than one. Therefore his payoff is strictly smaller than $v_i - c$. □

Proof *(Proof of Proposition 2).* "If" part. Suppose that $c \in \left(\frac{(v_2+k)^3}{(v_1+v_2+2k)^2}, v_1\right]$.

Consider strategies with the following features. Player 1 communicates, player 2 does not. Player 1 enters in subgame $(s_1, s_2) = (s, n)$ and in subgame $(s_1, s_2) = (s, s)$, while player 2 does not enter in these subgames.

Suppose 2 behaves as described above and consider 1. At the entry stage, it is optimal for 1 to enter whenever he has chosen $s_1 = s$ since in these subgames 2 stays out. Moreover, choosing $s_1 = s$ is optimal, again for the reason that in the resulting subgame 2 stays out.

Now suppose 1 behaves as described above and consider player 2. First consider the entry stage. Staying out if 1 has chosen $s_1 = s$ is optimal for 2 since

$$\frac{(v_2 + I_2)^3}{(v_1 + k + v_2 + I_2)^2} \le \frac{(v_2 + k)^3}{(v_1 + k + v_2 + k)^2} < c.$$

Second, consider the communication stage. Since 1 will enter even if 2 chooses $s_2 = s$, choosing $s_2 = n$ is optimal for 2 by Lemma 2.

It remains to consider the case where

$$c \in \left(\frac{v_2^3}{(v_1 + k + v_2)^2}, \min\left\{\frac{(v_2 + k)^3}{(v_1 + v_2 + 2k)^2}, v_1\right\}\right].$$

Here the construction above does not work since in subgame $(s_1, s_2) = (s, s)$, both players will enter. However, there still is an equilibrium with the desired features. It differs from the construction above only off the equilibrium path, in the subgame starting after $(s_1, s_2) = (s, s)$.

Consider strategies with the following features. Player 1 communicates, player 2 does not. Player 1 enters in subgame $(s_1, s_2) = (s, n)$ and in subgame $(s_1, s_2) = (s, s)$. Player 2 does not enter in the subgame $(s_1, s_2) = (s, n)$, but enters in subgame $(s_1, s_2) = (s, s)$.

In the entry stage, entry is optimal for player 1 in subgame $(s_1, s_2) = (s, n)$ since 2 stays out and $c \le v_1$. In the same subgame, staying out is optimal for player 2 since $c > \frac{v_2^3}{(v_1+v_2+k)^2}$. In the subgame $(s_1, s_2) = (s, s)$, entry is optimal for both since

$$c \le \frac{(v_2 + k)^3}{(v_1 + v_2 + 2k)^2} < \frac{(v_1 + k)^3}{(v_1 + v_2 + 2k)^2}.$$

In the communication stage, player 1 has no incentive to deviate since player 2 does not enter on the equilibrium path. Moreover, by Lemma 2, player 2 has no incentive to deviate in the communication stage, since player 1 enters after $(s_1, s_2) = (s, s)$.

"Only if" part. If $c > v_1$, clearly there is no equilibrium where 1 chooses $s_1 = s$. Moreover, using Lemma 2 it can easily be shown that there is no equilibrium where only 1 communicates if $c \le \frac{v_2^3}{(v_1+k+v_2)^2}$. □

Proof *(Proof of Proposition 3).* "If" part. Under condition (i), there is an equilibrium where only 2 communicates, only 2 enters after $(s_1, s_2) = (n, s)$, and only 2 enters after $(s_1, s_2) = (s, s)$.

Now suppose that $c \le \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$. Then this construction does not work since after $(s_1, s_2) = (s, s)$, player 1 will enter. However, if (ii) holds, there is an equilibrium where after $(s_1, s_2) = (s, s)$ both players enter. (On the equilibrium path, after $(s_1, s_2) = (n, s)$, only player 2 enters.) Entry after $(s_1, s_2) = (s, s)$ is optimal for both since $c \le v_2$ and

$$c \le \frac{(v_2 + k)^3}{(v_1 + v_2 + 2k)^2} < \frac{(v_1 + k)^3}{(v_1 + v_2 + 2k)^2}.$$

It remains to show that 1 has no incentive to choose $s_1 = s$. This follows from the fact that 2 enters even after $(s_1, s_2) = (s, s)$ together with Lemma 2.

