International Journal of Industrial Organization 31 (2013) 267–277

Contents lists available at SciVerse ScienceDirect

International Journal of Industrial Organization

journal homepage: www.elsevier.com/locate/ijio

Competitive effect of cross-shareholdings in all-pay auctions with complete information☆

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A R T I C L E I N F O

Available online 22 March 2012

JEL classification:
C7

Keywords:
All-pay auction
Cross-shareholding
Effort supply
Asymmetric bidders
Virtual bidding costs

A B S T R A C T

This paper investigates the competitive effect of cross-shareholdings in winner-take-all all-pay auctions with two asymmetric bidders. We show that cross-shareholdings may paradoxically create a "pro-competitive" effect and elicit more effort than a standard contest without cross-ownership. This observation runs in contrast to the anti-competitive effect that cross-shareholdings usually create in standard oligopolistic settings (such as Cournot or Bertrand competitions). Both bidding costs and the sizes of cross-shares affect the resultant total effort non-monotonically. Neither a cross-share nor a higher bidding cost necessarily decreases effort supply. A complete account of equilibrium bidding behaviors is provided and the necessary and sufficient conditions under which cross-shareholdings lead to higher or lower levels of overall effort are identified. However, the pro-competitive effect comes at a loss of efficiency.

1. Introduction

There is often a degree of congruence in the interests of competing parties. One salient example of such congruence is in the competition among cross-shareholders who each hold minority equity stakes in their rivals' companies. While each firm retains full control over its own operations, holding a passive equity stake in its rival allows it to obtain a share of the latter's profits. Such entangled interests are often found among the numerous Japanese firms which are historically linked to dominant business conglomerates (Keiretsu). Member firms of a Keiretsu each own small portions of the others' shares, thus forming an interlocking cross-shareholding network centered on a leading bank (e.g. Mitsubishi Trust and Banking) or a core firm (e.g. Toyota). Complex networks of cross-shareholdings are also widespread in the corporate sectors of various European countries (e.g. Italy, Sweden, and Germany) and such networks are viewed as an integral feature of the market system operating in these economies. Major pharmaceutical and biotechnology companies, such as Roche and L'Oréal, have similarly evolved into network organizations that rely heavily on extensive webs of strategic alliances in R&D through equity linkage. In China, the central government actively enforces an extensive level of cross-ownership among major state-owned companies in an effort to facilitate checks-and-balances and improve corporate governance.

Conventional wisdom tells us that cross-ownership mitigates competition between firms. Profit-sharing erodes a firm's incentive to behave competitively, and compels competing parties to internalize the externalities that they would have imposed on each other through their own strategic choices. Given this anti-competitive effect, the widespread existence of cross-shareholding networks in Germany, for instance, has long been viewed as a "drag on the nation's competitiveness" and on the operating efficiency of German firms.[2] Such concern has spawned substantial research efforts. The anti-competitive effect of cross-shareholdings has been formally identified in standard oligopolistic models (Cournot or differentiated product Bertrand competitions) in a number of economic studies, such as those by Farrell and Shapiro (1990), Reitman (1994), O'Brien and Salop (1999–2000), Dietzenbacher et al. (2000), and Gilo et al. (2006). This concern has also sparked long-lasting debates on regulatory rule-making. For instance, O'Brien and Salop (1999–2000) demonstrate that cross-ownership could cause a more severe anti-competitive effect than a full merger.

Despite the proliferation of the literature on cross-shareholdings in markets with Cournot or differentiated product Bertrand competitions, the role of cross-ownership in contests has yet to be systematically studied.[3] In a contest, economic agents expend costly resources to vie for a limited number of prizes. All participants forfeit their bids

☆ We are grateful to Michael Baye, Olivier Bos, Kai Konrad, Dan Kovenock and Casper de Vries for very helpful comments and suggestions. We thank Guest Editors Charles Knoeber and Theofanis Tsoulouhas, and two anonymous referees, for very constructive and helpful comments and suggestions on this paper. The authors gratefully acknowledge financial support from National University of Singapore on this research project [R-313-093-000-112 (Q. Fu) and R-122-000-155-112 (J. Lu)].

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[2] Source: Commentary: How Germany Inc. Is Loosening Up, by Jack Ewing, *BusinessWeek*, October 18, 2004.
[3] Konrad (2006) provides one notable exception.

0167-7187/$ – see front matter
doi:10.1016/j.ijindorg.2012.02.004

(efforts), and only the winners receive the prizes. These types of competitions are ubiquitous in the modern economic landscape. A wide array of competitive events resemble contests, including R&D races, procurement tournaments, architectural design competitions, policy lobbying, races in oil exploration, and firms' campaigns for market share.[4]

This paper attempts to answer the following question: Do cross-shareholdings necessarily hinder competition in contests? For instance, when firms with equity links explore promising oil fields in overlapping coastal waters, do their entangled interests attenuate the competition? Alternatively, when two R&D firms affiliated with the same parent network (e.g. Roche or L'Oréal) pursue similar innovations (such as a new drug), does their hidden dependence on each other serve as a disincentive? The incentive and payoff structures of the firms involved in these scenarios cause the competitions to function much like contests. However, as previously outlined, despite the prevalence of cross-shareholdings and the many regulatory concerns they raise, the contest literature has remained thin in this regard.

The winner-take-all characteristic of contests as well as the non-refundability of bidding costs leads to a discontinuous payoff structure, which drastically contrasts with the payoff structure of standard oligopolistic models. As a result, cross-shareholding yields strategic trade-offs that are not present in quantity or price competitions. To demonstrate the hidden trade-offs, consider a contest where two firms compete for a prize. Each firm is represented by a "bidder" who is the majority shareholder of the firm. At the same time, a minority share of the bidder's firm can be cross-owned by the bidder from the rival firm. A negative direct effect can be immediately foreseen from such cross-shareholding: Each bidder claims not only a share of the prize if he wins, but also secures a portion of the prize if the other party wins. The prize sharing effect dampens the bidders' incentive to supply competitive efforts. The cross-share, however, also obligates a minority stakeholder to share a portion of the rival bidder's effort cost, thereby allowing the latter to bear a "virtual bidding cost" that is only a portion of the actual bidding cost borne by the firm he represents. This "cost subsidy" tends to increase effort supply, and thus intensifies the competition. A formal analysis is necessary to delineate how a bidder balances the subsidized bidding cost against the suppressed prize differential between wins and losses.[5] Furthermore, cross-shareholdings trigger additional strategic interaction when one bidder bears a lower bidding cost than the other. Cross-ownership moderates bidders' stakes from the win, which may either add to, or offset the cost-asymmetry between the bidders. It tilts the balance of the playing field, and, in turn, affects both bidders' incentives to supply effort. Due to these competing effects, the overall impact of cross-shareholdings on the competition in contests remains unclear, and the results obtained from the Cournot or differentiated Bertrand models do not automatically apply.

Our paper investigates the competitive effect of cross-shareholdings in contests. Following the standard contest literature, we measure the degree of competition by the amount of overall effort exerted in the contest. We explore the ramifications of the ownership structure on bidders' incentives to supply effort, and on the resultant effort outlay. Following Konrad (2006), we consider a complete-information (winner-take-all) all-pay auction model (Hillman and Riley, 1989). Two bidders who are majority shareholders compete for a prize on behalf of their firms. They submit their bids (efforts) simultaneously, and the highest bidder wins. Both bidders are allowed to own positive cross-shares in the other firm, but the size of one party's cross-shares may differ from that of the other. As bidders can also bear different bidding costs, asymmetry can arise between the bidders as a result of both the cost structure and the cross-ownership structure. This characteristic constitutes a key aspect of the analysis, and distinguishes this paper from the existing literature.

We compare the equilibrium overall effort induced under each possible (exogenously given) cross-ownership structure to that of a standard contest without cross-shareholdings. The results are mixed. We show that cross-shareholdings always reduce competition when bidders are symmetric in their bidding costs, regardless of the structure of the cross-ownership. This result suggests that, in general, the negative *prize sharing effect* of cross-shareholding dominates the positive *cost subsidy effect* on effort supply. In contrast, when bidding costs are asymmetric, cross-shareholdings trigger substantial strategic reactions and the competitive effects are ambiguous. A positive *leveling effect* may emerge under certain cross-ownership structures. As previously mentioned, cross-ownership may also tilt the balance of the playing field. Consider for instance, the situation where the more efficient bidder holds a stake in his weaker opponent. In this scenario, a positive leveling effect emerges through two avenues. First, it decreases the more efficient bidder's marginal benefit from winning the competition, and weakens his incentive to supply competitive effort. This partially offsets his cost advantage and levels the competition. Second, the silent interests of the strong bidder in the weak firm reduces the weaker bidder's virtual bidding cost (through the implicit cost subsidy), and also mitigates the latter bidder's cost disadvantage. The leveled play field incentivizes the ex ante weaker bidder. The better motivated weaker bidder further prevents the stronger bidder from slacking off. The competition in the contest is thus intensified.

Our analysis provides a thorough and complete account of the equilibrium bidding behavior of cross-owned parties. A few interesting observations are highlighted below.

1. Cross-shareholdings can escalate competition in the contest and elicit greater effort. The cross-shares held by competing bidders affect the expected total effort non-monotonically. We identify the necessary and sufficient conditions under which cross-shareholdings lead to a higher or lower level of expected total effort (as compared to a standard contest without cross-shareholdings).
2. In a standard contest, the total expected effort always decreases as the (marginal) bidding cost further increases. However, the opposite may be observed in the presence of cross-shareholdings. Despite its (direct) negative effect, an increase in a bidder's marginal bidding cost may also serve to balance the distorted competitive environment created by the cross-ownership. This gives rise to a positive *leveling effect* as well, thereby creating more competition.
3. The more efficient bidder always contributes less effort when cross-shareholdings are present than he does in a standard contest without cross-shareholding. Although cross-shareholdings can improve competition, the additional effort can be elicited only from the less efficient bidder. Hence, the pro-competitive effect of cross-shareholding always comes at the loss of efficiency. This fact allows us to evaluate the competitive effect of cross-shareholdings in greater depth.

Besides the theoretical contribution, our paper also yields useful practical implications. The pro-competitive effect alludes to the potential utility of cross-ownership arrangement as an incentive device. We discuss this point in Section 3.3 in greater detail and with concrete examples.