"Only if" part. If $c > v_2$, there is clearly no equilibrium where 2 chooses $s_2 = s$ and enters. If $c \le \frac{v_1^3}{(v_1+k+v_2)^2}$, player 1 will enter after $(s_1, s_2) = (n, s)$; hence there is no equilibrium where only 2 communicates by Lemma 2. Finally, if $\frac{(v_2+k)^3}{(v_1+v_2+2k)^2} < c \le \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$, after $(s_1, s_2) = (s, s)$ only player 1 enters. Therefore, player 1 should deviate to choosing $s_1 = s$ by Lemma 3. □

B.2. Incomplete information

Proof *(Proof of Proposition 4).* Suppose there is such an equilibrium. If player 2 chooses weakly higher effort after observing $s_1 = s$ than after $s_1 = n$, the low-type player 1 should deviate to $s_1 = n$. Consider the following decision problem where the opponent's effort $x_2 \in (0, w)$ is a parameter:

$$\max_x \frac{x}{x+x_2}(w+k) - x - k.$$

The indirect utility function (i.e. the maximized value of this objective function) is $\left(1-\sqrt{\frac{x_2}{w+k}}\right)(w+k)-k$. Obviously, the indirect utility function decreases in k. It also decreases with x_2:

$$\frac{\partial}{\partial x_2}\left(1-\sqrt{\frac{x_2}{w+k}}\right)(w+k) = -\sqrt{w+k}\frac{\partial\sqrt{x_2}}{\partial x_2} < 0.$$

The two facts imply that player 1 has an incentive to send the message only if player 2 exerts a lower effort x_2 when observing communication. Player 1 has no incentive to bear the cost k otherwise.

Moreover, the indirect utility function decreases more rapidly if w is higher:

$$\frac{\partial^2}{\partial x_2 \partial w}\left(1-\sqrt{\frac{x_2}{w+k}}\right)(w+k) = -\left(\frac{\partial\sqrt{w+k}}{\partial w}\right)\left(\frac{\partial\sqrt{x_2}}{\partial x_2}\right) < 0$$

Thus, a reduction in the opponent's effort x_2 is more valuable if the value of winning is higher. Hence, whenever the low type (weakly) prefers to send the message, the high type must strictly prefer to send the message. This completes the proof. □

Proof *(Proofs of Proposition 5 and Corollary 1).* Suppose that a separating equilibrium where only the high type communicates is played. We begin the analysis by calculating equilibrium efforts and payoffs. Player 2 perfectly infers player 1's type from the observation of his message, and he forms the posterior $Pr(v_1 = v|s) = 1$ and $Pr(v_1 = v|n) = 0$. Upon the high type sending $s_i = s$, the two players in the subsequent contest would exert their efforts, respectively, $x_{1h} = \frac{(v+k)^2}{(v+k+1)^2}$, and $x_{2h} = \frac{(v+k)}{(v+k+1)^2}$.The high type receives an expected payoff $\pi_{1h}(k) = \frac{(v+k)^3}{(v+k+1)^2} - k$. Upon the low type sending $s_1 = n$, the two players' subsequent efforts are $x_{1l} = x_{2l} = 1/4$. The low-type player 1 receives an expected payoff $\pi_{1l} = 1/4$.

We then establish the incentive compatibility conditions which ensure that neither type has the incentive to misrepresent. If the high type deviates to $s_1 = n$, his opponent believes that he competes against a player with low valuation and he would choose $x_{2l} = 1/4$ in the subsequent contest. Player 1's optimal effort x_{hd} maximizes $\frac{x_{hd}}{x_{hd}+1/4}v - x_{hd}$. Thus he exerts the effort $x_{hd} = \frac{\sqrt{v}}{2} - \frac{1}{4}$. The payoff from this deviation is given by

$$\pi_{hd} = \frac{(\sqrt{v}/2)-(1/4)}{\sqrt{v}/2}v - \left(\frac{\sqrt{v}}{2} - \frac{1}{4}\right) = \left(\sqrt{v} - \frac{1}{2}\right)^2.$$

If the low type deviates by sending $s_1 = s$, his opponent chooses $x_{2h} = \frac{(v+k)}{(v+k+1)^2}$. The low type then chooses his effort x_{ld} to maximize his expected payoff $\frac{x_{ld}}{x_{ld}+((v+k)/(v+k+1)^2)}(1+k) - x_{ld}$. The optimal effort is given by $x_{ld} = \frac{\sqrt{(v+k)(1+k)}}{v+k+1} - \frac{v+k}{(v+k+1)^2}$. The payoff from deviation is thus given by

$$\pi_{ld}(k) = \left(1 - \frac{(v+k)/(v+k+1)^2}{(\sqrt{(v+k)(1+k)})/(v+k+1)}\right)(1+k) - k - \left(\frac{\sqrt{(v+k)(1+k)}}{v+k+1} - \frac{v+k}{(v+k+1)^2}\right)$$
$$= 1 - \frac{2\sqrt{(v+k)(1+k)}(v+k+1) - (v+k)}{(v+k+1)^2}.$$

The equilibrium requires

$$\pi_{1h}(k) \ge \pi_{hd} \text{ and } \pi_{1l} \ge \pi_{ld}(k).$$

Consider the high type. Define $G(k) := \pi_{1h}(k) - \pi_{hd} = \frac{(v+k)^3}{(v+k+1)^2} - k - \left(\sqrt{v} - 0.5\right)^2$. Note that $G(0) = \frac{v^3}{(v+1)^2} - \left(\sqrt{v} - 0.5\right)^2 > 0$ and $G'(k) = \frac{3(v+k)^2(v+k+1) - 2(v+k)^3}{(v+k+1)^3} - 1 = -\frac{3k+3v+1}{(k+v+1)^3} < 0$.

Moreover,

$$\lim_{k\to\infty} G(k) = \lim_{k\to\infty}\left(\frac{k^2v - 2k^2 + 2kv^2 - 2kv - k + v^3}{k^2 + 2kv + 2k + v^2 + 2v + 1}\right) - \left(\sqrt{v} - 0.5\right)^2 = \sqrt{v} - \frac{9}{4}$$

thus $\lim_{k\to\infty} G(k) = (<,>)0$ iff $v = (<,>)81/16$. Together with $G(0) > 0$ and $G'(k) < 0$, this implies that, if $v \geq 81/16$, then $G(k) > 0$ for all $k \geq 0$. On the other hand, if $v < 81/16$, there exists a unique $\hat{k} \in (0, \infty)$ such that $G\left(\hat{k}\right) = 0$, $G(k) > 0$ for all $k < \hat{k}$, and $G(k) < 0$ for all $k > \hat{k}$. In fact, $\hat{k}$ can be calculated explicitly. The equation $G(k) = 0$ has exactly one possibly positive solution, namely $\hat{k} = \frac{2-(v+1)(3-2\sqrt[4]{v})}{3-2\sqrt[4]{v}}$.

Even though it follows from the considerations above that $\hat{k} > 0$ if and only if $v \in (1, 81/16)$, we directly verify this here. This direct approach also yields the comparative static properties of $\hat{k}$. So suppose $v \in (1, 81/16)$. Then $3 > 2\sqrt[4]{v}$. It remains to show that $f(v) := 2 - (v+1)(3 - 2\sqrt[4]{v}) > 0$. We have $\lim_{v\downarrow 1} f(v) = 0$ and $\lim_{v\uparrow 81/16} f(v) = 2$. Differentiating $f(v)$ yields $f'(v) = (2v^{1/4} - 3) + \frac{1}{2}v^{-3/4}(v+1) = \frac{5}{2}v^{1/4} + \frac{1}{2}v^{-3/4} - 3$. Further differentiating $f'(v)$ yields $f''(v) = \frac{5}{8}v^{-3/4} - \frac{3}{8}v^{-7/4} = \frac{1}{8}v^{-3/4}(5 - \frac{3}{v})$, which must be strictly positive because $v > 1$. Further, $\lim_{v\downarrow 1} f'(v) = 0$. It implies that $f(v)$ must be increasing and therefore $f(v) > 0$ over the interval $(1, 81/16)$. Note also that these observations imply that $\hat{k}$ is increasing in v, and that $\lim_{v\downarrow 1}\hat{k} = 0$. If $v > 81/16$, then $3 < 2\sqrt[4]{v}$ and the $\hat{k}$ given in the formula above is clearly negative. To sum up this discussion:

Claim 1. *If $v > 81/16$, the high valuation type never has an incentive to deviate. On the other hand, if $v < 81/16$, then the high valuation type has no incentive to deviate if and only if $k \leq \hat{k}$. The critical value $\hat{k}$ is increasing in v, and* $\lim_{v\downarrow 1}\hat{k} = 0$

Next, we consider the incentives of the low valuation type. Define

$$H(k) := \pi_{1l} - \pi_{ld}(k) = -0.75 + \frac{2\sqrt{(v+k)(1+k)}(1+v+k) - (v+k)}{(1+v+k)^2}.$$

Note that $H(0) = -0.75 + \frac{2\sqrt{v}(1+v) - v}{(1+v)^2} < 0$. In addition, $H(k)$ can be written as

$$H(k) = -0.75 + \frac{2\sqrt{(v+k)(1+k)}}{(1+v+k)} - \frac{(v+k)}{(1+v+k)^2}$$

and hence

$$H'(k) = \frac{\sqrt{(k+1)(k+v)}\left(v^2 + kv + k + 1\right)}{(k+1)(k+v)(k+v+1)^2} + \frac{k+v-1}{(k+v+1)^3} > 0$$

(recall that $v > 1$). Moreover, note that $\lim_{k\to\infty} H(k) = 5/4$. Together with $H(0) < 0$ and $H'(k) > 0$, this implies that there is a unique $\tilde{k} \in (0, \infty)$ such that $H\left(\tilde{k}\right) = 0$, and $H(k) < (>)0$ if $k < (>)\tilde{k}$. To see how the critical value $\tilde{k}$ depends on v, note that $H(k)$ is decreasing in v, as can be shown by partial differentiation. From the implicit function rule,

$$\frac{d\tilde{k}}{dv} = -\frac{\frac{\partial}{\partial v}H(k)}{H'(k)} > 0.$$

Claim 2. *There is a unique $\tilde{k} \in (0, \infty)$ such that the low type has no incentive to mimic the high type if and only if $k \geq \tilde{k}$. The critical value $\tilde{k}$ is increasing in v.*

Now we are in position to consider existence of a separating equilibrium. The case where v is so big that the high type never wants to deviate is straightforward.

Claim 3. *If $v \geq 81/16$, a separating equilibrium exists whenever $k \geq \tilde{k}$.*

For the case where $v < 81/16$, we now show that $\hat{k} > \tilde{k}$. To see this, note that $H\left(\hat{k}\right)$ is strictly positive iff

$$2\sqrt{\left(v+\hat{k}\right)\left(1+\hat{k}\right)} > 0.75\left(1+v+\hat{k}\right) + \frac{v+\hat{k}}{\left(1+v+\hat{k}\right)}$$

$$\Leftrightarrow 4\left(v+\hat{k}\right)\left(1+\hat{k}\right) - \left(0.75\left(1+v+\hat{k}\right) + \frac{v+\hat{k}}{\left(1+v+\hat{k}\right)}\right)^2 > 0.$$

Insert $\hat{k}$ into the left hand side of this expression. The condition becomes

$$\frac{1}{4\sqrt{v}-12\sqrt[4]{v}+9}\left(8v-16\sqrt{v}+16\sqrt[4]{v}+16v^{3/2}+16v^{3/4}-32v^{5/4}-8\right) > 0$$

$$\Leftrightarrow v-2\sqrt{v}+2\sqrt[4]{v}+2v^{3/2}+2v^{3/4}-4v^{5/4}-1 > 0,$$

since $4\sqrt{v}-12\sqrt[4]{v}+9>0$, for $v<81/16$. The latter condition can be shown to be fulfilled for all $v>1$.