This paper fits in with the well established literature on the competitive effects of cross-shareholdings, including Farrell and Shapiro (1990), Reitman (1994), O'Brien and Salop (1999–2000), Dietzenbacher et al. (2000) and Gilo et al. (2006) among others. However, the focus of our paper is on winner-take-all competitions, which have different structural features from the standard oligopolistic competition models adopted by these pioneering studies. We show

[4] See Konrad (2009) for a comprehensive survey of the applications of contest models.

[5] Cross shareholdings imply that a bidder may win different (positive) portions of the prize depending on who wins.

in the current paper that cross-shareholdings may play a paradoxically positive role in influencing competitive behavior. In this aspect, our paper is also related to a recent study by Ghosh and Morita (2010). While adopting a model of Cournot competition, they also espouse the positive effect of cross-shareholding in terms of facilitating technology transfer when knowledge is of tacit nature.

This paper is most closely related to the work of Konrad (2006). He considers a complete-information all-pay auction with N asymmetric bidders. His setting allows only one particular bidder to own a share of one other firm. In particular, Konrad (2006) pursues a different course by studying how ownership structure affects the payoffs to the competitors, as well as the social efficiency of prize allocation. Our study complements his work by focusing on competitiveness and performance in terms of the expected total effort.

Methodologically, this paper is related to the work of Baye et al. (forthcoming). They present a unified framework of two-player symmetric complete-information all-pay auctions with various externalities. Their framework accommodates a family of externalities that depend linearly on the decision variables. This integrated framework can shed light on contests between cross-owned homogenous bidders who hold equal equity shares in each other. While the bidders remain symmetric in their analysis, this paper allows bidders to be asymmetric in two dimensions, i.e., their bidding costs and the sizes of their cross-shares.

The impact of cross-shareholdings has also been studied in auctions with incomplete information. Clark et al. (2007) have focused on all-pay auctions. Ettinger (2003), Dasgupta and Tsui (2004), and Chillemi (2005) investigate competition among cross-owned bidders for first price and second price auctions. These papers conclude that cross-ownership dampens competition and reduces the revenue of the auctioneer.[6]

The rest of this paper is organized as follows. Section 2 sets up the model and establishes the bidding equilibrium. Section 3 compares the amount of effort of a contest under cross-shareholdings with that of a contest without cross-shareholdings, and discusses the implications of the main results. Some concluding remarks on the caveats and extensions of the model are made in Section 4.

2. Model and equilibrium

2.1. The model

We consider a complete-information all-pay auction (Hillman and Riley, 1989). There are two firms, indexed by $i=1,2$. They compete for a prize v. Bidder i is the majority shareholder of firm i, and thus represents firm i in the competition. It is assumed that bidder i holds a cross share $s_i\in[0,\frac{1}{2})$ in firm j. This represents the "silent interest" that results from minority shareholdings. The two bidders simultaneously submit their bids x_1 and x_2 in order to maximize their individual profits. The highest bidder wins the prize for his firm. Without loss of generality, it is further assumed that bidder 1 (representing firm 1) is more efficient than bidder 2 (representing firm 2). The bid incurs a unitary marginal cost on firm 1, and a marginal cost c on firm 2, with $c\geq 1$.

A firm i's profit in the all-pay auction is denoted by π_i. π_i is given as follows:

$$\pi_1(x_1,x_2)=\begin{cases} v-x_1 & \text{if } x_1>x_2\\ \dfrac{v}{2}-x_1 & \text{if } x_1=x_2\\ -x_1 & \text{if } x_1<x_2\end{cases} \text{ and } \pi_2(x_2,x_1)=\begin{cases} v-cx_2 & \text{if } x_2>x_1\\ \dfrac{v}{2}-cx_2 & \text{if } x_2=x_1\\ -cx_2 & \text{if } x_2<x_1\end{cases}.$$

Due to the cross-shareholdings, a bidder i's overall payoff from the competition is $w_i(x_i,x_j;s_i,s_j)=(1-s_j)\pi_i+s_i\pi_j$. Each bidder strategically chooses his bid x_i in order to maximize his expected overall payoff $w_i(x_i,x_j;s_i,s_j)$. They then receive the following payoffs:

$$w_1(x_1,x_2;s_1,s_2)=\begin{cases}(1-s_2)(v-x_1)-s_1cx_2 & \text{if } x_1>x_2\\ (1-s_2)\left(\dfrac{v}{2}-x_1\right)+s_1\left(\dfrac{v}{2}-cx_2\right) & \text{if } x_1=x_2\\ -(1-s_2)x_1+s_1(v-cx_2) & \text{if } x_1<x_2\end{cases}, \quad (1)$$

and

$$w_2(x_2,x_1;s_2,s_1)=\begin{cases}(1-s_1)(v-cx_2)-s_2x_1 & \text{if } x_2>x_1\\ s_2\left(\dfrac{v}{2}-x_1\right)+(1-s_1)\left(\dfrac{v}{2}-cx_2\right) & \text{if } x_2=x_1\\ -(1-s_1)cx_2+s_2(v-x_1) & \text{if } x_2<x_1\end{cases}. \quad (2)$$

2.2. The equilibrium

Despite the presence of cross-shareholding, the game can be formulated alternatively through simple affine transformation. Denote by $F_i(x_i)$ the distribution of a bidder i's bid in his (mixed) bidding strategy. Bidder 1 chooses his bid x_1 to maximize his expected payoff $W_1(x_1,x_2;s_1,s_2)$

$$\begin{aligned}W_1(x_1,x_2;s_1,s_2)&=(1-s_2)vF_2(x_1)-(1-s_2)x_1-s_1cE(x_2)\\&\quad+s_1(1-F_2(x_1))v=(1-s_1-s_2)vF_2(x_1)-(1-s_2)x_1\\&\quad-s_1cE(x_2)+s_1v.\end{aligned}$$

Similarly, we obtain

$$W_2(x_2,x_1;s_2,s_1)=(1-s_1-s_2)vF_1(x_2)-(1-s_1)cx_2-s_2E(x_1)+s_2v.$$

The game is strategically equivalent to a standard two-bidder all-pay auction (Hillman and Riley, 1989), where bidders, respectively, maximize expected payoffs

$$\tilde{W}_1(x_1,x_2)=F_2(x_1)v_1-x_1, \text{ and } \tilde{W}_2(x_2,x_1)=F_1(x_2)v_2-x_2,$$

with *virtual valuations* of $v_1=\frac{(1-s_1-s_2)}{1-s_2}v$, and $v_2=\frac{(1-s_1-s_2)}{(1-s_1)c}$.[7]

The equilibrium of the game is subsequently analyzed in two cases: Case I, with $c(1-s_1)\geq(1-s_2)$ and Case II, with $c(1-s_1)\leq(1-s_2)$. Note that $v_1\geq v_2$ if and only if $c(1-s_1)\geq(1-s_2)$. In Case I, the game is equivalent to a standard contest where bidder values the prize more than his rival, i.e. $v_1\geq v_2$. By way of contrast, the game in Case II is equivalent to a standard contest where bidder 2 values the prize more, i.e. $v_1\leq v_2$. When $c(1-s_1)=(1-s_2)$, the two cases merge, and a symmetric contest results.

The canonical analysis of standard two-player all-pay auction, as suggested by Hillman and Riley (1989) and Baye et al. (1996), can then be exercised in this alternative context. The following results obtain immediately.

[6] Besides these studies, other authors have examined the impact of auction designs with various sources of externalities. These include the work of Linster (1993), Jehiel et al. (1996), Fehr and Schmidt (1999), Baye et al. (forthcoming), Goeree et al. (2005), Maasland and Onderstal (2007), Engers and McManus (2007), and Bos (2008), etc.

[7] We thank an anonymous referee for alerting us of the handy approach to solving for the equilibrium of the game.

Theorem 1.

(a) *In Case I* $\left(\frac{c(1-s_1)}{1-s_2}\geq 1\right)$, *a unique mixed strategy equilibrium exists. Bidder 1 randomizes his bid over the entire interval* $\left[0, \frac{1-s_1-s_2}{c(1-s_1)}v\right]$ *with c.d.f.* $F_1(x_1) = \frac{c(1-s_1)x_1}{(1-s_1-s_2)v}$, *while bidder 2 randomizes his bid over the interval* $\left[0, \frac{1-s_1-s_2}{c(1-s_1)}v\right]$ *with c.d.f.* $F_2(x_2) = \left[1-\frac{1-s_2}{c(1-s_1)}\right] + \frac{(1-s_2)x_2}{(1-s_1-s_2)v}$ *and bids zero with a positive probability* $F_2(0) = \left[1-\frac{1-s_2}{c(1-s_1)}\right]$.

(b) *In Case II* $\left(\frac{c(1-s_1)}{1-s_2}<1\right)$, *a unique mixed strategy equilibrium exists. Bidder 1 randomizes over the interval* $\left[0, \frac{1-s_1-s_2}{1-s_2}v\right]$ *with c.d.f.* $F_1(x_1) = \left[1-\frac{c(1-s_1)}{1-s_2}\right] + \frac{c(1-s_1)}{(1-s_1-s_2)v}x_1$ *and bids zero with a positive probability* $F_1(0) = \left[1-\frac{c(1-s_1)}{1-s_2}\right]$. *Bidder 2 randomizes over the interval* $\left[0, \frac{1-s_1-s_2}{1-s_2}v\right]$ *with c.d.f* $F_2(x_2) = \frac{(1-s_2)x_2}{(1-s_1-s_2)v}$.

For brevity, we do not provide detailed derivation for the results, as it directly follows Hillman and Riley (1989) and Baye et al. (1996).

The balance of the competition depends on not only bidders' cost differentials, but also the prevailing cross-ownership structure. In Case I, bidder 1 has an upper hand and behaves more competitively than bidder 2. By contrast, the balance of the contest is reversed in Case II. The excessively large share held by the ex ante more efficient bidder 1 erodes his incentive to win, which allows bidder 2 to prevail. The asymmetry between bidders is further witnessed by the following result.

Corollary 1. *The ratio of equilibrium bidding costs is equal to the inverse of the ratio of self-owned shares*, i.e. $\frac{E(x_1)}{cE(x_2)} = \frac{1-s_1}{1-s_2}$, *Similarly, the ratio of equilibrium effort is given by* $\frac{E(x_1)}{E(x_2)} = \frac{c(1-s_1)}{1-s_2}$, i.e. $E(x_1)\geq E(x_2)$ *if and only if* $c(1-s_1)\geq 1-s_2$.

The affine transformation also allows us to handily obtain bidders' ultimate payoffs in the contest by directly referring to Hillman and Riley (1989) and Baye et al. (1996).

Corollary 2.

(a) *In Case I* $\left(\frac{c(1-s_1)}{1-s_2}\geq 1\right)$, *bidder 1 receives an expected payoff* $(1-s_2)\cdot\left\{1-\frac{(1-s_1-s_2)}{c(1-s_1)}\cdot\frac{2(1-s_1)+s_1}{2(1-s_1)}\right\}v$, *while bidder 2 receives an expected payoff* $s_2\left[\frac{2c(1-s_1)-(1-s_1-s_2)}{2c(1-s_1)}\right]v$.

(b) *In Case II* $\left(\frac{c(1-s_1)}{1-s_2}<1\right)$, *bidder 1 receives an expected payoff* $s_1\left[\frac{2(1-s_2)-c(1-s_1-s_2)}{2(1-s_2)}\right]v$, *and bidder 2 receives an expected payoff* $(1-s_1)\left\{1-\frac{c(1-s_1-s_2)}{(1-s_2)}\cdot\frac{2(1-s_2)+s_2}{2(1-s_2)}\right\}v$.

3. Cross-shareholdings and effort supply

This section analyzes how the structural elements of this contest, i.e., s_1, s_2 and c, affect the resulting effort supply. The expected overall effort in a contest with cross-shareholdings is denoted by $X\equiv X(s_1,s_2,c)$. Further adapt the standard results of Hillman and Riley (1989), and we obtain

$$X = \begin{cases} \frac{(1-s_1-s_2)}{2c(1-s_1)}\left[1+\frac{1-s_2}{c(1-s_1)}\right]v & \text{if} \quad c(1-s_1)\geq(1-s_2)(\text{CaseI}); \\ \frac{(1-s_1-s_2)}{2(1-s_2)}\left[1+\frac{c(1-s_1)}{1-s_2}\right]v & \text{if} \quad c(1-s_1)\leq(1-s_2)(\text{CaseII}). \end{cases} \quad (3)$$

When $s_1=s_2=0$, i.e., when cross-shareholdings are absent, the contest elicits an expected total effort $X_0 = \frac{1}{2c}\left(1+\frac{1}{c}\right)v$.

3.1. Overview

A comparison of X with X_0 is to be conducted later in this section. Some key properties of the effort function X with respect to the main structural parameters are first discussed, which provides useful intuitions and insights for our formal analysis.

In Case I, the impact of s_1 on X is indefinite and can be nonmonotonic. Evaluating X with respect to s_1 yields

$$\frac{\partial X}{\partial s_1} = \frac{[(1-s_2)-cs_2](1-s_1)-2s_2(1-s_2)}{2c^2(1-s_1)^3}v. \quad (4)$$

It can be seen from Eq. (4) that X increases with s_1, i.e., $\frac{\partial X}{\partial s_1}>0$, if and only if $s_1<1-\frac{2s_2(1-s_2)}{[(1-s_2)-cs_2]}$ and $(1-s_2)-cs_2>0$. Various competing effects come into play when s_1 increases. First, a greater s_1 reduces bidder 1's additional payoff from winning the contest, which tends to weaken his incentive to supply effort. Second, it decreases bidder 2's bidding cost, which encourages him to step up effort. Third, a "weaker" bidder 1 balances the playing field in the competition. This leveling effect further incentivizes bidder 2. These positive effects, as will be formally demonstrated, may more than offset the negative direct effect.

However, s_2 plays an unambiguously negative role, as X strictly decreases with s_2. A greater s_2 not only (directly) weakens bidder 2's incentive to supply effort, but also offsets the positive leveling effect exercised by s_1 (by further upsetting the balance in the competition). Hence, any positive indirect effect of s_1 must be tempered by s_2. To see it more formally, note that an increase in s_2 always drives down $\frac{\partial X}{\partial s_1}$, because $\frac{\partial^2 X}{\partial s_1 \partial s_2} = \frac{-(1+c)(1-s_1)+(4s_2-2)}{2c^2(1-s_1)^3}v<0$.

When s_1 exceeds the cutoff $1-\frac{1-s_2}{c}$, Case II arises. Bidder 2 appears to be the more competitive contender because bidder 1 holds an excessively large stake in his rival. The roles played by s_1 and s_2 are alternated: Obviously, X would strictly decrease with s_1, but the size of s_2 impacts the overall effort indefinitely.

3.2. Effort comparison

The above discussion briefly depicts the ramifications of cross-ownership structure on effort supply in the contest. We now formally compare X with X_0. Before the comprehensive analysis is carried out, the following important fact deserves to be noted.

Proposition 1. *When bidders are symmetric in terms of their bidding costs, cross-shareholdings always lead to less total effort*, i.e., $X<X_0$ *if* $max(s_1,s_2)>0$.

Proposition 1 simply states that cross-shareholdings always reduce effort supply when bidders are symmetric, regardless of the prevailing cross-ownership structure. A dedicated proof is unnecessary. It is well known in the literature that a standard all-pay auction with symmetric bidders fully dissipates the rent, i.e., total effort $X_0=v$. However, rent is never fully dissipated whenever cross-shareholdings are in place, i.e., $s_1>0$ or $s_2>0$. Cross-shareholdings can only adversely affect the resultant effort supply when bidders are symmetric in their bidding costs. First and foremost, the aforementioned (positive) leveling effect would not loom large in an otherwise even race. Second, the positive effect due to "cost subsidy" is inadequate to offset the negative effect from "prize sharing".

However, when bidders are asymmetric, the cross-ownership structure intertwines with the asymmetric cost structure, and complicates the comparison tremendously. Three factors, s_1, s_2 and c, are involved in determining the value of total effort X. For analytical convenience, we treat the total effort $X\equiv X(s_1;s_2,c)$ as a function of s_1 while treating s_2 and c as parameters throughout the subsequent analysis. The comparison between X and X_0 is systemized by characterizing the function $X(s_1; s_2, c)$ under (all possible) given s_2 and c.

To streamline the presentation, the subsequent analysis is broken into three steps, with each meant to answer a specific question. Together, the three steps provide a complete account of the effort comparison.

3.2.1. Step one

We first answer the following questions: *Would cross-shareholding necessarily lead to lower effort? When would X uniformly fall below X_0*, i.e., $X \equiv X(s_1; s_2, c) \le X_0$, $\forall s_1 \in [0, \frac{1}{2})$?

To proceed, we define two cutoffs for c and s_2, respectively. Let $\bar{c} \equiv \frac{3+\sqrt{17}}{4}$ denote a cutoff for bidder 2's bidding cost c. We further define a cutoff $\bar{s}_2(c) < \frac{1}{1+c} \le \frac{1}{2}$ as follows for s_2[8]:

$$\bar{s}_2(c) \triangleq \begin{cases} \dfrac{(c+3)-\sqrt{(c+3)^2-4}}{4} & \text{if } c \ge \bar{c}, \\ \dfrac{c-1}{(2c-1)(c+1)} & \text{if } c \le \bar{c}. \end{cases}$$

Note that for given s_2 and c, $1-\frac{1-s_2}{c}$ is the cutoff for s_1 that separates Case I and Case II. This brings us to the following result.

Theorem 2.

(a) *For given c and s_2, $X < X_0$ for all $s_1 \in [0, \frac{1}{2})$ if and only if $s_2 \in (\bar{s}_2(c), \frac{1}{2})$.*

(b) *When $s_2 \in [0, \bar{s}_2(c)]$, there exists a unique cutoff $s_1^l(s_2, c) \in \left[0, min\left(1-\frac{1-s_2}{c}, \frac{1}{2}\right)\right]$ such that $X \leqq X_0$ if $s_1 \leqq s_1^l(s_2, c)$ and $X > X_0$ if $s_1 \in \left(s_1^l(s_2, c), min\left(1-\frac{1-s_2}{c}, \frac{1}{2}\right)\right]$. Specifically, $s_1^l(s_2, c) = \frac{1+(1+c)s_2}{2} - \sqrt{\left[\frac{1-(1+c)s_2}{2}\right]^2 - s_2(1-s_2)}$, which leads to that $s_1^l(s_2, c) = 0$ if and only if $s_2 = 0$.*

Proof. See Appendix A. ■

The results of Theorem 2(a) and Theorem 2(b) are illustrated in Figs. 1 and 2, respectively. In Fig. 1, the curve of $X \equiv X(s_1; s_2, c)$ lies completely below that of X_0. Theorem 2(a) unambiguously demonstrates the negative effect exercised by the cross-share owned by bidder 2: whenever s_2 is sufficiently large and exceeds the threshold $\bar{s}_2(c)$, cross-shareholding must lead to a lower total effort ($X < X_0$), regardless of the size of s_1. The negative effects of a large s_2 are twofold. First, a large s_2 reduces bidder 1's share of prize when he wins, which discourages his effort supply. Second, a large s_2 reduces bidder 1's virtual marginal bidding cost and further upset the balance in the contest, which negatively affects the effort supply of both bidders.

Theorem 2(b) states that X can exceed X_0 if s_2 remains moderate (i.e. $s_2 \le \bar{s}_2(c)$) and there is a sufficiently large s_1, i.e., $s_1 > s_1^l(s_2, c)$, that exercises a positive leveling effect and counteracts the negative effect of s_2. As shown in Fig. 2, X_0 and X intersect at the cutoff $s_1^l(s_2, c)$. X surpasses over X_0 when s_1 exceeds the cutoff. It should be noted that when $s_1 = s_1^l(s_2, c)$, Case I still prevails because $s_1^l(s_2, c) \le \min\left(1-\frac{1-s_2}{c}, \frac{1}{2}\right)$. Beyond $s_1^l(s_2, c)$, the curve of X remains above that of X_0 within Case I.

Corollary 3. *The cutoff $s_1^l(s_2, c)$ must strictly increase with s_2.*

Proof. See Appendix A. ■

Corollary 3 further demonstrates the roles played by s_2, which have also been illustrated in Section 3.2.1. When s_2 increases, a greater s_1 is required to exercise the leveling effect and counteract the negative effects of s_2. No feasible s_1 exists to overcome the negative effect of s_2 if s_2 gets excessively large, i.e., $s_2 > \bar{s}_2(c)$.