Since $H(\hat{k}) > 0 = H(\tilde{k})$ and $H'(k) > 0$, it follows that $\hat{k} > \tilde{k}$. Therefore

Claim 4. *If $v < 81/16$, a separating equilibrium exists if and only if $k \in [\tilde{k}, \hat{k}]$.*

□

Proof *(Proof of Lemma 4).* Suppose that both types of player 1 send the message. By standard arguments, there is a unique Bayesian Nash equilibrium. Player 2, whose type is known to be $v_2 = 1$ solves

$$\max_{x_2} \lambda \frac{x_2}{x_{1l}+x_2} + (1-\lambda)\frac{x_2}{x_{1h}+x_2} - x_2,$$

where x_{1l} is the effort of the low-type player 1, and x_{1h} of the high type. The first order condition is

$$\lambda \frac{x_{1l}}{(x_{1l}+x_2)^2} + (1-\lambda)\frac{x_{1h}}{(x_{1h}+x_2)^2} = 1.$$

Note that the payoff function is globally concave in x_2 and hence the first order condition is sufficient for a global maximum.

Player 1 with private information solves

$$\max_{x_{1t}} \frac{x_{1t}}{x_{1t}+x_2}(v_t+k) - x_{1t} - k \quad t = h, l$$

where $v_h = v$ and $v_l = 1$. The first order condition is

$$\frac{x_2}{(x_{1t}+x_2)^2} = \frac{1}{v_t+k}.$$

Solving the three first order conditions simultaneously leads to the formula for x_2 given in the lemma.

To establish the comparative static of x_2, let $z = \frac{1}{\sqrt{(1+k)}}$ and $y = \frac{1}{\sqrt{(v+k)}}$. Then we can write

$$\sqrt{x_2} = \frac{\lambda z + (1-\lambda)y}{1+\lambda z^2 + (1-\lambda)y^2}$$

We show that this expression increases in y, keeping all else constant.

$$\frac{\partial}{\partial y}\left(\frac{\lambda z+(1-\lambda)y}{1+\lambda z^2+(1-\lambda)y^2}\right) = (1-\lambda)\frac{\left(1+\lambda z^2+(1-\lambda)y^2\right) - (\lambda z + (1-\lambda)y)2y}{\left(1+\lambda z^2+(1-\lambda)y^2\right)^2}$$

which is strictly positive if and only if $1 > (1-\lambda)y^2 + \lambda z(2y-z)$, which is true since

$$z(2y-z) < \max_{\zeta} \zeta(2y-\zeta)) = y^2 < 1.$$

It follows that x_2 is decreasing in v. A similar argument shows that

$$\frac{\partial}{\partial z}\left(\frac{\lambda z+(1-\lambda)y}{1+\lambda z^2+(1-\lambda)y^2}\right) > 0.$$

Together with the observation above, this implies that x_2 decreases in k. Finally,

$$\frac{\partial}{\partial\lambda}\left(\frac{\lambda z+(1-\lambda)y}{1+\lambda z^2+(1-\lambda)y^2}\right)=\frac{(z-y)(1-yz)}{\left(1+\lambda z^2+(1-\lambda)y^2\right)^2}>0$$

where the inequality holds since $y<z<1$. It follows that x_2 increases in λ. □

Proof *(Proof of Proposition 6).* Suppose both types of player 1 send a message of confidence. The first order condition of type $t\in\{l,h\}$ of player 1 in such a pooling equilibrium gives $x_{1t}=\sqrt{x_2 v_t}-x_2$, where $v_h=v+k$ and $v_l=1+k$. Thus, the payoff of type t of player 1 is

$$\begin{aligned}\pi_t^* &= \frac{x_{1t}}{x_{1t}+x_2}v_t-x_{1t}-k\\ &= \frac{(-x_2+\sqrt{x_2v_t})v_t-k\sqrt{x_2v_t}}{\sqrt{x_2v_t}}+x_2-\sqrt{x_2v_t}=(v_t-k)+x_2-2\sqrt{x_2v_t}.\end{aligned}$$

where x_2 is given in Lemma 4.

To demonstrate that the pooling equilibrium exists, we have to show that neither type of player 1 wants to deviate by not sending a message of confidence. We thus have to consider the out-of-equilibrium belief of player 2 if she unexpectedly observes no message. Denote this belief by $\mu'=\Pr(t=l|n)$. The derivation of payoffs when player 1 deviates from the pooling equilibrium proceeds again in the same way as outlined in the proof of Lemma 4. Just replace λ by μ' and let k approach 0. Player 2 exerts effort

$$x_2(\mu')=\left(\frac{\mu' v+(1-\mu')\sqrt{v}}{v+\mu' v+(1-\mu')}\right)^2$$

while the two types of player 1 receive, respectively,

$$\pi_l^{/}=1+x_2(\mu')-2\sqrt{x_2(\mu')},\quad \pi_h^{/}=v+x_2(\mu')-2\sqrt{x_2(\mu')v}.$$

The pooling equilibrium with both types of player 1 sending a message of confidence exists if and only if there exists $\mu'\in[0,1]$ such that $\pi_t^*\geq\pi_t^{/}$ for both types. The conditions depend on the out-of-equilibrium belief μ' and the corresponding effort $x_2(\mu')$ of player 2. It should be noted that by the claim of Proposition 4, the incentive compatibility condition for the high type is redundant. We then focus on the low type. The condition is written as

$$1+x_2-2\sqrt{x_2(1+k)}\geq 1+x_2(\mu')-2\sqrt{x_2(\mu')},$$

or

$$x_2-x_2(\mu')\geq 2\left(\sqrt{x_2(1+k)}-\sqrt{x_2(\mu')}\right).$$

The equilibrium conditions cannot hold if $\mu'=\lambda$ and subsequently $x_2(\mu')=x_2$. Player 1 has an incentive to send the message in the pooling equilibrium only if he would be punished by a more unfavorable belief ($\mu'>\lambda$) otherwise, which leads player 2 to exert higher effort (note that $x_2(\mu')$ is increasing in μ'). The harshest punishment for deviation is therefore a belief with $\mu'=1$ and $x_2(\mu')=1/4$.

Hence, the pooling equilibrium exists if and only if the equilibrium payoff of the low-type player 1 is bigger than 1/4, his deviation payoff when player 2 holds belief $\mu'=1$ and chooses effort $x_2(\mu')=1/4$. The corresponding condition $\pi_l^*=1+x_2-2\sqrt{x_2(1+k)}\geq 1/4$ is equivalent to $(\sqrt{x_2})^2-2\sqrt{1+k}(\sqrt{x_2})+3/4\geq 0$, or

$$-2\sqrt{1+k}\frac{[\lambda(v+k)\sqrt{1+k}+(1-\lambda)(1+k)\sqrt{v+k}]}{(1+k)(v+k)+\lambda(v+k)+(1-\lambda)(1+k)}+\frac{3}{4}\geq 0. \tag{1}$$

$$-2\sqrt{1+k}\frac{[\lambda(v+k)\sqrt{1+k}+(1-\lambda)(1+k)\sqrt{v+k}]}{(1+k)(v+k)+\lambda(v+k)+(1-\lambda)(1+k)}+\frac{3}{4}\geq 0. \tag{1}$$

Now consider the limit $v\to 1$. Here, condition (1) simplifies to

$$\frac{[\lambda\sqrt{1+k}+(1-\lambda)\sqrt{1+k}]^2}{[(1+k)+\lambda+(1-\lambda)]^2}-2\sqrt{1+k}\left(\frac{[\lambda\sqrt{1+k}+(1-\lambda)\sqrt{1+k}]}{(1+k)+\lambda+(1-\lambda)}\right)+\frac{3}{4}\geq 0$$

$$\Leftrightarrow\frac{1+k}{[2+k]^2}-\frac{2+2k}{2+k}+\frac{3}{4}\geq 0\Leftrightarrow\frac{4+4k-8(1+k)(2+k)+3[2+k]^2}{4[2+k]^2}\geq 0$$

$$\Leftrightarrow\frac{-8k-5k^2}{4[2+k]^2}\geq 0.$$

Obviously, the condition can never be fulfilled regardless of λ. No pooling equilibrium exists since equilibrium payoffs are approaching those from the complete information case.

Similarly, consider the limiting case of $\lambda \to 1$. In this case, meeting a high-type player 1 becomes a zero-probability event, so x_2 does not depend on v. As a result, the size of v does not affect the low type player's incentive to misrepresent his type. We have verified in the complete-information setting that communication does not occur. Hence, a pooling equilibrium does not exist regardless of the size of v.

We learn that the size of x_2 (under prior beliefs) matters critically. When λ approaches 0 and v tends to infinity, x_2 approaches 0 and π_l^* approaches 1. The low-type player 1 clearly has an incentive to play the equilibrium strategy. By continuity, a pooling equilibrium must exist if v is big enough and λ is sufficiently small.