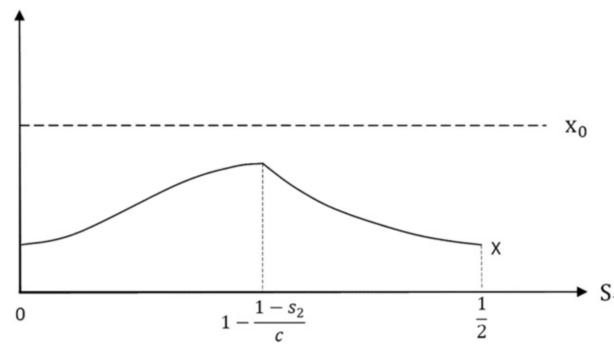

Fig. 1. $X < X_0$ for all $s_1 \in [0, \frac{1}{2})$.

3.2.2. Step two

As illustrated by Fig. 2, X would rise above X_0 to the right of $s_1^l(s_2, c)$ if s_2 remains moderate ($s_2 \le \bar{s}_2(c)$). However, when s_1 continues to increase and eventually exceeds the cutoff $1-\frac{1-s_2}{c}$, Case II emerges. In this case, bidder 1 becomes the weaker bidder. Hence, a greater s_1 not only weakens bidder's incentive to exert effort, but also softens the competition by further upsetting the balance in the contest as a higher s_1 reduces bidder 2's marginal virtual bidding cost. Therefore, X would drop with s_1. The next question we attempt to explore is: *When s_1 continues to increase, would X fall below X_0?* The following is obtained.

Theorem 3. *If and only if $c < \bar{c}$, a cutoff $\hat{s}_2(c) \in [0, \bar{s}_2(c))$ for s_2 exists, which satisfies the following properties: (i) If $s_2 \in (\hat{s}_2(c), \bar{s}_2(c)]$, a unique cutoff $s_1^h(s_2, c) \in \left(1-\frac{1-s_2}{c}, \frac{1}{2}\right)$ for s_1 exists such that the total effort $X > X_0$ for $s_1 \in (s_1^l(s_2, c), s_1^h(s_2, c))$ and $X \le X_0$ for $s_1 \in [s_1^h(s_2, c), \frac{1}{2})$. (ii) If $s_2 \le \hat{s}_2(c)$, such cutoff $s_1^h(s_2, c)$ does not exist, which means $X > X_0$ for any $s_1 \in (s_1^l(s_2, c), \frac{1}{2})$.*

Proof. See Appendix A. ■

We illustrate the result of Theorem 3 in Figs. 3 and 4. For $s_1 > 1-\frac{1-s_2}{c}$ (where Case II applies), X will eventually fall below X_0 if and only if the two conditions are met.

1. Bidders are not excessively asymmetric in terms of bidding costs, i.e., $c < \bar{c}$.
2. Cross-share s_2 is sufficiently large, i.e., $s_2 \in (\hat{s}_2(c), \bar{s}_2(c)]$.

When both of the two conditions are met, X falls below X_0 on the right of the intersection s_1^h, as shown in Fig. 3. When $s_2 \le \bar{s}_2(c)$ but either $c < \bar{c}$ or $s_2 > \hat{s}_2(c)$ is not met, we see in Fig. 4 that X remains

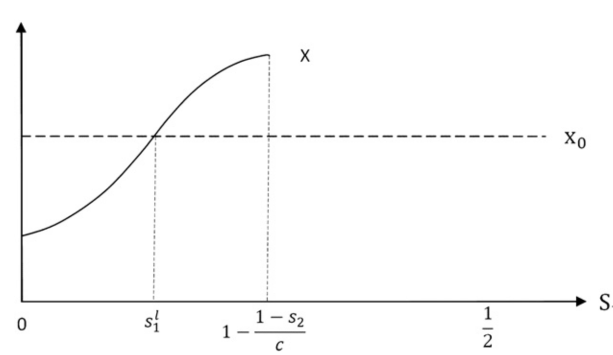

Fig. 2. $X > X_0$ for $s_1 \in (s_1^l(s_2, c), \min(1-\frac{1-s_2}{c}), \frac{1}{2})$.

[8] The reader is referred to the proof of Corollary 3 for the proof of $\bar{s}_2(c) < \frac{1}{1+c}$.

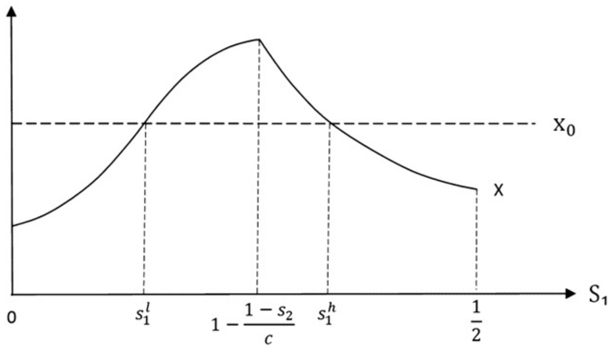

Fig. 3. X drops below X_0 as s_1 continues to increase.

above X_0 for all $s_1 > s_1^l$. It will not fall below X_0 even when s_1 continues to increase and approaches its maximum $\frac{1}{2}$.

Theorem 3 further identifies the role played by s_2. A large share held by bidder 2 ($s_2 > \hat{s}_2(c)$) makes X drop more rapidly. Hence, we observe from Fig. 3 in that case that X eventually drops below X_0 when s_1 continues to increase.

Although the theorem explicitly focuses on the role of s_2, the analysis alludes to the implicit role played by c. The logic underlying our results will further unfold when we examine the impact of c. Two facts deserve to be highlighted.

First, the cutoff $\hat{s}_2(c)$ for s_2 is meaningful if and only if c falls below $\bar{c}$. That is, when c is sufficiently large such that it exceeds $\bar{c}$, X will not fall below X_0. The size of s_2 does not make a difference as long as it remains moderate, i.e. $s_2 \leq \bar{s}_2(c)$. The second fact is stated more formally as the follows.[9]

Proposition 2. *There exists a unique* $\underline{c} \in (1, \bar{c})$ *such that (1)* $\hat{s}_2(c) = 0$ *if* $c \leq \underline{c}$ *and (2)* $\hat{s}_2(c) > 0$ *if* $c \in (\underline{c}, \bar{c})$.

Proposition 2 states that the cutoff $\hat{s}_2(c)$ boils down to zero once c drops below $\underline{c}$. That is, when c is sufficiently small such that it falls below the lower bound $\underline{c}$, any positive s_2 would satisfy $s_2 > \hat{s}_2(c)$. Hence, X must eventually fall below X_0 regardless of s_2.

The implications of the two facts are straightforward: X is more likely to eventually drop below X_0 (the scenario in Fig. 3) when c is smaller. The cost parameter c exercises subtle effects when the contest involves cross-ownership, which deserves to be remarked upon more carefully. The following paradoxical observation is made.

Proposition 3. *In Case I* ($c(1-s_1) \geq (1-s_2)$), *the total expected effort* X *strictly decreases with* c. *In Case II* ($c(1-s_1) \leq (1-s_2)$), *the total expected effort* X *strictly increases with* c.

These results can be immediately seen from Eq. (3), so a dedicated proof is unnecessary. Proposition 3 states that higher bidding cost may surprisingly lead to more intense bidding. In Case I, a greater c unambiguously reduces X. Not only does it directly erode bidder 2's incentive to bid, but it also further upsets the balance in the competition and offsets the positive leveling effect exerted by s_1. However, when $c(1-s_1) \leq (1-s_2)$ (Case II), bidder 1 behaves less competitively, while bidder 2 appears to be more aggressive. An increase in c places a "handicap" on bidder 2, and "restores" the balance between the two bidders. This indirect effect therefore mitigates the negative incentive effect that results from the distorted cross-ownership structure, and it overcomes the direct negative effect of an increasing c.

The interaction of these effects ultimately contributes to Theorem 3 and Proposition 2. When c is excessively small and it

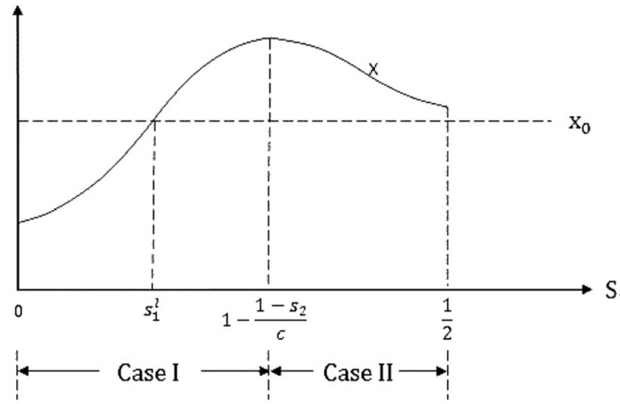

Fig. 4. X does not drop below X_0 when s_1 continues to increase.

drops below $\underline{c}$, its positive effect is limited and it is insufficient to offset the negative effect inflicted by the excessively large s_1 in Case II. We can then derive the reason that X must fall below X_0 for a sufficiently large s_1 if $c < \underline{c}$. By way of contrast, the opposite can be observed when c is sufficiently large ($c \geq \bar{c}$). In this case, a strong "leveling effect" (due to large c) prevails and X never falls below X_0. When its size remains in an intermediate range, i.e. $c \in [\underline{c}, \bar{c}]$, the outcome depends on not only the size of c, but also the size of s_2, as stated by Theorem 3.

3.2.3. Step three

Finally, the results we have obtained thus far point clearly towards the answer to the following question: *When would X consistently stay above* X_0, i.e., $X(s_1; s_2, c) \geq X_0$, $\forall s_1 \in [0, \frac{1}{2})$? By previous results, we conclude the following.

Theorem 4. $X \geq X_0$ *regardless of* s_1 *if and only if* $s_2 = 0$ *and* $c \geq \underline{c}$.

Proof. See Appendix A. ■

The result of Theorem 4 is illustrated in Fig. 5. The curve of X always stays above that of X_0 when the conditions $s_2 = 0$ and $c \geq \underline{c}$ are met. The result in fact directly follows the previous analysis. When the necessary condition of $s_2 = 0$ is met, the negative effect of s_2 would not take place in Case I when s_1 is small (see Theorem 2 and Corollary 3). In addition, it requires nontrivial asymmetry between the two bidders (in terms of bidding cost). Hence, a sufficiently large c would balance the playing field and exercise a leveling effect when Case II applies, and prevents X from dropping too rapidly as s_1 becomes large (see Theorem 3 and Proposition 2).