Consider the limiting case of $\lambda \to 0$. Then the condition boils down to

$$\frac{1}{\left[\sqrt{(v+k)}+1\right]^2} - 2\sqrt{1+k}\frac{1}{\sqrt{(v+k)}+1} + \frac{3}{4} \geq 0. \tag{2}$$

We have verified that when v approaches one, the condition cannot hold. However, when v approaches infinite, the condition must hold because $\frac{1}{\sqrt{(v+k)}+1}$ reduces to zero. Hence, there must exist a unique $\underline{v} > 1$, which satisfies $\frac{1}{\left[\sqrt{(\underline{v}+k)}+1\right]^2} - 2\sqrt{1+k}\frac{1}{\sqrt{(\underline{v}+k)}+1} + \frac{3}{4} = 0$. Hence, no pooling equilibrium exists if v falls below the cutoff.

When λ approaches zero, $v = \underline{v}$ would make the equilibrium conditions hold. Note that LHS of the inequality decreases with $\sqrt{x_2}$. Hence, it increases with v and decreases with λ. When v exceeds the cutoff $\underline{v}$, there must exist a unique cutoff $\lambda(v)$, such that the condition continues to hold as long as $\lambda \leq \lambda(v)$. The cutoff probability is determined by the equation

$$-2\sqrt{1+k}\frac{[\lambda(v)(v+k)\sqrt{1+k}+(1-\lambda(v))(1+k)\sqrt{v+k}]}{(1+k)(v+k)+\lambda(v)(v+k)+(1-\lambda(v))(1+k)} + \frac{3}{4} = 0. \tag{3}$$

$$-2\sqrt{1+k}\frac{[\lambda(v)(v+k)\sqrt{1+k}+(1-\lambda(v))(1+k)\sqrt{v+k}]}{(1+k)(v+k)+\lambda(v)(v+k)+(1-\lambda(v))(1+k)} + \frac{3}{4} = 0. \tag{3}$$

□

Proof *(Proof of Proposition 7)*. Note that analysis of the benchmark case, in which no communication is involved, can be borrowed directly from that of Lemma 4, by setting $k = 0$. Using players' best response functions, we obtain $\frac{x_2}{x_{1h}+x_2} = \frac{x_2}{\sqrt{x_2 v}}$ as player 2's winning probability against a high-type player 1. In a separating equilibrium, player 2 wins with probability $\frac{1}{1+v}$.

We now claim $\frac{x_2}{\sqrt{x_2 v}} > \frac{1}{1+v}$, or equivalently $\sqrt{x_2} > \sqrt{v}\frac{1}{1+v}$. Plug in x_2 from Lemma 4 (when $k = 0$) to obtain

$$\frac{\lambda v + (1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v + v} > \sqrt{v}\frac{1}{1+v}.$$

Dividing by $\sqrt{v}$ and rearranging it, the inequality is rewritten as

$$\left(\lambda\sqrt{v}+(1-\lambda)\right)(1+v) > (1-\lambda)+\lambda v + v.$$

Since

$$\left(\lambda\sqrt{v}+(1-\lambda)\right)(1+v) - ((1-\lambda)+\lambda v+v) = \lambda\sqrt{v}\left(\sqrt{v}-1\right)^2 > 0$$

the condition $\frac{x_2}{\sqrt{x_2 v}} > \frac{1}{1+v}$ is met. Since $\frac{1}{1+v} > \frac{1}{1+v+k}$ (which is the win likelihood of player 2 against a high-type player 1 in the separating equilibrium) the claim is established. □

Proof *(Proof of Proposition 8)*. Allocative efficiency is an issue only if player 1 turns out to be of the high type. Consider the pooling equilibrium and suppose that player 1 has a high type. Using the first order condition of the high type of player 1, the probability that he wins is

$$\frac{x_{1h}}{x_{1h}+x_2} = \frac{\sqrt{x_2(v+k)}-x_2}{\sqrt{x_2(v+k)}} = 1 - \sqrt{\frac{x_2}{v+k}}.$$

From Lemma 4, we know that x_2 decreases in k. Thus the probability that 1 wins, conditional on his type being high, increases in k. □

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