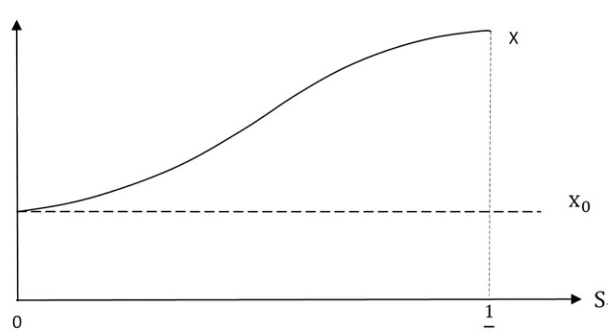

Fig. 5. $X \geq X_0$ for all $s_1 \in [0, \frac{1}{2})$.

[9] This result has been formally shown in the proof of Theorem 3.

Table 1
Effort comparison results.

c	s_2	s_1	Effort comparison
$c\geq 1$;	$s_2\in(\bar{s}_2(c),\frac{1}{2})$;	$s_1\in[0,\frac{1}{2})$;	$X<X_0$.
$c\geq 1$;	$s_2\in[0,\bar{s}_2(c)], 1-\frac{1-s_2}{c}\geq\frac{1}{2}$;	$s_1\in[0,s_1^l(s_2,c)]$;[a]	$X\leq X_0$.
$c\geq 1$;	$s_2\in[0,\bar{s}_2(c)], 1-\frac{1-s_2}{c}\geq\frac{1}{2}$;	$s_1\in(s_1^l(s_2,c),\frac{1}{2})$;	$X\geq X_0$.
$c\geq 1$;	$s_2\in[0,\bar{s}_2(c)], 1-\frac{1-s_2}{c}<\frac{1}{2}$;	$s_1\in\left(s_1^l(s_2,c),1-\frac{1-s_2}{c}\right]$;	$X\geq X_0$.
$c\geq\bar{c}$;	$s_2\in[0,\bar{s}_2(c)), 1-\frac{1-s_2}{c}<\frac{1}{2}$;	$s_1\in\left(1-\frac{1-s_2}{c},\frac{1}{2}\right)$;	$X\geq X_0$.
$c\in(\ c,\bar{c})$;	$s_2\in[0,\hat{s}_2(c)), 1-\frac{1-s_2}{c}<\frac{1}{2}$;	$s_1\in\left(1-\frac{1-s_2}{c},\frac{1}{2}\right)$;	$X\geq X_0$.
$c\in(\ c,\bar{c})$;	$s_2\in[\hat{s}_2(c),\bar{s}_2(c)), 1-\frac{1-s_2}{c}<\frac{1}{2}$;	$s_1\in\left(1-\frac{1-s_2}{c},s_1^h(s_2,c)\right]$;	$X\geq X_0$.
$c\in(\ c,\bar{c})$;	$s_2\in[\hat{s}_2(c),\bar{s}_2(c)), 1-\frac{1-s_2}{c}<\frac{1}{2}$;	$s_1\in(s_1^h(s_2,c),\frac{1}{2})$;	$X<X_0$.
$c\in[1,\ c]$;	$s_2\in[0,\hat{s}_2(c)), 1-\frac{1-s_2}{c}<\frac{1}{2}$;	$s_1\in\left(1-\frac{1-s_2}{c},\frac{1}{2}\right)$.[b]	$X\geq X_0$.

[a] By Theorem 2(b), when $s_2=0$, $s_1^l(s_2,c)$ reduces to zero In this case, $X=X_0$ if and only if either $s_1=0$ or $s_1=s_1^l(s_2,c)$. X drops below X_0 for all $s_1\in(0,s_1^l(s_2,c))$.
[b] By Theorem 3(ii), $s_1^h(s_2,c)$ exceeds $\frac{1}{2}$ in this case.

3.2.4. Summary

Our analysis on equilibrium effort comparison is then completed. All possible cases are summarized in Table 1. Note that in the table, $\hat{s}_2(c)<\bar{s}_2(c)$, $s_1^l(s_2,c)<min\left\{\frac{1}{2},1-\frac{1-s_2}{c}\right\}$, $s_1^h(s_2,c)\in\left(1-\frac{1-s_2}{c},\frac{1}{2}\right)$ and $1<\ c<\bar{c}$.

3.3. Discussion

Our result on effort comparison stems out of the complex interaction among various forces. The main results allow us to go further and explore the contest in greater depth. In a contest with an equity structure $\{s_1, s_2\}$, bidder 1 exerts an expected effort

$$\hat{x}_1\equiv\hat{x}_1(s_1,s_2)=\begin{cases}\dfrac{(1-s_1-s_2)}{2c(1-s_1)}v & \text{if } c(1-s_1)\geq(1-s_2);\\ \dfrac{c(1-s_1)(1-s_1-s_2)}{2(1-s_2)^2}v & \text{if } c(1-s_1)\leq(1-s_2).\end{cases} \quad (5)$$

Further denote by $\hat{x}_1^0\equiv\frac{v}{2c}$ his expected effort in the benchmark contest without cross-shareholdings. Having learned the impact of cross-ownership on overall effort, we now turn to its role in mediating individual bidders' incentives to supply effort. We compare $\hat{x}_1$ with $\hat{x}_1^0$ and obtain the following.

Theorem 5. *Bidder 1 always exerts less effort under a contest with cross-shareholding than he does in the benchmark contest* (*without cross-shareholdings*), *regardless of the prevailing cross-ownership structure*, i.e., $\hat{x}_1\leq\hat{x}_1^0$, *for all* $\{s_1, s_2\}$ *with* $max(s_1,s_2)\in\left(0,\frac{1}{2}\right)$.

Proof. See Appendix A. ■

Theorem 5 states that the efficient bidder 1 always supplies less effort when cross-shareholdings are in place. Despite its technical simplicity, Theorem 5 has rich implications.

First, Theorem 5 thus further illuminates the nature of the "pro-competitive" effect of cross-shareholdings. Despite that cross-shareholding may lead overall effort of the contest to exceed over that in a standard contest, the effort increase must be contributed by the less efficient contestant! This "pro-competitive" effect looms large when the ownership structure weakens bidder 1's advantage and creates a more even race. It therefore better incentivizes bidder 2 and leads him to bid more competitively. Although a more aggressive rival also further incentivizes bidder 1 and tends to prevent him from slacking off, this second-order positive effect cannot offset the first-order negative effect. As a result, cross-shareholdings always disincentivizes the more efficient bidder.

Bidder 2, however, can be either incentivized or disincentivized by the cross-shareholdings, depending on the specific structure of the cross-ownership. It can be easily verified that the inefficient bidder supplies less effort in this scenario than he does in the benchmark contest whenever $s_2>s_1$. This sufficient condition further illustrates the critical force that mediates effort supply in the contest. Specifically, the large share owned by the inefficient bidder further weakens his incentive to win and the cross-ownership structure further unbalances the competition, thus leading the leveling effect to backfire.

Second, by revealing the source of the pro-competitive effect, Theorem 5 provides a more comprehensive view on the pro-competitive effect of cross-shareholdings. The additional competition created by cross-shareholdings must result in a loss of efficiency. A consequence of the increase in total effort involving the cost-efficient firm to reduce effort is that total costs are also increasing.

Our analysis provides a complete account of the effects of cross-shareholding on bidding behaviors in contests. Besides the theoretical contribution, our results also provide interesting and useful practical implications. The pro-competitive effect implies that the cross-shareholding can be practiced as an incentive device by strategically manipulating the ownership structure. Our analysis thus provides a useful guideline to exploit the positive effect.

For instance, consider a business group or network organization, e.g. Roche. It solicits an innovative technology by posting contingent rewards to its R&D subsidiaries. Major pharmaceutical companies, for instance, have conventionally sponsored such contests within their networks to facilitate in-house R&D. The equity linkage within a network could either promote or mitigate the competition. To make the most out of its budget, the parent organization could strategically vary the down-stream ownership structures of the affiliated research entities to incentivize R&D effort.

As aforementioned, the central government of China requires its major state-owned enterprises to cross-own the shares of one another. This strategy, in an attempt to establish internal mechanism for checks-and-balances, has long been accused of reducing competition. Our results, nevertheless, implies that the reform on equity structure may not have to sacrifice market competition, if the cross-ownership structure is properly arranged.

Despite that our analysis is conducted in a stylized setting, the results provide a qualitatively useful guideline for exploiting the pro-competitive effect. First, our results point out that it does not pay off to let a weaker firm cross-own its stronger rival. Such an arrangement handicaps the weaker firm and upset the balance of the competition even further, which disincentivizes the bidders. Second, we show that the parent organization should not seek the pro-competitive effect through cross-ownership arrangement when the competing subsidiaries have a close match. Cross-ownership creates more competition only if it is properly structured to balance the play field. This effect would be more pronounced when the asymmetry is more severe between firms in terms of their competence. Theorem 4 clearly illustrates these arguments: with nontrivial ability differential ($c\geq\ c$), effort supply can be improved when the stronger bidder owns a share in the other, regardless of the size of its share! Conversely, as witnessed by Proposition 1, when contestants are symmetric, cross-shareholdings always jeopardize the efficiency of the competition

4. Concluding remarks

This paper investigates the competitive effect of cross-shareholdings in a winner-take-all all-pay auction with complete information. A complete account of bidding behaviors has been provided in a setting where two bidders own passive equity stakes in each other. The analysis yields interesting predictions, which contrast sharply with those obtained in standard quantity or price competition settings. When bidders are asymmetric in their bidding abilities, cross-shareholdings mediate their competitive incentives in a complex way. Cross-ownership may turn out to exercise a pro-competitive effect and stimulate effort supply. Both positive and negative competition effects may loom large, depending on the particular scenario.

However, cross-ownership can only better motivate the inefficient bidder, while it must disincentivize the efficient bidder. Its pro-competitive effect sacrifices efficiency, and comes at an increasing overall bidding cost.

One natural extension to our analysis is to allow more than two bidders to participate in the competition. Konrad (2006) considers a setting with N asymmetric bidders, with one bidder owning a cross-share in one of the others. He demonstrates in particular a case in which the second strongest bidder owns a sufficiently large cross-share only in the third strongest bidder. In this case, a standard equilibrium with only two active bidders no longer exists and the elegant approach of Hillman and Riley (1989), and Baye et al. (1996) no longer applies. The solution to this game demands an innovative approach which has yet to be developed in the literature, and the technical complexity of the task is exacerbated further if multiple bidders are allowed to own asymmetric stakes in each other. The lack of handy approaches to solving this problem limits our study to the two-player setting.

The problem poses a difficult but important challenge to future researchers in this area. The existing literature, however, has remained thin in this regard. Siegel (2009) studies separable all-pay contests[10] that capture a wide variety of asymmetries that could stem from valuations, bidding costs and sunk investments. He provides a closed-form formula for players' equilibrium payoffs and analyzes players' participation while staying away from efforts to solve for equilibrium bidding strategies. However, an all-pay auction with cross-shareholdings, in general, does not fall into this category of separable all-pay contests. While we believe that most of the insights derived from the current setup would continue to apply to more general settings, we aim to pursue this technically challenging extension in future efforts.

We explicitly assume that neither of the two bidders holds a majority share in rival firms, i.e., $s_1, s_2 < \frac{1}{2}$. It should be noted that our equilibrium analysis does not depend on this restriction. The restriction simply serves to maintain a practically plausible competitive setting: A firm's bidding decision is made by the majority shareholder, and the two firms are controlled by rival bidders. Theorem 1 continues to hold even if $s_i > \frac{1}{2}$, as long as bidder j makes bidding decision on behalf of firm j. Some of the main results on effort comparison would remain robust if the restriction is relaxed. Consider, for instance, Theorem 2. It establishes that the benchmark contest always elicits more effort than a contest with cross-shareholdings if s_2 exceeds the cutoff $\bar{s}_2(c)$. Because $\bar{s}_2(c) < \frac{1}{2}$, the benchmark contest elicits more effort than a contest with cross-shareholdings for all $s_1 < \frac{1}{2}$ even if s_2 may exceed $\frac{1}{2}$. However, X may not monotonically decrease with s_1 when s_1 exceeds $\frac{1}{2}$, as implied by proof of Theorem 2. Additional complications can then be caused and a more definitive conclusion demands a substantially more sophisticated analysis. This extension goes beyond the scope of the current study, but it will be attempted by the authors in future effort.

Another interesting extension is to investigate firms' incentives to enter cross-shareholding networks. This issue is important, while it goes beyond the scope of the current study. Equity linkage yields various strategic benefits, and firms are often subject to multi-faceted objectives when they engage in equity-swap transactions. For instance, the complex cross-shareholding networks created by Japanese conglomerates and their European counterparts have long been viewed as devices with which to protect member firms from hostile takeover.[11] At the same time, firms may be facing other concerns such as knowledge transfer (the Russian oil giant Rosneft's recent deal with BP and ExxonMobil), market entry (Roche's entry into Japan through Chugai), and the coordination of market competition, when forming cross-shareholding relations. A serious study of the endogenous formation of equity-linked networks demands a substantially richer framework than the existing one allows if it were to include the wide array of strategic concerns. The modeling choice adopted in such an endeavor would have to be sensitive to the particular contexts of interest. It will be attempted by the authors in a future effort.

Appendix A

A.1. Proof of Theorem 2

Our proofs require the following Lemma A0, which will be useful for subsequent proofs. Its proof is relegated to Appendix B.

Lemma A0. *$X \leqq X_0$ for $s_1 = 0$ if and only if $s_2 \geqq 0$.*

We now continue with the proof of Theorem 2.

Proof of Theorem 2. Define $\Phi(s_1; s_2, c) = \frac{(1-s_1-s_2)}{2c(1-s_1)}\left[1 + \frac{1-s_2}{c(1-s_1)}\right]$, which is the coefficient of v in X for Case *I*. We first consider Case *I* and compare the value of $\Phi(s_1; s_2, c)$ with $\frac{1}{2c}\left(1 + \frac{1}{c}\right)$, which is the coefficient of v in X_0. We have

$$\Phi(s_1; s_2, c) - \frac{1}{2c}\left(1 + \frac{1}{c}\right) = \frac{1}{2c^2(1-s_1)^2}\left\{-(1-s_1)^2 + [1-(1+c)s_2](1-s_1) - s_2(1-s_2)\right\}.$$

Note that when $1-(1+c)s_2 \leq 0$, i.e., $c \geq \frac{1-s_2}{s_2}$ or $s_2 \geq \frac{1}{1+c}$, we must have the above difference is negative definite for all eligible s_1. Clearly, in this case $X \leq X_0$ for all eligible s_1.

The quadratic term in the bracket $-(1-s_1)^2 + [1-(1+c)s_2](1-s_1) - s_2(1-s_2)$ equals 0 if and only if $s_1 = \frac{1+(1+c)s_2}{2} \pm \sqrt{\left[\frac{(1-(1+c)s_2}{2}\right]^2 - s_2(1-s_2)}$. We define $s_1^l(s_2, c) \equiv \frac{1+(1+c)s_2}{2} - \sqrt{\left[\frac{(1-(1+c)s_2}{2}\right]^2 - s_2(1-s_2)}$ and $s_1^h(s_2, c) \equiv \frac{1+(1+c)s_2}{2} + \sqrt{\left[\frac{(1-(1+c)s_2}{2}\right]^2 - s_2(1-s_2)}$. Note that $s_1^l(s_2, c) > 0$ and $s_1^h(s_2, c) \geq \frac{1}{2}$ if they are real roots. Thus, by Lemma A0, we conclude $\Phi(s_1; s_2, c) - \frac{1}{2c}\left(1 + \frac{1}{c}\right) > 0$ if and only if $s_1 \in \left[s_1^l, \frac{1}{2}\right]$, $\left[\frac{(1-(1+c)s_2}{2}\right]^2 \geq s_2(1-s_2)$ and $1-(1+c)s_2 \geq 0$. When $\left[\frac{(1-(1+c)s_2}{2}\right]^2 - s_2(1-s_2) < 0$, the equation we considered has no real roots. This is because the LHS is negative definite.

Hence, we conclude that (1) $X < X_0$ for all $s_1 \in \left[0, \frac{1}{2}\right)$ if and only if $X < X_0$ for $s_1 = min\left(1 - \frac{1-s_2}{c}, \frac{1}{2}\right)$[12]; (2) $X > X_0$ for all s_1 such that $s_1^l(s_2, c) < s_1 \leq min\left(1 - \frac{1-s_2}{c}, \frac{1}{2}\right)$, where $1 - \frac{1-s_2}{c}$ is the cutoff for s_1 separating Case *I* and Case *II*.

We further examine the properties of the two roots, $s_1^l(s_2, c)$ and $s_1^h(s_2, c)$. $s_1^l(s_2, c)$ is apparently positive if it exists. Because $X < X_0$ when $s_1 = s_2$, we also obtain $s_1^l(s_2, c) > s_2$. In addition, it is straightforward to verify $s_1^h(s_2, c) > \frac{1}{2}$ if it exists.

[10] The reader is referred to Siegel (2009) for a more thorough definition of separable contests.

[11] Source: Japan's Cross-Shareholding Legacy: the Financial Impact of Banks, by Country Analysis Unit, Federal Reserve Bank of San Francisco, *Asia Focus*, August, 2009.

[12] Note that if $X < X_0$ for $s_1 = 1 - \frac{1-s_2}{c} \left(< \frac{1}{2}\right)$, then X will continue staying below X_0 when s_1 increases up to $\frac{1}{2}$, because X decreases with s_1 in Case II.

We now look at the cutoff between Case I and Case II, where $c(1-s_1)=(1-s_2)$ holds. We have $X=\frac{(1-s_1-s_2)}{2(1-s_2)}\left[1+\frac{c(1-s_1)}{1-s_2}\right]v$ for Case II. Then the total effort X strictly decreases with s_1 for $s_1 \geq 1-\frac{1-s_2}{c}$. Thus, to see whether X consistently falls below X_0, it suffices to compare $s_1^l(s_2,c)$ with $min\left(1-\frac{1-s_2}{c},\frac{1}{2}\right)$. Alternatively, it suffices to compare X with X_0 at the cutoff $s_1=1-\frac{1-s_2}{c}$ when $c<2(1-s_2)$, or at $s_1=\frac{1}{2}$ when $c\geq 2(1-s_2)$. Thus, we conduct the subsequent analysis in two possible scenarios.

Scenario 1. $c\geq 2(1-s_2)$.

In this setting, only Case I would apply. We compare X with X_0 for $s_1=\frac{1}{2}$. We have $X=\frac{\frac{1}{2}-s_2}{\frac{1}{2}2c}\left[1+\frac{2(1-s_2)}{c}\right]v=\frac{(1-2s_2)[(c+2)-2s_2]}{2c^2}v$. We only need to compare $(1-2s_2)[(c+2)-2s_2]$ with $1+c$, which yields

$$\begin{aligned}&(1-2s_2)[(c+2)-2s_2]-(1+c)\\&\quad=4s_2^2-2(c+3)s_2+1.\end{aligned}$$

Thus, we have $X<X_0$ when $s_1=\frac{1}{2}$ if and only if $s_2\in\left(\frac{(c+3)-\sqrt{(c+3)^2-4}}{4},\frac{(c+3)+\sqrt{(c+3)^2-4}}{4}\right)$. Because $\frac{(c+3)+\sqrt{(c+3)^2-4}}{4}>\frac{1}{2}$, the condition is equivalent to $s_2>\frac{(c+3)-\sqrt{(c+3)^2-4}}{4}$. Note that $\frac{(c+3)-\sqrt{(c+3)^2-4}}{4}$ is smaller than $\frac{1}{2}$.

Scenario 1 also requires $s_2\geq 1-\frac{c}{2}$. Which lower bound applies? Compare $\Delta=\frac{(c+3)-\sqrt{(c+3)^2-4}}{4}-(1-\frac{c}{2})$, and we obtain $\Delta=\frac{(3c-1)-\sqrt{(c+3)^2-4}}{4}$ 0 if and only if c $\frac{3+\sqrt{17}}{4}$. Hence, we have $X<X_0$ regardless of s_1 if (1) $s_2\in\left(\frac{(c+3)-\sqrt{(c+3)^2-4}}{4},\frac{1}{2}\right)$ when $c\geq\frac{3+\sqrt{17}}{4}$ or (2) $s_2\in\left[1-\frac{c}{2},\frac{1}{2}\right)$ when $c\leq\frac{3+\sqrt{17}}{4}$.

Scenario 2. $c<2\,(1-s_2)$.

In this scenario, we have $c\leq 2$. In this case, for any $s_1\in\left[0,\frac{1}{2}\right)$, there exists a unique $s_2\in\left[0,\frac{1}{2}\right)$ such that $s_1=1-\frac{1-s_2}{c}$. At the cutoff, we have

$$\begin{aligned}X&=\frac{(1-s_1-s_2)}{2c(1-s_1)}\left[1+\frac{1-s_2}{c(1-s_1)}\right]v\\&=2\cdot\frac{1-s_2-\left(1-\frac{1-s_2}{c}\right)}{2c\cdot\frac{1-s_2}{c}}v\\&=2\cdot\frac{(c+1)(1-s_2)]-c}{2c(1-s_2)}v.\end{aligned}$$

Thus, we only need to compare $\frac{2\cdot(c+1)(1-s_2)]-c}{(1-s_2)}$ with $\frac{1+c}{c}$. We obtain $\frac{2\cdot(c+1)(1-s_2)]-c}{(1-s_2)<\frac{1+c}{c}}$ if and only if $s_2>\frac{c-1}{(2c-1)(c+1)}$. However, Scenario 2 requires $s_2<1-\frac{c}{2}$. Note that $\frac{c-1}{(2c-1)(c+1)}$ strictly increases with c because $\frac{d\frac{c-1}{(2c-1)(c+1)}}{dc}=\frac{(2c-1)(c+1)-(c-1)(4c+1)}{(2c-1)^2(c+1)^2}=\frac{2c(2-c)}{(2c-1)^2(c+1)^2}>0$, while $1-\frac{c}{2}$ strictly decreases with c. In addition, because $\frac{c-1}{(2c-1)(c+1)}<1-\frac{c}{2}$ when $c=1$, and $\frac{c-1}{(2c-1)(c+1)}>1-\frac{c}{2}$ when $c=2$, there must exist a unique cutoff $\overline{c}\in(1,2)$ such that $\frac{c-1}{(2c-1)(c+1)}=1-\frac{c}{2}$ when $c=\overline{c}$.

When $c=\frac{3+\sqrt{17}}{4}$, we have $\frac{c-1}{(2c-1)(c+1)}=2\cdot\frac{\sqrt{17}-1}{(\sqrt{17}+1)(\sqrt{17}+7)}$ and $1-\frac{c}{2}=1-\frac{3+\sqrt{17}}{8}=\frac{5-\sqrt{17}}{8}$. At this point, we have $\frac{c-1}{(2c-1)(c+1)}=1-\frac{c}{2}$. To see that, we have $2\cdot\frac{\sqrt{17}-1}{(\sqrt{17}+1)(\sqrt{17}+7)}=\frac{2(\sqrt{17}-1)}{8(\sqrt{17}+3)}$, which is equal to $\frac{5-\sqrt{17}}{8}$, because $\left(\sqrt{17}+3\right)\left(5-\sqrt{17}\right)=15-17+2\sqrt{17}=2\left(\sqrt{17}-1\right)$. Hence, we understand that $\overline{c}=\frac{3+\sqrt{17}}{4}$.

Combine the findings from the two scenarios, and we have Theorem 2 established. ■

A.2. Proof of Corollary 3

Proof. We claim $\overline{s}_2(c)<\frac{1}{1+c}\leq\frac{1}{2}$. When $c\leq\overline{c}$, $\overline{s}_2(c)-\frac{1}{1+c}=\frac{c-1}{(2c-1)(c+1)}-\frac{1}{1+c}=\frac{1}{c+1}\left(\frac{c-1}{2c-1}-1\right)<0$. When $c\geq\overline{c}$, $\frac{(c+3)-\sqrt{(c+3)^2-4}}{4}-\frac{1}{1+c}\propto(c+3)-\sqrt{(c+3)^2-4}-\frac{4}{1+c}\propto\left[(c+3)-\frac{4}{1+c}\right]^2-\left[(c+3)^2-4\right]=(c+3)^2-\frac{8(c+3)}{1+c}+\frac{16}{(1+c)^2}$. Hence, to verify the claim, one only need to prove $\frac{8(c+3)}{1+c}-\frac{16}{(1+c)^2}>4\Leftrightarrow\frac{2(c+3)}{1+c}-\frac{4}{(1+c)^2}>1\Leftrightarrow\frac{2}{1+c}\left(c+3-\frac{2}{1+c}\right)>1$. This is obvious because $c+3-\frac{2}{1+c}\geq c+2$.

The above fact guarantees $1-(1+c)s_2>0$. Hence, it is straightforward to see $s_1^l(s_2,c)$ strictly decreases with s_2. Firstly, $\frac{1+(1+c)s_2}{2}$ increases with s_2. Secondly, $\sqrt{\left[\frac{1-(1+c)s_2}{2}\right]^2-s_2(1-s_2)}$ decreases with s_2: $\left[\frac{1-(1+c)s_2}{2}\right]^2$ apparently decreases with s_2 because $1-(1+c)s_2>0$; while $s_2(1-s_2)$ increases with s_2 because $s_2\leq\overline{s}_2(c)<\frac{1}{1+c}\leq\frac{1}{2}$. ■

A.3. Proof of Theorem 3

The proof requires three intermediate results: Lemmas A1–A3. The proofs of these Lemmas are relegated to Appendix B.

Because X strictly decreases with s_1, we only need to compare X with X_0 when s_1 approaches its limit $\frac{1}{2}$. If $X<X_0$ for $s_1=\frac{1}{2}$, we then conclude that X would drop below X_0 eventually. By Intermediate Value Theorem, there must exist a unique cutoff $\overline{s}_1^h\in\left(1-\frac{1-s_2}{c},\frac{1}{2}\right)$ such that $X=X_0$ when $s_1=\overline{s}_1^h$. X drops below X_0 when s_1 exceeds this threshold.

Case II emerges only if $1-\frac{1-s_2}{c}<\frac{1}{2}$ or $c<2(1-s_2)$, i.e., $s_2<1-\frac{c}{2}$. Note that the defining condition for Case II $c<2(1-s_2)$ implicitly requires $c\leq 2$. Apparently, we do not have to consider the case of $s_2\geq\overline{s}_2(c)$. In that case, X would not exceed X_0 in the first place. We obtain the following result.

Lemma A1. *When $c\in[\overline{c},2]$, the cutoff $\overline{s}_1^h$ does not exist and $X\big|_{s_1=\frac{1}{2}}\geq X_0$, $\forall s_2\in[0,\overline{s}_2(c))$.*

Lemma A1 indicates that as long as c exceeds the threshold $\overline{c}$ and s_2 remains below the cutoff $\overline{s}_2(c)$, X would not fall below X_0 even when s_1 continues to increase. In subsequent analysis, we then focus on the case of $c<\overline{c}$ in which $\overline{s}_2(c)=\frac{c-1}{(2c-1)(c+1)}$, and $\overline{s}_2(c)$ must be strictly less than $1-\frac{c}{2}$.[13] We further have the following:

Lemma A2. *There exists a unique cutoff $\underline{c}\in(1,\overline{c})$ such that if $c\in\left[1,\underline{c}\right)$, $X\big|_{s_1=\frac{1}{2}}<X_0$, $\forall s_2\in[0,\overline{s}_2(c))$.*

Lemma A2 states that when c is excessively small, i.e., when bidders are less asymmetric in terms of bidding efficiency, X must fall below X_0 when s_1 is large. The cutoff $\overline{s}_1^h\in\left(1-\frac{1-s_2}{c},\frac{1}{2}\right)$ must exist.

When $c\in\left(\underline{c},\overline{c}\right)$, the result of the comparison depends on the interaction between c and s_2. As we show in Appendix B (the proof of Lemma A3), for any given $c\in\left(\underline{c},\overline{c}\right)$, there exists a unique threshold

[13] The claim is obvious for $c\leq 1.5$ as $(2c-1)(c+1)\geq 2$. For $c>1.5$, $(2c-1)(c+1)\geq 7.5$. Note that $c-1\leq 7.5(1-\frac{c}{2})$ requires $c\leq 1.789$, which is higher than $\overline{c}$.

$\tilde{s}_2(c)\in\left(0,1-\frac{c}{2}\right)$ for s_2 that solves $X\big|_{s_1=\frac{1}{2}}=\frac{1}{c}\left(1+\frac{1}{c}\right)$. We then have $X\big|_{s_1=\frac{1}{2}}\ \frac{1}{c}\left(1+\frac{1}{c}\right)$ if and only if $s_2\ \tilde{s}_2(c)$.

$\hat{s}_2(c)$ is defined as

$$\hat{s}_2(c)=\begin{cases}0 & \text{if } c\le \underline{c},\\ \tilde{s}_2(c) & \text{if } c\in\left(\underline{c},\overline{c}\right).\end{cases}$$

Lemma A3. *$\tilde{s}_2(c)<\overline{s}_2(c)$ when $c\in\left(\underline{c},\overline{c}\right)$, which further renders $\hat{s}_2(c)<\overline{s}_2(c)$ when $c\in(1,\overline{c})$.*

A.4. Proof of Theorem 4

Proof. $s_2=0$ is necessary. When $s_2>0$, we must have $X=\frac{(1-s_2)}{2c}\left[1+\frac{1-s_2}{c}\right]v<X_0$ for $s_1=0$. We only need to focus on the case of $s_2=0$. It implies $s_1^l=0$. Hence, X must increase for $s_1\in\left[0,1-\frac{1}{c}\right]$, where Case I applies. We then look at the conditions under which X does not drop below X_0 when s_1 exceeds the cutoff $1-\frac{1}{c}$. When $c\ge\overline{c}$, we understand by Lemma A1 that $X\big|_{s_1=\frac{1}{2}}\ge X_0$ must hold. Furthermore, Lemma A2, Proposition 2 and Theorem 3 imply that, when $c\in(\ \underline{c},\overline{c})$, $X\big|_{s_1=\frac{1}{2}}$ would fall below X_0 only if s_2 exceeds a positive cutoff. Given $s_2=0$, this condition never holds. However, when $c<\ \underline{c}$, any $s_2\le\overline{s}_2(c)\left(<\frac{1}{2}\right)$ would make $X\big|_{s_1=\frac{1}{2}}$ drop below X_0. ■

A.5. Proof of Theorem 5

Proof. First consider Case I with $c(1-s_1)\ge(1-s_2)$. $\frac{(1-s_1-s_2)}{2c(1-s_1)}\ v\le\frac{v}{2c}$, because $1-s_1-s_2\le 1-s_1$. Then consider Case II with $c(1-s_1)\le(1-s_2)$. $\hat{x}_1-\hat{x}_1^0=\frac{v}{2c}\left[\frac{c^2(1-s_1)(1-s_1-s_2)}{(1-s_2)^2}-1\right]$. Because $1-s_1-s_2\le 1-s_1$ and $c(1-s_1)\le(1-s_2)$, $\frac{c^2(1-s_1)(1-s_1-s_2)}{(1-s_2)^2}=\frac{c(1-s_1)}{(1-s_2)}\cdot\frac{c(1-s_1-s_2)}{(1-s_2)}\le 1$. ■

Appendix B. Proofs of Lemmas A0–A3

B.1. Proof of Lemma A0

Proof. The result can be seen immediately from Corollary 2(a). When $s_1=0$, any positive s_2 leads to a strictly lower expected total effort as compared to X_0. As aforementioned, a positive s_1 is required to counteract the adverse effect of s_2. When $s_1=0$, the positive indirect effect is missing and no force exists to offset the negative effect exercised by a positive s_2. ■

B.2. Proofs of Lemmas A1–A3

The proofs of these lemmas involve some common setup. We have $X\big|_{s_1=\frac{1}{2}}=\frac{(1-2s_2)}{4(1-s_2)}\left[1+\frac{c}{2(1-s_2)}\right]v$, which is no less than X_0 if and only if $\frac{(1-2s_2)}{2(1-s_2)}\left[1+\frac{c}{2(1-s_2)}\right]\ge\frac{1}{c}\left(1+\frac{1}{c}\right)$. Defining $y\equiv\frac{1}{1-s_2}$, we rewrite the condition as

$$Z(y;c)\triangleq-\frac{c}{4}y^2+\frac{c-1}{2}y+1\ge\frac{1}{c}\left(1+\frac{1}{c}\right),$$

where $y\in\left[1,\frac{2}{c}\right)$ as $s_2\in\left[0,1-\frac{c}{2}\right)$. Note that $Z(y;c)$ strictly decreases with y in this interval because $\frac{dZ}{dy}=-\frac{c}{2}\ y+\frac{c-1}{2}=-\frac{c}{2}\ (y-1)-\frac{1}{2}<0$.

B.3. Proof of Lemma A1

Proof. Because $Z(y;c)$ strictly decreases with y, we must have $Z(y;c)>Z\left(\frac{2}{c}\ ;c\right)=2-\frac{2}{c}$ because $y<\frac{2}{c}$. When $c=\overline{c}=\frac{3+\sqrt{17}}{4}$, we must have $2-\frac{2}{c}=\frac{1}{c}\left(1+\frac{1}{c}\right)$. We have $LHS=2-\frac{2}{\frac{3+\sqrt{17}}{4}}=2\left(1-\frac{4}{3+\sqrt{17}}\right)=\frac{2(\sqrt{17}-1)}{3+\sqrt{17}}$, while $RHS=\frac{4}{3+\sqrt{17}}\cdot\frac{7+\sqrt{17}}{3+\sqrt{17}}=\frac{4}{3+\sqrt{17}}\ \frac{\sqrt{17}-1}{2}=\frac{2(\sqrt{17}-1)}{3+\sqrt{17}}$ as $\left(3+\sqrt{17}\right)\left(\sqrt{17}-1\right)=14+2\sqrt{17}=2\left(7+\sqrt{17}\right)$. Thus $Z(y;\overline{c})\ \ge\ 1\overline{c}\ (1+1\overline{c})$ for $y\in[1,2\overline{c})$.

In addition, $Z(y;c)$ strictly increases with c because $\frac{dZ(y;c)}{dc}=-\frac{1}{4}\ y^2+\frac{1}{2}\ y=-\frac{1}{4}y(y-2)>0$ for any $y\in\left[1,\frac{2}{c}\right)$, while $\frac{1}{c}\left(1+\frac{1}{c}\right)$ decreases with c. Thus, when $c\in(\overline{c},2]$, we must have $Z\left(\frac{2}{c}\ ;c\right)>Z\left(\frac{2}{c}\ ;\overline{c}\right)\ge 1\overline{c}\ (1+1\overline{c}\ )>\frac{1}{c}\left(1+\frac{1}{c}\right)$. Thus, we see that whenever $c\in[\overline{c},2]$, we must have $X\big|_{s_1=\frac{1}{2}}>X_0$, $\forall s_2\le\overline{s}_2(c)\left(<\frac{1}{2}\right)$. ■

B.4. Proof of Lemma A2

Proof. Because $Z(y;c)$ strictly decreases with y, we then have $Z(y;c)\le Z(1;c)=-\frac{c}{4}+\frac{c-1}{2}+1=\frac{2c-2-c}{4}+1=\frac{c+2}{4}$, which increases with c. We compare it with $\frac{1}{c}\left(1+\frac{1}{c}\right)$, which decreases with c. We must have $Z(1;c)\ \frac{1}{c}\left(1+\frac{1}{c}\right)$ if and only if $(c+1)[c(c+1)-5]+1\ 0$. When $c=1$, $Z(1;c)<\frac{1}{c}\left(1+\frac{1}{c}\right)$, but when $c=\overline{c}$, $Z(1;c)<\frac{1}{c}\left(1+\frac{1}{c}\right)$. There exists a unique $\ c\in(1,\overline{c})$, which solves $Z(1;c)=\frac{1}{c}\left(1+\frac{1}{c}\right)$. When $c\in[1,\ \ c)$, we must have $X\big|_{s_1=\frac{1}{2}}<X_0$, $\forall s_2\le\overline{s}_2(c)\left(<\frac{1}{2}\right)$. ■

B.5. Proof of Lemma A3

Proof. For any given $c\in\left(\underline{c},\overline{c}\right)$, $Z(1;c)>\frac{1}{c}\left(1+\frac{1}{c}\right)$ and $Z\left(\frac{2}{c}\ ;c\right)<\frac{1}{c}\left(1+\frac{1}{c}\right)$. There must exist a unique $\tilde{y}$ that solves $Z(y;c)=\frac{1}{c}\left(1+\frac{1}{c}\right)$. Because y strictly increases with s_2, it implies that for any given $c\in\left(\underline{c},\overline{c}\right)$, there exists a unique threshold $\tilde{s}_2(c)\in\left(0,1-\frac{c}{2}\right)$ for s_2 that solves $X\big|_{s_1=\frac{1}{2}}=\frac{1}{c}\left(1+\frac{1}{c}\right)$.

To prove the claim, we only need to prove $Z(y;c)<\frac{1}{c}\left(1+\frac{1}{c}\right)$ when $y=\frac{1}{1-\overline{s}_2(c)}$. When $c<\overline{c}$ and $s_2<\overline{s}_2(c)$, we must have $1-\frac{1-s_2}{c}<\frac{1}{2}$ holds. By the definition of $\overline{s}_2(c)$, we then have $X=X_0$ when $s_1=1-\frac{1-s_2}{c}$ for $s_2=\overline{s}_2(c)$. Because X would strictly decrease with s_1 when s_1 exceeds the cutoff $1-\frac{1-s_2}{c}$, we then have $X<X_0$ when $s_1=\frac{1}{2}$, which thus gives $Z(y;c)<\frac{1}{c}\left(1+\frac{1}{c}\right)$ for $y=\frac{1}{1-\overline{s}_2(c)}$. ■

References

Baye, M.R., Kovenock, D., de Vries, C.G., 1996. The all-pay auction with complete information. Economic Theory 8, 291–305.
Baye, M.R., Kovenock, D., de Vries, C.G., forthcoming. Contests with Rank-Order Spillovers, Economic Theory.
Bos, O., 2008. Charity Auctions for the Happy Few. Working paper.
Chillemi, O., 2005. Cross-owned bidders competing in auctions. Games and Economic Behavior 51, 1–19.
Clark, D., Konrad, K., Riis, C., 2007. The all-pay auction with cross shareholdings. Working paper.
Dasgupta, S., Tsui, K., 2004. Auctions with cross-shareholdings. Economic Theory 24, 163–194.
Dietzenbacher, E., Smid, B., Volkerink, B., 2000. Horizontal integration in the Dutch financial sector. International Journal of Industrial Organization 18, 1223–1242.
Engers, M., McManus, B., 2007. Charity Auctions. International Economic Review 48, 953–994.
Ettinger, D., 2003. Efficiency in auctions with crossholdings. Economics Letters 80, 1–7.
Farrell, J., Shapiro, C., 1990. Horizontal mergers: an equilibrium analysis. American Economic Review 80, 107–126.
Fehr, E., Schmidt, K.M., 1999. A theory of fairness, competition, and cooperation. Quarterly Journal of Economics 114, 817–868.
Ghosh, A., Morita, H., 2010. Partial Equity Ownership and Knowledge Transfer. Working paper.

Gilo, D., Mosse, Y., Spiegel, Y., 2006. Partial cross ownership and tacit collusion. The Rand Journal of Economics 37, 81–99.

Goeree, J.K., Maasland, E., Onderstal, S., Turner, J.L., 2005. How (Not) to raise money. Journal of Political Economy 113, 897–926.

Hillman, A.L., Riley, J.G., 1989. Politically contestable rents and transfers. Economics and Politics 17–39.

Jehiel, P., Moldovanu, B., Stacchetti, E., 1996. How (not) to sell nuclear weapons. American Economic Review 86, 814–929.

Konrad, K., 2006. Silent interest and all-pay auctions. International Journal of Industrial Organization 24, 701–713.

Konrad, K., 2009. Strategy and Dynamics in Contests. Oxford University Press.

Linster, B., 1993. A generalized model of rent-seeking behavior. Public Choice 77, 421–435.

Maasland, E., Onderstal, S., 2007. Auctions with financial externalities. Economic Theory 32, 551–574.

O'Brien, D., Salop, S., 1999–2000. Competitive effects of partial ownership: financial interest and corporate control. Antitrust Law Journal 67, 559–614.

Reitman, D., 1994. Partial ownership arrangements and the potential for collusion. The Journal of Industrial Economics 42 (3), 313–322.

Siegel, R., 2009. All-pay contests. Econometrica 77 (1), 71–92.